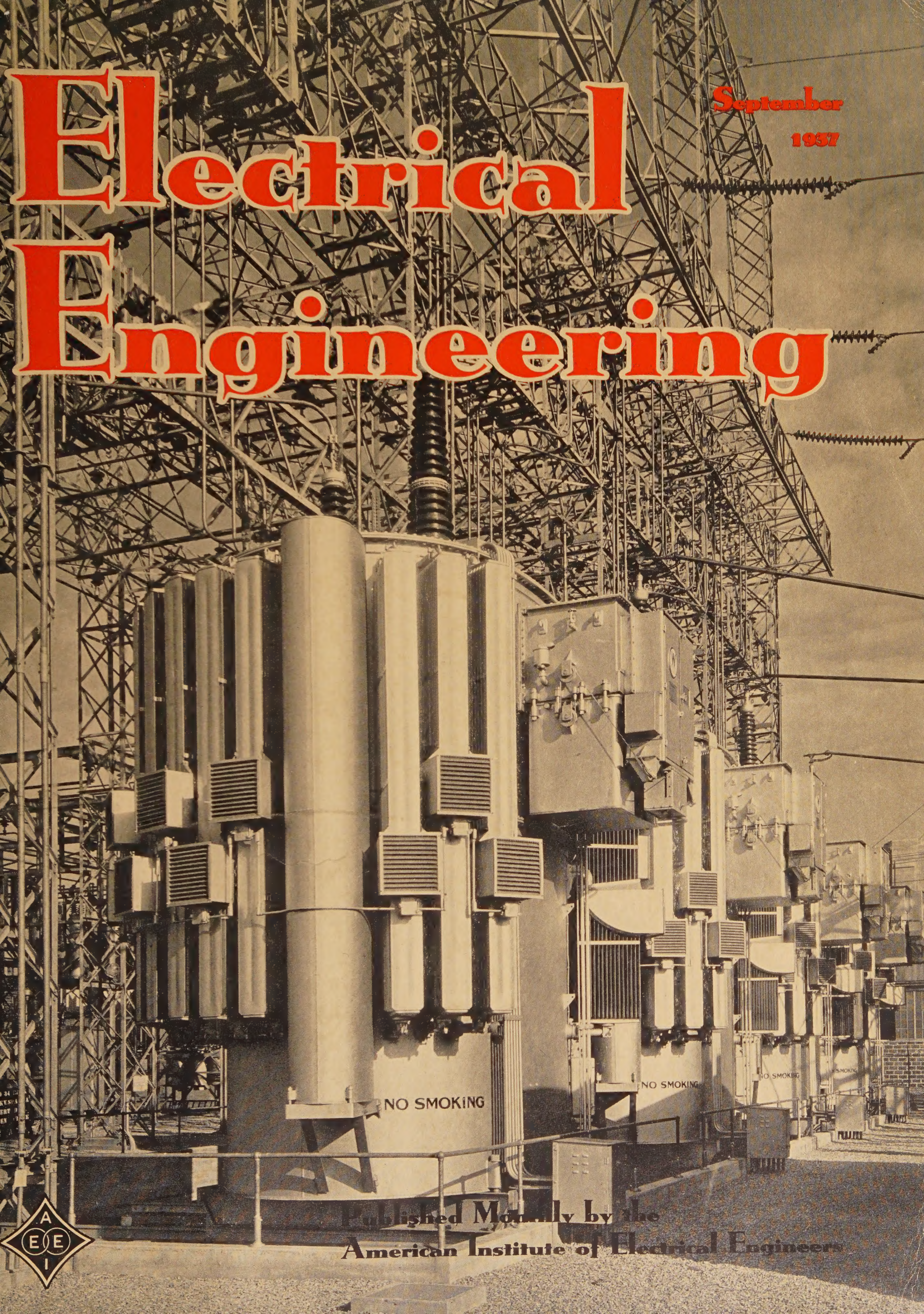
Electrical Engineering

September

1937



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These Companies Know *that FP-19 Reclosers Pay for Themselves*

in less maintenance
in service continuity
in increased revenue

COMPANY A:
One recloser operated 24 times in two months without a lockout.

COMPANY B:
One recloser operated 15 times in 45 days. Finally a lockout caused by a line falling to the ground.

COMPANY C:
A recloser operated 16 times in one night—no lockouts.

COMPANY D:
Three reclosers. One operated 54 times in two months; one 68 times in three months; and one 50 times in three months.

ALL of these reclosers were equipped with operation counters, and it was easy to tell just how many times they had restored service to customers on rural power lines.

Assuming a cost of only $10 to send out an emergency crew to locate trouble and restore service, these reclosers have made a big saving.

No maintenance was required on any of these reclosers. Except in the case of the fallen line (Company B), service was maintained. So, no revenue was lost. These experiences are typical. General Electric Co., Schenectady, New York.

THEY PAY MOST DURING THE LIGHTNING SEASON

890-34

GENERAL ELECTRIC

Electrical Engineering

Registered U. S. Patent Office

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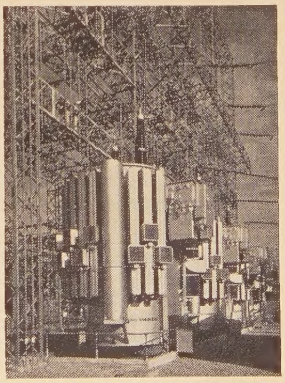
Westport 230/33-kv substation of the Consolidated Gas, Electric Light and Power Company, Baltimore, Md. The transformers are 42,000-kva units



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High Lights

Coupling Between Earth-Return Circuits. Induced voltages recorded in communication circuits paralleling d-c electric railways, at times of short circuit on the railway, have shown marked divergencies from computed values. Because of the numerous factors that might contribute to these divergencies, experimental studies of the transient coupling between parallel earth-return circuits, free from the effects of shielding conductors, and with concentrated rather than distributed grounds, have been made in order to provide a better understanding of the problem (*pages 1159–64*).

American Engineering Council. From its very beginning, in 1920, American Engineering Council has aimed to aid the engineering profession in presenting a united front in matters of interest to government and to the public. A review of some of the problems, policies, and activities of AEC, prepared by the president of Council, appears in this issue (*pages 1082–4*). A report on coal-price fixing and the Bituminous Coal Act of 1937, from a current "news letter" of AEC is included in the news section (*pages 1211–12*).

Electricity in Steel Mills. This issue contains 4 papers on the applications of electricity in the manufacture of steel products. One summarizes the entire range of electrical applications in steel mills (*pages 1115–23*); a second describes a 72-inch strip mill (*pages 1105–14*); another describes more specific uses of electricity in the measurement and control of tension in cold-strip mills (*pages 1141–4*); and a fourth indicates the important characteristics of carbon brushes for steel-mill equipment (*pages 1165–8*).

Limiters. One solution to the problem of providing short-circuit protection for secondary distribution networks consists of the use of "limiters," which are reduced metallic sections incorporated in connectors, lugs, or other connecting devices used in normal cable installation procedure. A large eastern metropolitan power company has installed 16,000 of these devices on its distribution system during the past year (*pages 1191–6*).

Inductances for Ultrahigh Frequencies. With short-wave (or ultrahigh-frequency) radio receivers becoming common, it is important that the inductances for these receivers be properly designed. From the results of experimental and analytical studies of the resistance of small single-layer inductances for frequencies between 4 and 25 megacycles, a procedure has been deduced for the "optimum" design of such inductances (*pages 1169–76*).

Middle Eastern District Meeting. Akron, Ohio, world's largest rubber manufacturing center, will be the scene of a 3-day meeting of the AIEE Middle Eastern District, October 13–15, 1937. A program including 23 technical papers, an excellent variety of inspection trips to local industries, and sports and social events has been arranged. A District student conference also will be held during the meeting (*pages 1205–07*).

Engineers' Income. The earning power of engineers varies widely, according to information obtained in a comprehensive survey of the profession in which incomes for 1929, 1932, and 1934 were reported. Striking differences exist within each professional class, and within each group classified on the basis of educational background (*pages 1089–1104*).

Matrices in Engineering. The matrix not only is "a convenient notation that summarizes in a natural and convenient form whole groups of operations, but . . . it is . . . possible to solve the sets of differential or algebraic equations written in matrix form in a most convenient manner" (*pages 1177–90*).

Resistance Welder Control. Fully automatic repeat control devices are needed to obtain maximum operating speeds in resistance-welding machines without sacrificing the quality of the welds. Operating speeds of 350 or more welds per minute are said to be made practical with a new type of control equipment(*pages 1145–8*).

Selection and Training of Engineers. Many members of the Institute will recall having heard Retiring - President MacCutcheon speak on this subject at their local Section meetings during the past year. In response to many requests, Mr. MacCutcheon has prepared this address for publication (*pages 1085–8*).

Noise Test Code. A proposed test code for apparatus noise measurement has been developed by an AIEE subcommittee. The purpose of this code is to establish uniform methods of conducting and reporting sound-level tests on apparatus. Discussion is invited (*pages 1079–82*).

Silver-Tellurium Alloys. Compensation of temperature errors in electrical instruments may be accomplished by the use of series resistors having negative temperature coefficients of resistance. A method of making such resistors of tellurium and silver has been described (*pages 1128–33*).

Electric Shock. That low-voltage house circuits can give fatal shocks is shown by the death of from 100 to 200 persons annually in the United States from this cause. Preventive measures will reduce the hazard, and resuscitation will save many victims (*pages 1077–9*).

Alternator Short Circuits. Formulas for armature currents in the faulted phases and for the main field current at any time after the short circuit have been developed for a double line-to-neutral short circuit of an alternator (*pages 1149–55*).

Engineering Education. An eminent engineering educator has prescribed a new type of engineering curriculum for undergraduate students, intended to produce engineers interested in assuming places in public leadership (*pages 1073–6*).

Prize Awards. National and some District prize awards for 1936 AIEE technical papers were announced in the June issue; additional awards, for Districts 5 and 8, have now been announced (*page 1208*).

Flywheels for A-C Units. A compendium of the theory of flywheels for a-c generating units and of the parallel operation of such units is included in a paper in this issue (*pages 1158–60*).

The 2-Reaction Theory. Extensions of the 2-reaction theory as applied to multiphase synchronous machines are included in 2 papers in this issue (*pages 1124–7 and 1197–1201*).

Rectifier Regulation. Several factors may affect the "angle of delay" which is used in calculating the regulation of grid-controlled rectifiers, and their effect should be included in the determination (*pages 1134–40*).

Letters to the Editor. Comments on AIEE publications and incandescent-lamp voltages appear in the "letters" columns of this issue (*pages 1212–13*).

DISCUSSIONS

Appearing in this issue are discussions
of the following papers:

Watt-Hour Meter Bearings—Kinnard and Goss **1202**

First Report of Power System Stability—Subcommittee Report **1204**

Statements and opinions given in articles and papers appearing in Electrical Engineering are the expressions of contributors, for which the Institute assumes no responsibility. Correspondence is invited on all controversial matters. ¶ Subscriptions—$12 per year to United States, Mexico, Cuba, Porto Rico, Hawaii, Philippine Islands, Central and South America, Haiti, Spain, Spanish Colonies; $13 to Canada; $14 elsewhere. Single copy $1.50. ¶ Address changes must be received by the fifteenth of the month to be effective with the succeeding issue. Copies undelivered because of incorrect address cannot be replaced without charge. ¶ Electrical Engineering is indexed annually by the Institute, weekly and monthly by *Engineering Index*, and monthly by *Industrial Arts Index;* abstracted monthly by *Science Abstracts* (London). ¶ Copyright 1937 by the American Institute of Electrical Engineers. Number of copies this issue—19,800

Engineering Education and Democracy

By ROBERT E. DOHERTY
FELLOW AIEE

Most engineers are agreed that the engineering profession is not sufficiently active in civic and governmental affairs; that engineers are not sufficiently insistent in voicing their opinions to public officials who administer affairs vital to the profession. With a view to the future, an eminent educator here prescribes one form of engineering education intended to build engineers interested in assuming their places as effective citizens in our democratic government.

PREVIOUSLY I have had the privilege of discussing before the Institute[1] and elsewhere[2,3,4,5] certain problems of engineering education, especially those relating to the undergraduate program. On such occasions I have stressed the importance of a thorough understanding of fundamental science and have expressed concern over the relatively high degree of specialization in undergraduate work at the expense of such understanding; I have contended, moreover, that educators should give to students a start toward that rounded-out education which is so essential to professional activity on a high plane, and which is indispensable in placing engineers in effective leadership alongside other professional men. This latter phase I shall now discuss further, and emphasize as much as possible the critical necessity for prompt attention to this matter. Educational machinery for readjustment moves slowly; the process of bringing young men from freshman year to professional maturity requires from 15 to 20 years; and social and economic change continues at an accelerated rate with little or no intelligent control. Hence time is important.

Increasing complexity confounds us. Technology with all its blessings yet subverts traditional habits and attitudes of mind; narrow specialists multiply endlessly; interdependence of individuals and of communities increases apace; pressure groups spring up on all sides; rackets thrive; selfish interest reigns supreme. Surrounded by all of this, we find ourselves, our hopes for our children, and the great possibilities of American democracy, primarily in the hands of confused lawyers and economists. They are not to be condemned; on the contrary, they are to be given great credit for at least some serious thought regarding the matter. Merely, things have moved too fast for them. But to what extent has the engineering profession taken a hand? It has added fuel to the technological flame that has illuminated and warmed the whole social community with physical comfort and convenience, but apparently it has not occurred to the profession that the flame, though beautiful and interesting, may yet consume us. Engineers, who understand technology, must give more thought to consequences. They, in common with lawyers, economists, and other responsible professional people, must consider a new kind of educational preparation for their oncoming generations, for if these generations grow up with the limited understanding, and with the attitudes that characterize present professional groups, it would seem our hopes are forlorn, because the enemy's strength now is growing faster than ours.

Believing, as I do, that new professional attitudes are essential and that these can be created in sufficient strength only by education of the professional generation yet in college, or just graduated; and believing further, that the accomplishment of adequate changes in educational programs will require, first, the adoption of a significantly modified philosophy by both educators and industrialists; and still further, that to change attitudes in older generations regarding education itself is difficult—believing these things, I have grown to be less optimistic. In any case, the first basis for optimism will be a concerted move toward a readjusted philosophy.

What are the fundamental attitudes from which our thinking should proceed? More specifically, what is *the* fundamental attitude? What others are implied?

Unquestionably we must start from the premise that competent citizenship is the first responsibility, and that the more the individual's educational opportunity is extended, the greater this responsibility. This aspect of educational philosophy, although recognized here and there, is almost wholly overlooked in higher education. Great credit and gratitude are due those educators that preside over primary and secondary education for their attitude in this connection, but the matter must not rest at that point. Competency as a citizen requires more than a preliminary knowledge of the origin, purpose, and form of our government, the early disposition to co-operate and accept majority opinion, and the other attitudes cultivated in the first levels of education. It requires, as well, especially of the better minds that have had the privilege of higher education, an understanding of principles and forms of government, a cultivated ability and attitude to weigh the complex issues that arise; and some feeling of obligation to decide intelligently and give support to

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Essentially full text of an address presented at a meeting of the AIEE Lehigh Valley Section, Bethlehem, Pa., May 14, 1937.

ROBERT E. DOHERTY is president of Carnegie Institute of Technology, Pittsburgh, Pa. Doctor Doherty served as a designing engineer for the General Electric Company, Schenectady, N. Y., from 1910 until 1920, when he became an assistant to Doctor C. P. Steinmetz (A'90, M'91, F'12, and past-president). Upon the death of Doctor Steinmetz in 1923, Doctor Doherty became consulting engineer for the General Electric Company, and served in that capacity until he became professor of electrical engineering at Yale University in 1931. In 1932 he was appointed dean of the school of engineering, and in 1936 was elected president of Carnegie Institute of Technology. He has served on many of the Institute's technical and general committees, and has contributed much to technical literature.

[1] For all numbered references, see list at end of article.

the decision. Our experiment in democracy rests four-square upon the assumption that the citizens themselves are competent to settle issues wisely. My question is: If most of those, including engineers, whose minds are trained to think carefully, are to be directed merely to enterprises that create still more of the social and economic complexities that confound us, who will do the thinking in our community and national life? For, after all, are we not interested quite as much as a nation in where we are going and taking our children, as in the thrill and fascination of the journey itself? If we are not interested as trustees of American freedom, then are we not interested as parents of our children? I can answer the question. If more professional people, who presumably are trained to think, do not take interest in these matters, then the future talent exercised upon the problems of state will be unchanged, and the result is clear. The American experiment is too great, its ramifications too complicated, its growth in complexity too rapid, its values too precious to be guided by emotion and selfish interest. A successful issue of the experiment demands guidance that is based on intelligent, dispassionate study, and this cannot come from a mere handful of men; it must, by the very nature of democracy, come from large numbers. If this nation is to endure as a free nation, we must not forget, as we certainly have done, the fundamental philosophy of 1776 that competent citizenship is our first responsibility, and that this responsibility becomes proportionally greater as the educational privilege is extended.

If this point of view is once again seriously accepted, then new attitudes are implied in higher education and business. Professional education must devote to this end more of the precious time in its programs, and this means either that the programs must be extended, or that industry and business must be willing to accept graduates with training less specialized then they now have. I wish to discuss the problems into which we should thus be led, confining myself to engineering education and to industries that absorb engineering graduates.

What would engineering education do if industry gave it a free hand? The following is what it might do, and what I think it ought to do: The indispensable parts of engineering education, in my opinion, are an understanding of mathematics and the basic physical sciences; a characteristic habit of mind engendered by dealing with the purpose of utility and with definite facts, laws, and predictable outcomes; and thus a combination of these which has been termed "the engineering method." Science alone is not enough; neither is a superficial dabbling with the method without the science. Both are necessary. So the first step is to provide these, or else we shall be beyond the limits of engineering education. But having provided them, we shall have a curriculum that qualifies by definition; and anything further on the technical, scientific side is to be regarded as specialization.

But I am now proposing that we change the definition. It should include as well the requirement of an adequate background and discipline for the new responsibility: that of dealing with problems involving social elements. Both as a citizen and as a professional man active in matters that affect environmental relationships in society, the engineer is under obligation to be prepared for this responsibility, and he should not be regarded as an engineer in the professional sense without it. The civic aspect of this responsibility already has been discussed. I now add the professional, since the educational preparation for the one is also for the other.

Some two-thirds of all engineering graduates eventually have entered work involving personnel management, and perhaps half that number have assumed this reponsibility in a major way. The dark labor situation that recently has sprung up throughout the country indicates the extent to which engineers, even though involuntarily, are rapidly being thrown by the onrushing technological tide into positions where their plans and decisions may have pervasive social implications. No other group is in such a key position as are engineers to make great contributions to industrial peace. By virtue of such a position, which yearly is growing in importance, engineers' professional responsibility is an additional reason for their active interest and competency in dealing with social and economic problems, and in framing an educational program these aspects of responsibility should be recognized as on equal footing with technical responsibility so far as their being essential factors is concerned.

What, then, are the practical implications regarding the curricula? Stated generally, they are the necessity for a redistribution of time and emphasis, and the question of whether industry is willing to assume a share of the responsibility by accepting graduates with broader and more thorough training, but with less specialization.

Before entering these particular matters I wish, in the interest of clarity of discussion, to dispense with certain questions that are closely related. In the first place the thesis of this discussion should not become confused or lost by the mention of the 5-year curriculum. In the view of that portion of past industrial generations that built rapidly and inefficiently, the way to get increased output was merely to extend the plant. With allowances there is a parallel here which might be profitably drawn when the 5-year plan is pressed. Unquestionably some students desire, and can afford the time and money, to spread their education over 5 or 6 years at undergraduate level, and they should be able to do so. Moreover, a relatively small part of them should extend formal education into graduate study in higher scientific levels. These would constitute the supply of professional technical specialists. All of this seems clear. However the great mass of those who study engineering should not be required to take an added year or two out of their professional lives either because we feel, like the past generation of industrialists, that the only way to change is to add, or because certain phases of present industries require at graduation specialized skills that could equally or better be learned on the job. I am convinced that if industry will do its part, graduates can be prepared, along the lines I have indicated, within the 4-year period.

I believe industry will do its part. There are many leaders of industry whose views I do not know, but practically every one of them whose opinion I have read or heard has not urged specialization, but breadth of view. The classical report of the Society for the Promotion of Engineering Education (1923–29) includes composite opinions of industrial leaders and practicing engineers regarding the character of education desired in an engineer; and questionnaires of similar purpose have been analyzed and reported in numerous other publications. One certainly cannot conclude from the well-known results of these inquiries that in general industry demands specialized training at the expense of fundamental breadth and humane understanding. Yet the necessity of retaining existing courses of specialized nature seems to hinge on the idea that a graduate will not be able to get a job if he does not have them. To whatever extent it is true that industry, by insisting upon the teaching of special, routine skills, prevents us from educating our students along broader lines that seem to be so clearly necessary, then to that extent it becomes our responsibility to convince industry that its position is ill-advised. The stakes are too large in this matter to permit a narrow view of a small part of industry to dictate general educational policy. In any case, I set down as the problem before us, a 4-year program that will recognize the educational policy outlined, and yet be a preparation for entering those fields that now employ engineering graduates.

The first critical question is the distribution of time between the technical and social phases, or stems, of the curriculum. This will depend in a limited way upon such factors as the field of activity for which students are preparing. For instance, a course looking primarily to management would demand a larger portion of the curriculum for the social phase than one looking primarily to technical engineering. Moreover, if the students were divided into general course and honors groups, as presumably they will be more and more, the programs of the honors group probably would include more time for the social stem. Naturally, individual judgment as to the division of time also will vary. My own opinion is that the social stem should be accorded from one quarter to perhaps one third or more of the total time, depending upon the factors referred to.

There is an additional specification regarding the time distribution: the social stem should be continuous. The purpose is to build up a habit of mind; to create a desire to become broadly educated; to cultivate a *stem* which will continue to grow after graduation. And I am clear that this cannot be accomplished by discontinuous effort. The program must be continuous and sequential, beginning in the freshman year and extending through the senior year into professional life.

The second critical question is the specific nature of the social and economic side of the program. What subjects should be included? How should they be taught? I have found only one way in which to answer these questions. It is first to state specifically the outcomes desired, and then frame the program so that it will achieve these outcomes. Perhaps the program cannot be defined in traditional terms; merely of subjects to be studied and courses to be taken. The things that cultivate those definite outcomes must be considered, and traditional courses in history, English, economics, government, and sociology, will not accomplish the purpose. In the first place, there would not be enough time for the student to cover these subjects in the traditional way, and secondly, even if he did so, he would not achieve the ends.

What, then, should be the outcomes? I recommend (1) a clear historical understanding of the parallel growths of science and engineering, on the one hand, and social customs, relations, and institutions, on the other, and how these have reacted on each other. In other words, a historical perspective of social evolution, especially since the invention of the machine: (2) The ability to read purposefully in order to make a critical analysis of a problem involving social and economic elements, and arrive at an intelligent opinion about it. The ability to analyze technical problems does not, as sometimes assumed, imply an ability to analyze these others: (3) The ability to organize thoughts logically and with purpose: (4) The capacity to use the English language and thus express those thoughts effectively: (5) Appreciation of English literature: And (6) a continuing interest in all these matters, and thus the desire to become educated.

These outcomes imply the general character of the social stem of the curriculum. They imply extensive and carefully selected reading and seminar or class discussion during the first 2 years, in order to acquire general background and to establish clear conceptions of both the decisive turning points in social development and the attempts that have been made by social scientists to generalize from the recorded experience. This preliminary study must not be too detailed. It must be directed toward establishing an over-all pattern that can be examined as a whole. What is essential is a clear picture of the general features of the highway system by which civilization has reached its present stage—the junction points, the sign posts and traffic signals, the most general character of the main routes, and possibly some of the important detours. The less essential features of the individual highways, of the scenery and towns along the way, and of detailed social customs must be left to future study when more time may be spent in a particular section. I cannot emphasize too strongly the importance I attach to such an integrated view. It must be achieved at all costs, but it does not cost much. There is adequate time in 2 years if only one can restrain the habitual insistence of many textbook authors upon the gorging of details, and of instructors thus to administer them. With such a general picture in mind, subsequent study of problems and subject matter regarding a particular area then becomes both intelligent and interesting.

In the last 2 years only a few special problems should be studied; that is, definite social and economic questions about which the student would be required to read under competent guidance, and come to an intelligent opinion. He should have 3 or 4 of these in the junior year, one or 2 in the senior, each being more difficult than the last, and have the responsibility of defending his solution, presented in an essay before a small seminar—not more than 7 or 8 participating students, each of whom had read the essay. Such a program would not involve many scheduled classes, especially in the last year. There would be seminars only at times when problems were ready for review, and a few other meetings found to be necessary to keep students under way. These latter would occur more frequently at the beginning of the junior year, and almost not at all in the second semester of the senior year. But the transition from frequent meetings to few is necessary. The rest of the time would be devoted to reading, study, and writing.

I wish to stress especially this feature of independent study of a question. It gives actual experience in *doing*. One can read for background, take course after course in this and that, but until one has actually gone through the process of reading around a question and has one's self thus formed an intelligent judgment that must stand the test of critical discussion—until then, one has missed a fundamental first step in achieving a disciplined intellect. And if that educational experience is not acquired in college, it may never be.

May I add one or 2 more comments about this particular feature of the plan? I may be accused of proposing graduate activity in undergraduate work. Perhaps I am. Undergraduate students may seem immature, but I feel that the character of much undergraduate work tends to keep them so. I am convinced that the sooner they can be introduced to the *method*, however simple the problem, of high-grade graduate activity, the more we shall help students achieve intellectual maturity.

Or it may be said I am proposing merely an honors course, after the English model. Perhaps I am. There is this distinction, however, that simplifies our problem: Here the student's reading centers around a specific question, not a whole field. Indeed, he may be led to read in 2 or 3 fields to solve his problem, but his entire activity has a perfectly definite focus, and the character of the problem always can be adjusted to his ability and available time.

Finally, it may be suggested, and with some reason, that such a program would be too expensive and administratively complicated, but it is not as bad as it appears. The first 2 years would be largely class work as to time and schedule, but with emphasis different from the usual. And in the last two, although much of the guidance may be individual in character, the conferences would not be frequent—perhaps biweekly. And those aspects of the work which, in the earliest part of the junior year may seem to require more frequent meetings, are those which can be handled in group meetings. Moreover, I acknowledge that a portion of the class may require group guidance for an extended period, and these should be so guided. Although the program may involve careful planning, the values are too great to forego, especially for those students who can pursue the work with a minimum of guidance.

Returning now to the other features of the program implied in the proposed outcomes, I have mentioned the ability to organize thought and to express it effectively. A rigorous discipline in achieving logical relationships of ideas, and in writing, should be a characteristic of the program throughout the whole 4 years. Through the medium of essays, very simple at first, the student's mind should be kept in constant challenge as to logical thought and statement.

With respect to an appreciation of English literature, I confess I see no better way to foster this than the best type of English courses. The purpose should be to create an interest and desire to read for pleasure; it should not be to cover as much literary ground as possible. By the way, one theory I have heard for covering much ground is that it is the last chance to tinge the poor engineer with a little culture! Thus the requisites I would specify are interesting subject matter, interesting class discussion, interesting essays.

Summarizing, I have thus outlined the nature of the program I would recommend as the social stem of the engineering curriculum. The educational values are as follows: All reading and study would be focused upon a definite, clearly visible objective; counsel of a mature mind; experience in pursuing independent critical study, in discriminating between the important and the trivial, in forming intelligent judgments, and in organizing thought logically and with purpose; the discipline of intelligent criticism by other minds; and the unmatched incentive to fruitful effort by the student of his "coming to bat" before, and in competition with, his associates. And I think I am justified in adding that it will create in many students the desire to continue reading and study after graduation. Experimentation for 4 years with such a program in junior and senior years gives me confidence that the outcomes implied in the proposed educational policy can be achieved reasonably; they certainly can be fully achieved with the better minds.

Is it worth while? Is America worth while? I do not suggest that this particular program is the final answer to the problem. It is still at experimental stage. And there may be many other ways to accomplish the purpose. But I do insist that we cannot escape the responsibility of striving toward the outcomes I have indicated, and insist, further, that the present typical educational programs do not achieve them. And even if adequate programs may cost slightly more, and require sacrifices in the specialized courses on the technical side, I repeat the question, Are they not worth while? For what is to be gained by preparing engineers to continue their building of a physically magnificent structure only to have it torn down ultimately by the uncontrolled, unintelligent forces of selfish interests and vandalism?

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Electrical Accidents

By W. B. KOUWENHOVEN
FELLOW AIEE

Six factors influence the result of an electric shock: type of circuit, voltage, resistance, current, current pathway, and duration of contact. Preventive measures will reduce the hazard, and resuscitation will save many victims, according to information summarized in this article, which is based on reports previously published.

MAN-MADE electric shocks are not modern. The first shock recorded in literature was reported by Galvani in 1780. The first fatal electrical accident occurred in France in 1879. In the United States the present death rate due to electrical accidents is one per 100,000 of the population. Approximately one-half of the electrical accidents reported result in fatalities. Some fatal accidents occur on low-voltage circuits and the house circuits account for the deaths of between 100 and 200 persons annually. With the increased availability of electric power and the new uses that are advocated, there is the possibility that electrical accidents may increase. It is the duty of engineers to acquaint the public with the dangers that do exist, so that undue hazard may be avoided.

Maclachlan[1] has published an excellent study of 479 cases of electric shock for which complete data are available. He showed that men are more commonly injured than women and that the ages of the greatest incidence of shock lie between 15 and 30, perhaps owing to the greater carelessness of youth. Accidents are more common in the summer months, as the result of the smaller protection offered by the clothing and the presence of perspiration, which lowers the skin resistance. Maclachlan found that accidents are more common at the end of the working day when fatigue is at a maximum. He reported that the current path through the body is generally from the right hand to the feet.

Factors Involved

In every electrical accident there are 6 factors which influence the result. These are, first, the type of circuit with which contact is made; second, the voltage of the circuit; third, the resistance offered to the flow of current; fourth, the value of the current; fifth, the current pathway through the body; and sixth, the duration of the contact with the circuit.

TYPE OF CIRCUIT

The type of circuit[2] with which contact is made affects the resulting injury. For example, low-voltage alternating current is more dangerous than the corresponding continuous voltages. On the contrary, d-c circuits above 1,000 volts are more dangerous than a-c circuits of the same voltage. Experimental studies[3] have shown that there is little if any difference between commercial 25- and 60-cycle circuits.

The tetanic contraction of the striated musculature of the body caused by contact with d-c circuits is not as severe as that produced by alternating current. The latter will sometimes throw the victim clear of the circuit. Experiments made with the lightning discharge of an impulse generator on animals showed no evidence of musculature contraction.

VOLTAGE

High-voltage circuits are recognized as dangerous and are carefully avoided by everyone. The public considers the low-voltage house circuits as perfectly safe; this, however, is not the case, as already has been stated. The contraction of the muscles when good contact is made with a low-voltage circuit is usually insufficient to throw the victim clear. He is held in contact with the circuit and death may result. Many of these low-voltage accidents could be avoided if the public appreciated the danger and realized that electricity, although a good servant, is a bad master.

RESISTANCE

Greatest protection against electric shocks, particularly those caused by low-voltage circuits, lies in the skin. The resistance that dry skin offers to the flow of electricity ranges from 70,000 to 100,000 ohms per square centimeter. Its value depends to a certain extent upon the callousness of the skin. Once the current enters the body, the tissues offer only a relatively low resistance to its flow.

When the skin is thoroughly moistened by washing or perspiration, its resistance falls to less than one one-hundredth of its dry value. It is under these conditions that contact with the low-voltage circuit is dangerous. Contact with electric circuits should be made impossible in kitchens, bathrooms, cellars, and industrial locations where moisture is present.

Considerable heat is generated in the epidermis at the points of contact with the circuit, because of its high resistance. If the victim cannot release his hold and contact with the circuit continues for even a short time,[4] the temperature may be raised to a sufficient degree to produce blisters. These break down the skin and destroy completely any protection that it had initially offered to the flow of current.

An address presented at the exercises commemorating the 25th anniversary of the engineering school of The Johns Hopkins University in February 1937.

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[1] For all numbered references, see list at end of paper.

CURRENT

The sensitivity to electric currents varies widely. Some individuals can detect the presence of currents of one milliampere or less, while others require several times that amount in order to be certain that a circuit is alive. In any electrical accident, Ohm's law determines the current that flows, and it is the passage of this current through the body that determines the injury produced. The question naturally arises: What value of current is dangerous? The writer believes, because of the results of laboratory experiments and field data, that an alternating current of one-tenth of an ampere at commercial frequencies is sufficient to cause death provided it flows through the vital organs. This does not mean, however, that a current of 20 amperes will make death more certain. In fact the opposite is more often the case. The reason for this apparently anomalous statement is due to the fact that the flow of a small current through the chest may be sufficient to destroy the normal rhythm of the heart and to throw that organ into ventricular fibrillation. Under these conditions, the circulation of the blood ceases and death ensues as the human heart rarely recovers its regular beat spontaneously. A large current, however, produces a severe contraction of the body musculature including that of the heart and holds that organ at rest. Then when the circuit is broken, provided the duration of the contact is brief, the heart automatically resumes its activity.

CURRENT PATHWAY

The path that the current follows in its passage through the body is of great importance. In general, if no vital organ lies in the pathway, no serious injury results. For example, in experiments with rats when the electrodes were attached to the 2 hind legs, all but 2 of the animals survived a 2-second shock from a given circuit without injury. When, however, the electrodes were shifted to the forelegs all of the animals died. The currents and voltages were the same in both experiments. Once the current is inside the body, it spreads out in a more or less fusiform shape, assuming uniform specific resistance for the tissues. It is difficult to believe that any appreciable current passes through parts of the body that lie far outside of the main current pathway. There are, however, accident cases on record where unconsciousness followed the passage of a current between 2 fingers on the same hand or 2 points on the same leg. In such cases, no current flows through the central nervous system and the action must have been caused by the sudden shock and the resulting vasomotor changes.

The brain appears to be relatively easily injured by the passage of an electric current, and in legal electrocution one electrode is placed on the head. Fortunately, in most industrial accidents the brain does not lie in the current path; however, the heart and respiratory apparatus are often injured. In many electrical accidents the current in passing through the body produces a block in the nervous system. This prevents the normal stimuli from the brain reaching a given organ. If that organ is the breathing mechanism, the victim will die of asphyxia, unless artificial respiration is given promptly.

DURATION OF CONTACT

The duration of the contact with the circuit is extremely important in determining whether an individual will survive or not. There is usually little data available in cases of human accidents as to the length of time that the current flowed through the body. In our first series[5] of experiments on animals in which they were subject to shocks of different voltages, the importance of the time factor was clearly demonstrated. In general, the higher the voltage, the shorter the time during which the circuit may remain closed and there be hope of resuscitation. Fortunately, due to the violent contraction of the muscles when contact is made with high-voltage circuits, the shock is usually of short duration.

Preventive Measures

It is the duty of qualified persons to acquaint the public with the rules for the safe handling of electrical circuits and devices. It is the duty of the designing engineer and architect to reduce the low-voltage hazard by specifying highest grade equipment and so installing it as to make humanly impossible, as far as practicable, contact with a live circuit, especially in damp locations. The floors of such places are usually of concrete or similar electrically semiconducting material. Keeping in mind the fact that in most accidents the current flows from the hands to the feet, the building designer might well consider the question of providing an insulating floor covering. The manufacturer of power-operated hand tools and their accessories should use ample amounts of the best grades of insulation because the best is none too good.

Resuscitation

The problem of the resuscitation of a shock victim divides itself into 2 fields, namely, the treatment of the fibrillating heart and of cases of paralysis of respiration due to nerve shock.

COUNTERSHOCK

The statement has been made that small amounts of current are sufficient to throw the heart into ventricular fibrillation, but that when a large current flows, the heart is held quiet because of the strong musculature contraction. Experiments were carried out in which the heart was first thrown into ventricular fibrillation and then a larger current applied for a short time in order to ascertain if it were possible to arrest the fibrillation and recover normal heart action. These experiments[6] were successful. Ventricular fibrillation was stopped in dogs whose hearts had been left in that state for periods up to 5 minutes by the use of a "countershock" of 6 or 7 amperes at 330 volts applied by means of electrodes located on either side of the animals' chests. In these cases, the hearts have resumed normal operation and the animals have recovered. Ferris and Williams[7] have also experimented with the countershock method of resuscitating the fibrillating heart, and, using larger animals, have duplicated these results.

In the experiments carried on over a period of time at

The Johns Hopkins University, fibrillation has been stopped after having been allowed to continue for periods of more than 5 minutes. Although circulation has been re-established for a time in some of these cases, experimenters have not succeeded in saving the animals.

[**Editor's Note:** The method of countershock has not yet passed beyond a laboratory stage. Immediate application of artificial respiration is the recommended procedure.]

Artificial Respiration

The most common result in electrical accidents is a physiological block or break in conduction in the nervous system controlling the operation of respiration. The victim will be apparently dead; he will not be breathing and his heart beat may be so faint as to make it impossible to detect the pulse. The nerves have been paralyzed by the current and no longer transmit the stimuli from the brain to the lungs. The block will often disappear in time if given a chance. Artificial respiration by the Schaeffer prone-pressure method should be applied at once and be continued either until the victim recovers or until rigor mortis sets in.

Remember that death from electric shock is often only apparent and that lives have been saved by the prolonged application of artificial respiration.

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A Proposed Test Code for Apparatus Noise Measurement

An Advance Report Presented for Purposes of Discussion

PREPARED under the auspices of the subcommittee on sound* of the AIEE standards committee, this preliminary report is intended to provide some basis for standardized technique in the relatively new field of apparatus noise measurement. Because of the newness of apparatus noise investigations, and the importance of the subject, the subcommittee anticipates some possible inadequacies in this report, and solicits criticisms and comments from readers. To facilitate specific discussions, paragraphs are numbered.

1. This test code is intended for use as a guide in the measurement of sound levels and the investigation of the various elements that contribute to the total noise produced.

2. Often, after apparatus is installed, questions arise as to the true causes of the observed sound level, and the best way in which to reduce it. Also, it is often desirable to design apparatus for a given installation to meet a specified sound level. These objectives require knowledge of the sound produced by the apparatus and of the effects on the sound level of reflections and absorption due to the ambient conditions. In this connection, it is necessary to make numerous measurements of sound levels, both in the factory and in the field.

3. The purpose of this code is, therefore, to establish reasonably uniform methods of conducting and reporting such sound level tests, so that wherever or by whomever tests are made, the results will be of value for record and will be truly comparable with tests made at other times and places.

4. It is recognized that additional codes, sponsored by appropriate industry committees, may be desirable in the future for the conduct of sound level tests on specific types of apparatus, such as fans, motors, gears, etc. The present code contains only general recommendations, which will serve as a guide in the development of the technique of sound-level testing and a common foundation for future codes for specific types of apparatus.

Scope

5. This test code contains general instructions for conducting and reporting sound-level tests on apparatus. It is based upon the 1936 American standards $Z24.1$ (Acoustical Terminology), $Z24.2$ (Noise Measurement), and Z-24.3 (Sound-Level Meters).

General Considerations in Apparatus Noise Measurement

6. To appreciate the necessity of a test code for sound measurement, it is well to review a few of the important

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* Personnel of Subcommittee on Sound of AIEE Committee on Standards, 1936–37: P. L. Alger, *chairman;* A. P. Fugill, C. R. Hanna, H. B. Marvin, R. G. McCurdy, and H. M. Turner.

physical phenomena attending the transmission and radiation of sound which have to be considered in formulating the code.

RADIATION OF SOUND

7. The distribution of sound about sources other than a few simple types is usually very complex. The out-of-phase as well as the multifrequency vibrations which exist on the surfaces of large sounding bodies such as transformers, motors, and generators, and other apparatus give rise to sound patterns that defy mathematical analysis. In practice, however, several simplifying assumptions can often be applied to help us understand the nature of sound radiation from apparatus. If the wave length of the emitted sound is large compared with the dimensions of the source the sound will be radiated quite uniformly in all directions. But if the wave length is short compared with the dimensions of the source, important directive effects will appear. This suggests that for large sound sources with complex radiating surfaces, a considerable number of sound measurements at different points about the source should be made and averaged to determine the sound level of the source.

8. Many small sound generators such as fractional horsepower motors, small contactors, electric clocks, etc., are inefficient and nondirective radiators of low-frequency sound, but they can be excellent radiators of sound of higher frequencies. Such sounding bodies can often produce marked increases in sound of lower frequency when their vibrational energy is transmitted to surfaces large enough to be efficient radiators. This principle, along with resonance, is often used to amplify the feeble tone of a tuning fork. A conventional horn-type loud speaker reproducing music outdoors provides an excellent example for illustrating directive radiation. The lower-frequency components of the music can be heard about equally well in all directions. The higher-frequency components, since they are effectively directed, can be heard best along the axis of the speaker.

9. A study of the radiation of single-frequency sounds from generators of simple shape reveals several interesting facts that should be kept in mind when noise measurements are made. If we assume a vibrating disk of dimensions such that the wave length of the emitted sound is short compared to the diameter of the disk, we find that for a range of distances along the axis, dependent upon the wave length and the distance, the intensity of the sound goes through a series of maxima and minima before a distance is reached beyond which the intensity decreases inversely as the square of the distance. If we assume a pulsating sphere which is small compared to the wave length of the emitted sound the intensity in free space will decrease inversely as the square of the distance. In this latter case, the sound level will decrease 6 decibels each time the distance is doubled.

ROOM EFFECTS

10. Particular attention should be given to sound radiated from room surfaces in response to vibrations transmitted through the apparatus supports. Sound produced in this way can be controlled by providing suitable vibration isolating supports for the apparatus.

11. When sound generators are placed within rooms the sound will usually be reflected back and forth many times from the surfaces, and so give rise to a complicated distribution of intensity within the space. At any point within the room the sound level is the resultant of 2 effects; namely, the direct sound which reaches the point without first being reflected, and the indirect sound which arrives after suffering one or more reflections from surrounding surfaces. In general, there will be little correlation between intensity and distance to the source. The intensity may not decrease inversely as the square of the distance; instead it may even be found that at certain points remote from the source the sound is more intense than at positions much closer.

12. The effect of room reflections upon sound measurements can usually be made small by placing the microphone close to the source, and by taking a number of measurements about the source. The use of effective sound-absorbent materials on the interior surfaces of the room will help materially in reducing the effect of indirect sound.

13. An analysis based on reverberation theory leads to the conclusion that if a given amount of sound is generated within a room, the average energy density due to diffusely reflected sound is inversely proportional to the total absorbing power of the room. Hence, assuming that a given absorbing power is constant with respect to the important frequencies in the sound, and converting the expressions relating the intensities at a point in the room where the sound is mostly indirect to the corresponding decibel expressions, the following relation approximately applies:

$$(SL)_1 - (SL)_2 = 10 \log_{10}\left(\frac{A_2}{A_1}\right)$$

where $(SL)_1$ is the sound level in decibels in a room having a total absorption of A_1, and $(SL)_2$ is the sound level when the absorption is A_2.

Ambient Sound Levels

14. Measurements made in many locations have shown that the sound levels existing in factories, offices, and residences usually fall within the ranges given below:

	Sound Level in Decibels
Factories	60–90
Offices	40–75
Residences	30–55

Instrument Standards

15. A sound level meter in accordance with American tentative standards for sound-level meter Z24.3 should be used for measuring sound level.

16. Three response curves are provided for in the standards, curve *A* for a 40-decibel sound level, curve *B* for a 70-decibel level, and a flat response curve *C* for higher

Table I. Correction for Ambient Sound Level

Difference in decibels between ambient plus apparatus sound level and ambient sound level	2	3	4	6	8	10
Correction to be applied to ambient plus apparatus sound level to obtain apparatus sound level	−4.3	−3.0	−2.2	−1.3	−0.8	−0.4

levels. Each of these curves is desirable for use under proper conditions, but it is recommended that curve *A* be used for usual apparatus noise measurements, curve *C* for very high intensities where low frequency noise is predominant, and that curve *B* be used only in special cases.

17. This recommendation is supported by the fact that it is important to have sound-level readings representative of the level that is actually heard under operating conditions. If the apparatus sound level is high, the apparatus is usually placed remote from the point where quietness is desirable. Therefore, the sound level actually heard is much lower than that measured, and hence it is desirable to make the measurements with the scale corresponding to the lower level, that is, the *A* scale.

Factory Tests

18. Factory noise measurements on apparatus are to be made under such conditions that the increase in the measured sound level produced by reflections from surrounding surfaces is small. Also, the microphone should be placed in a position free from disturbing air currents, vibration, electric or magnetic fields, or other external influences, which may affect the readings obtained. The ambient sound level at the test locations should be at least 6 decibels, preferably 10 decibels or more, lower than the sound level due to the apparatus. In general, this requires a special noise box or room with efficient sound-absorbing material placed on the floor, walls, and ceiling. Corrections for ambient levels in factory tests are not generally permissible due to the uncertainties introduced.

Note: Free field conditions for the test room should be attained in so far as it is practicable, but when reflections of importance cannot be avoided their effects upon the measured apparatus sound level may be checked by making a series of sound level measurements with the apparatus placed in various parts of the room, maintaining constant at all times the position of the microphone with respect to the apparatus. If the source is constant the variation in the magnitude of the sound level measured is an approximate measure of the effect of room reflections. The effect of room reflections may also be checked by comparing measurements made outdoors in an open space with measurements made inside of the room.

19. The apparatus under test may be mounted, at the manufacturer's discretion, so that the sound transmitted through the supports will be negligible.

20. Make measurements at several locations near the apparatus to obtain a close approximation to the average sound level produced by the apparatus under free field conditions. When a given type of apparatus consistently produces the same general sound pattern, it is often possible to make production tests at some one point which has proved to be representative of the sound level as determined by averaging the measurements about the machine. The arithmetic average value of the readings is usually satisfactory, but when point to point variations exceeding 10 decibels occur, the weighted energy average, or 10 times the logarithm of the average of the antilogarithms of $^1/_{10}$ the separate decibel values, should be employed.

Note: *Example*

Microphone Position	Sound Level (Decibels)	Antilogarithm of 1/10-Decibel Sound Level
A	60	1,000,000
B	50	100,000
C	40	10,000
		3)1,110,000 total
		370,000 average

$\log_{10} 370{,}000 = 5.54$

$10 \log_{10} 370{,}000 = 55.4$ or the desired decibel value

21. When reporting sound-level measurements, the dimensions of the room and the nature of the floor, walls, and ceiling, the sound level of the room, the condition of operation of apparatus under test, the position of the microphone, and the frequency response of the sound-level meter should be recorded. The type of mounting and other information indicative of the degree of transmission of sound to the supports should also be reported.

22. To promote uniformity and facilitate the comparison of measurements, it is suggested that one of 3 standard distances from the microphone to the apparatus be adopted in making factory noise measurements, these distances being 6 inches, one foot and 3 feet. For very small apparatus with a sound level of less than 35 decibels and major dimensions less than one foot over-all, the 6 inches distance is suggested. For fractional-horsepower motors in general and for measurements where the apparatus is large compared to the space in which measurements are made, the distance of one foot is suggested. For large rotating machines, office equipment, domestic applicances, etc., the 3-foot distance is suggested. In locating the microphone with respect to the apparatus, the distance should be measured to the nearest major surface, minor surface projection being disregarded. When directional microphones are used, the method of orientation with respect to the apparatus outlined in the introduction of the American tentative standards for sound level meters *Z*24.3 should be employed. It is recognized that definite values for these distances, the number of readings to be taken, etc., will ultimately be decided by those interested in formulating codes for sound-level measurements on each specific line of apparatus. The suggestions here are offered for consideration as a logical starting point in formulating such specific codes.

Standard Factory Measurement Positions

23. When measurements at more than one position are made, and it is desired to obtain one over-all value to represent the noise produced by the apparatus, it is recom-

mended that the readings taken at several points be averaged in accordance with the method outlined under "Factory Tests," paragraph 20.

Field Investigations

24. The first step should be to measure the sound level produced while the apparatus is in operation, with the microphone placed at the standard factory test positions, if possible. In any case, the positions should be chosen so near the apparatus producing the noise that the effects of wall reflections or other environmental conditions are a minimum. Then shut down the apparatus and measure the ambient sound level of the room.

25. Further procedure will depend on which of 4 different types of situations are encountered.

(*a*). Apparatus noise much lower than the ambient noise, indicated by no change in reading when the apparatus is shut down. The readings of the apparatus noise in this case are not correct, and indicate only that the apparatus noise does not exceed ambient noise level.

(*b*). Apparatus noise higher than ambient noise. If the apparatus noise is more than 6 decibels above the ambient level (as will be the case when the apparatus plus ambient, and the ambient, levels differ by more than 7 decibels) the apparatus noise is affected but slightly by ambient. In this case, it will be desirable to compare the ambient noise level in this particular location with the ambients of similar installations elsewhere, and also the apparatus noise level with that of similar apparatus when measured in the factory under standard test conditions.

(*c*). Apparatus noise of same order as ambient noise. Under these conditions the sound level of the apparatus may be approximated by the use of the following table which is based upon the fact that the sound level meter adds the weighted energies of the various sounds that reach the microphone.

(*d*). The effects of wall reflections and other environmental conditions may be determined by comparing readings at different positions in the vicinity of the apparatus with those at the standard positions. If the reduction of sound level with respect to distance deviates considerably from that obtained under free field conditions, or if marked variations in sound level occur from point to point, sufficient measurements should be made to establish the general character and location of local increases in sound level. These may be produced by reflecting surfaces, structure-borne noise transmission, focal points of reflections, etc.

The American Engineering Council

An Outline of Some of the Problems, Policies, and Activities of This Jointly Sponsored Engineering Agency

By A. A. POTTER

President, American Engineering Council

INTEREST in co-operation among different engineering groups and the need for an agency to provide, on behalf of our profession, tangible service to public agencies, prompted, in 1920, the organization of the American Engineering Council with Herbert Hoover, then Secretary of Commerce, as its first president. From its very beginning it has been an organization of organizations intended to aid the engineering profession in presenting a united front in matters of interest to our government and to the public. While the Founder and other major engineering societies have been aiding in professional development of the engineer, the American Engineering Council has been supplementing their efforts by relating the engineering profession to public welfare and by making our government, as well as the public at large, conscious of the value of the engineers' services and contributions. It has served during the past 16 years as a clearing house for questions of public nature which require technological knowledge and engineering experience, and it has acted as an advisory body for the various engineering societies in connection with problems of government. At the same time the American Engineering Council has been an agency of inquiry into economic and social problems as affected by technology.

Four Major Functions

The major functions of the American Engineering Council are performed through four groups of committees, which are concerned with public affairs, engineering economics, operations within the Council, and united action of its member societies. These committees, through 17 subcommittees, are in close contact with trends in the conservation and utilization of our national resources, relation of the engineer in private practice to government, patent legislation, public works, aeronautics, surveys and maps, rural electrification, merit system in public service, inquiries in the fields of social and economic activities of

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A report to the Council of the American Society of Mechanical Engineers, read at its semi-annual meeting, Detroit, Mich., May 17, 1937. Published by permission A.S.M.E.

Doctor Potter, dean of the schools of engineering and director of the engineering experiment station, Purdue University, Lafayette, Ind., and past-president of the American Society of Mechanical Engineers and of the Society for the Promotion of Engineering Education, now is serving his second term as president of American Engineering Council.

our country, and similar problems in which engineers may have opportunities for co-ordinated action.

The problems of the American Engineering Council have taken a new slant since 1934. Instead of the normal process of legislation on engineering matters, the AEC has had to work during the past 3 years with a group of administrative departments, whose legislative authority and funds had been established by administrative order. Government departments and bureaus had to be "reeducated" about the American Engineering Council, and its work had to be changed from an orderly process of committee reports and committee hearings and procedures to the specific acts of keeping up with rapidly changing conditions.

Much of the work of the American Engineering Council had to go unreported because it had to do with contacts and associations with individuals in the government, and one of its main tasks had been to make and keep friends with the constantly changing and greatly expanding agencies of the government. Much of the work had to be confidential and some intangible, but a few of the high points may be discussed.

The American Engineering Council has had a constant demand for engineers from government agencies and a large portion of the time of the staff had to be devoted to employment problems for the government bureaus and departments. This service included not only the recommendation of engineers to the various government agencies but also aid to administrative agencies in planning the handling of such work as emergency mapping under the WPA, Rural Electrification Administration programs, Rural Resettlement plans, and NRA codes.

The American Engineering Council is constantly being consulted as to salaries to be paid to engineers and is guided in such requests by the reports of the Founder member societies. It is also invited to suggest names of consultants for various enterprises.

The American Engineering Council has been instrumental in placing numerous engineers in important consulting and line engineering governmental positions, those appointed to such positions seldom knowing the part that the AEC has played in placing them, as it has always been careful to submit a number of names and not to promote the employment of any particular individual.

As part of this employment service, the American Engineering Council had attempted also to analyze and to make a survey of the employment of engineers by government, classified by divisions. Government bureau chiefs are fully appreciative of this service and realize that advice and recommendations by the American Engineering Council are based solely upon the ability and technical competence of engineers and not on questions of political preferment.

During the past few weeks, at a request of one of the national member societies, the American Engineering Council made an exhaustive examination of those laws, regulations, interpretations, and procedures affecting the engagement of technical talent and engineering service by the federal government outside of the civil service. Conditions in more than 30 of the principal agencies known to be using technical talent were investigated, and the results when finally compiled, should prove of great value to the engineering profession.

Advice on Policy

The American Engineering Council has been called for service in connection with matters of policy affecting emergency government setups. A concrete illustration is its relation to the National Resources Committee. Immediately after this committee was set up, as the National Resources Board, the executive secretary made contact with its chairman and director. This was followed by a conference with the chairman, arranged by the American Engineering Council, at which were present the secretaries of the member national societies and from which grew out contacts which closely related engineers to the work of this committee.

Many of the earlier reports of the American Engineering Council on such questions as flood control and the policies for the development of water resources were transmitted to the National Resources Committee and the recommendation of this new agency of government has reflected, in part at least, the recommendations of the member organizations of the American Engineering Council and of its committees. Specifically, the proposal for improvement in collecting water-resources data and the whole question of the development of water resources on the basis of conception of the "water shed," rather than on the basis of states alone, were in the earlier reports of the American Engineering Council.

The authors of the number of reports issued by the National Resources Committee were recommended by the American Engineering Council. Even the most recent report, "Technological Trends and Their Social Implications," has been influenced by advice from the American Engineering Council.

Mapping and Surveying

This is a matter which is having the active interest of the American Engineering Council and has been helpful in educating the Budget Bureau on the value of an adequate mapping program. The American Engineering Council has worked with the Federal Bureau of Surveys and Maps in rounding up the opinions of organizations with reference to this program. Its aid to individual states has resulted in definite progress. A concrete example is Indiana which has appropriated a considerable sum at the 1937 session of its legislature for mapping.

Various Interests

The Upstream Engineering Conference, the Third World Power Conference, and the plans for the World Management Conference are examples of activitites by the American Engineering Council in the interest of united action.

The American Engineering Council, through a committee of which a secretary of a Founder society was chairman, was instrumental in this study by the Bureau of Labor Statistics of the United States Department of Labor. This study made available data regarding 60,000 engineers with reference to such questions as employment conditions, compensation, and distribution in different fields.

Engineering Economics

The activities of the American Engineering Council in studies in engineering economics have been helpful in projecting the engineering profession into the fields of thinking and action relating to our national social, economic, and industrial questions. Among the early contributions of the Council in this field may be mentioned the reports on "Waste in Industry," which had a profound effect upon the conduct of industry the world over; the "Twelve-Hour Shift in Industry" (1921), which demonstrated the merits of the shorter working day; "Civil Aviation" (1924), which focused the national point of view upon the development of commercial aviation; "Radio Broadcasting," which was instrumental in bringing about present policies and practices; and "Water Resources," which was helpful in appropriations for the installation of the national hydraulic laboratory.

Among the recent engineering economic studies of major value may be mentioned the inquiry of the balancing of forces which has greatly influenced economic thought and action during recent years.

There is need for accurate knowledge to show the real effect of science and technology upon social security; such information should go a long way in overcoming the present tendency to make the engineer the major scapegoat of our social ills, by showing that engineers create employment, wealth, comfort, values, and other factors which definitely contribute to social security. The American Engineering Council is now endeavoring to ally itself with representative economists, sociologists, and statisticians in order to study questions of this type and to guide in the public interest some of the questions of larger policy which lie in the fields of both engineering and economics.

There is also need for factual reports, for the benefit of the public, by committees of leading engineers on the engineering projects of our government. Engineers have an opportunity to show, through nonpartisan and unbiased services, their competency and integrity, and to make the public more conscious of engineering contributions. The American Engineering Council hopes that before long it will be in a position to finance the preparation and distribution of more reports, on behalf of our profession, which bear upon public welfare.

National Problems

The American Engineering Council is concerned with national questions to which engineers may contribute co-operatively and in problems in which the engineering societies may have opportunity for co-ordinated action. It is not a super organization intended to manage or to interfere with the autonomy or with the specialized fields of it member societies. It is striving to serve the public and to bring about a better appreciation on the part of all, and particularly of government agencies, of the importance and value of the engineer's contributions. It is endeavoring to bring about concerted action on the part of engineers in public questions of national scope, and a definite interest on the part of the engineering profession in a well-balanced social structure and good government. By representing the engineering profession in matters of public welfare, government, and of joint interest to all engineering societies, the American Engineering Council serves the engineer and the public.

The Making of Industrial Physicists

THE human material for the construction of an industrial scientist may be qualitatively described with considerable ease; but to write for it an engineering specification with inspection techniques and with prescribed tolerances is at present impossible. Unless, however, we would credit evolution with having changed human nature within the times of recorded history we will agree that, despite his specialized training, modern scientific point of view, and wealth of precise instruments, the industrial scientist of today is the man of yesterday and of yesteryear and so on back into prehistoric days. What did this type of human being do in the past? Was he king or priest? Captain or explorer? Medicine man or scribe? Trader or pirate? None of these. He was thinker and craftsman. His was the type that invented the wedge, the inclined plane, the lever, and the screw. He surveyed and built the pyramids; designed and constructed triremes, roads, and aqueducts; shaped the tools of his day; and led the way from stone to bronze to steel.

His urge was the instinct of workmanship, a pleasure in the act of creation or construction which is its own reward. Neither driven by acquisitive instinct nor maddened by lust for power or fame, his satisfaction was his work; he sought little reward; and died unknown. More practical than the philosophers; spending little of his time in teaching or writing; and no follower of the Muses, he burned with a quiet steady flame of the same divine fire that suffused the great teachers and poets of his time. The captains and the kings depart; but what he wrought is the hidden basis of our material grandeur.

From a paper presented by John Mills (A'11, F'23) director of publication, Bell Telephone Laboratories, Inc., at the annual meeting of the Society for Promotion of Engineering Education, Cambridge, Mass., July 1, 1937.

The Selection and Training of Engineers

A company may choose an engineer, or the engineer may choose a company; certain qualities should be sought in either instance

By A. M. MacCUTCHEON
PRESIDENT AIEE 1936–37

IN CONSIDERING this subject a clear, concise, definite, and comprehensive conception of an engineer is essential. The dictionary definition "one versed in or practicing any branch of engineering," does not meet the requirements. Several practicing engineers submitted definitions, the most amusing of which was, "an engineer is a man who can do with one dollar what any man can do with 2 dollars." An engineer's wife felt that "an engineer is one with whom it is rather difficult to live but quite impossible to live without." Of the definitions submitted, the following was the best: "an engineer is one who through training, study, and practice successfully adapts and controls the materials and forces of nature to the benefit and advantage of himself, his fellow engineers, and the rest of the human family." This is quite similar to that prepared by the American Engineering Council, "engineering is the science of controlling the forces and utilizing the materials of nature for the benefit of man and the art of organizing and directing human activities therewith."

Two young men were presenting to a general in the Army Engineering Corps the design of a new and novel pump, and hoping for its adoption. He was not interested because the pump had not been proved. In answer to their inquiry he assured them that he considered himself an engineer and one of the young men stated, "an engineer is one who can tell whether a thing will work before it has been tried, for any fool can tell afterwards."

Each young engineer in training and each engineer engaged in their training should have a clear conception of "an engineer" for that is what their combined effort is intended to produce. Engineers are often charged with being poor salesmen and in some cases this criticism is justified, since they are poor salesmen of themselves and their services. Their engineering should be to the advantage of themselves but this viewpoint should not be overemphasized. Today each engineer should ask himself whether his engineering is in the interest of the human family and if it is not, is he in the truest sense an engineer. Society should recognize that the engineer should be suitably rewarded for his engineering efforts and achievements, since otherwise able young men will not continue to enter the profession.

Selection is defined as the act of choosing. Shall the young engineer be chosen by the company, or the company chosen by the young engineer? This will depend upon the relative standing of each, compared with the others in their field. To a young engineer, it is most important whether he will select a job or whether he will seek one. If his standing and achievements among his fellows are high, he will usually have the privilege of selection. Following are the qualities which a company seeks in a young engineer, listed in approximately the relative order of importance: integrity, dependability, determination, resourcefulness, forcefulness, adaptability, co-operativeness, diplomacy, friendliness, broadmindedness, and knowledge. While the last quality is of great importance to a company selecting a man, the first 10 may be even more important. The more knowledge possessed by a man, the more valuable he will be but of all these 11 qualtities a deficiency in knowledge is most easily remedied. If the young engineer has the privilege of selection, it is just as important that the company he selects should possess the first 10 qualifications and to these should be added: an able and experienced personnel and leadership, a good financial standing, and a definite record of accomplishment. In either case the decision should be based on the probable situation 10 years later. No young man should be satisfied to be an average engineer unless his definition is analogous to Sam's definition of an average tip.

A clergyman, unused to traveling, was making the trip from Birmingham to New York. He was pleased to find that the Pullman porter had, at one time, been the sexton of his church. Throughout the journey Sam took good care of the minister and as Sam was brushing him off, while crossing the Jersey meadows, he asked Sam about what was the average tip that a traveler gave in going from Birmingham to New York. Sam answered, "Well boss, the average tip is about $5." As the minister left the train he handed Sam a $5 bill. Sam's eyes popped and he said, "Boss, you is the first man what ever come up to the average."

When a company selects a young man, graduating from technical school, it should consider the record of his accomplishment even as far back as when he was in high school. It should consider his standing among his fellow students. If he is respected, admired, and followed it is likely that his business associates will respect him and follow him. During his time in college what did he do for his class, and what did he do for his college? This is of importance as an indication of what he is likely to do for his company. His scholastic standing is of much importance for he has gone to a technical college with a definite purpose and if his standing has not been good he has

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Essentially full text of an address presented before many AIEE Sections, and published in response to numerous requests.

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failed in his purpose. This is an indication that he will not be successful in his purpose after graduation. Membership in such societies as Tau Beta Pi or Eta Kappa Nu is of importance as an evidence of accomplishment but scholarship is not the only qualification to be considered for he may be an excellent scholar and definitely lack in the other 10 qualifications.

If the young man is to have the privilege of selection he should consider the accomplishment of the company in the business which it has been carrying on, and its standing with its competitors, for if the competitors do not think well of a company it is questionable whether it is truly successful. He should consider the company's activities in connection with the professional and trade societies having to do with and endeavoring to advance the interests of that class of business. If a company is not interested in such activities it is not doing its part in the development of the broad field of its activity. Lastly, he should consider the salary which is being offered by the company. While this is important it should not be overemphasized. Of far greater importance is the salary which he will receive 10 years later.

The company should offer a salary competitive with other companies in its line of activity. While many individuals give far too much consideration to the matter of salary, some give too little. The salary received is the most tangible evidence of accomplishment. While an engineer should receive a salary commensurate with the contribution that he makes to the success of the company, he should recognize that fair salaries cannot be paid unless the company is successful. A broadminded engineer may willingly accept a relatively low salary over a considerable period and without dissatisfaction because he believes in the ultimate success of the company and that when this success is achieved, the company will suitably recognize his contribution in making the company successful. If an individual is not satisfied with his financial return, and is convinced that the company is not and will not be fair to him, he should not stay with that company for one of the greatest deterrents to a man's success is a long continued dissatisfaction with his financial return, about which he does nothing constructive. If a young engineer selects the right company and makes himself so valuable that he cannot be replaced without a considerable penalty, he will usually have to give very little consideration to the matter of financial returns. Great weight should be given to the financial standing of a company, and of particular importance is the record during such a period of business depression as existed from 1930 to 1934.

In the selection of young engineers, company representatives should make trips to the educational institutions and study the young men in their student environment. Many companies have done this for years, and the practice is becoming almost universal. It is equally as desirable that the young engineer should visit companies in order to prepare himself for the making of an intelligent decision. Because of limited finances, university restrictions, and lack of experience, the young man is handicapped in such an investigation. Twenty years ago a technical graduate was not discouraged if he had not located, by the time of graduation. This time of selection has gradually moved backward. Those men and companies, whose records of accomplishment permit them to be the selectors, have usually made their decision prior to the first of April. Certain advantages would result from a mutual agreement between the technical schools and industrial organizations that December should be a period of contact and investigation with decision to be reached early in March. The Christmas holidays offer an opportunity for the visiting of plants and the period immediately before and during the midyear examinations should be free from interruption. Some specialization during the last semester would be possible. Several companies are now offering to the young engineer who is so fortunate as to be in that group which has the privilege of selecting, the opportunity of visiting the plant that he may familiarize himself with the activities and the personnel of the company, and decide whether the association would be mutually advantageous.

Training Before Graduation

The larger part of the technical training of the young engineer is received in the technical school prior to his selection and necessarily independent of the company with which he is to be associated or the particular type of work in which he will engage. Such generalized training is broadening but not so immediately useful as training of a more specialized type. There should be a sufficient amount of specialized training as to develop a proper conception of the intimate relation between training and accomplishment. Broad technical training should be continued after graduation and still closer contact and co-operation between educators and industrial managements is most desirable that the training before and after graduation may be fully co-ordinated, the one supplementing the other.

Comments made by one connected with a manufacturing company, on the training prior to graduation, are made from a narrow viewpoint and are offered only as suggestions for the consideration of the engineer educators whose lives are devoted to the training of young engineers and who are intensively studying the technique of engineering education.

Admiral Sims, in his story, so effectively illustrates the influence of a varying viewpoint. A young American was ordered to England to take charge of the London office. He met and married a delightful English lady whose mother was strongly pro-British. Many arguments resulted between the young man and his mother-in-law as to the relative merits of the 2 countries. In the course of time a daughter was born and some 3 years later the father was ordered back to the States. On the evening before they sailed, the mother-in-law requested the privilege of putting her granddaughter to bed. She descended the stairs smiling broadly and said to her son-in-law, "You should have heard your little daughter's good-night prayer. 'Goodbye God, we're going to America tomorrow.'" "But," said the son-in-law, "what she meant was—'Good, by God we're going to America tomorrow.' "

To one in manufacturing it sometimes seems that the technical schools still lay more emphasis on the giving and acquiring of information than on the methods of proper engineering attacks, engineering analysis, and the securing of engineering information. While the imparting of knowledge to the young engineer is of great importance, instruction in the proper application of the "engineering method" is of still greater importance. Because of the necessary generalization much of the information gained is not usable after graduation. Training in the engineering methods is most valuable no matter with what company the young engineer may be associated. As a simple example of a somewhat unusual training, consider the course given in Columbia by Professor Lucke to a group of electrical engineers who had requested a course on gas engines. Most of the work was done in the laboratory where there were 10 gas engines. Under Professor Lucke's direction the laboratory assistant did something to each of these gas engines so that they would not properly function. The students were divided into 10 squads; each squad assigned to a particular engine. The problem was to determine what was wrong with the engine, make the necessary correction, and leave the engine running properly. Text books, specifications, and diagrams were available for consultation. Both Professor Lucke and the laboratory attendant would answer any questions which might be asked with one exception, "What is wrong with this gas engine?" There was an intense competition as to which group would first have their engine functioning properly. These electrical men received an unusual training in how to attack an engineering problem, analyze it, and solve it. The qualities of determination, resourcefulness, forcefulness, adaptability, co-operation, diplomacy, and friendliness were being developed in each one of the students.

In each engineering school should there not be a course "how to get results. How to handle other people and get them to do what you wish them to do?" Such a course would give valuable training to any man, particularly to engineers; a course just as valuable to the design or research engineer as to the engineer who would later serve in an executive capacity. The ability to handle people and get results is subject to development through training and this training is greatly needed by all engineers. Such a course might be given by the case method so successfully used in the training of lawyers. Successful executives would undoubtedly co-operate by giving lectures, citing unusual instances of results secured through properly approaching, informing, and co-operating with others. The best engineering plan has little value unless the engineer can convince others of the soundness of the plan and persuade them to use it. The following is a simple illustration of securing results and properly handling other people. An important new machine was being designed. Successively the designer compelled himself to view the design as a member of the sales department, the manufacturing department, the service department, and, most important of all, as the user. He prepared a series of briefs summarizing the design from each viewpoint. He presented the appropriate brief to at least one in each of these groups and asked for criticism and suggestions. He adopted any suggestions that seemed to him practicable. He avoided any prolonged arguments, but stated in writing why certain suggestions could not be adopted. All of this was done before the routine delivery of drawings and specifications. Each interested group was prepared for and committed to solving their problems in connection with the design before the problems actually reached them. Throughout the manufacture, tests, installation, and final operation, the design engineer stood ready to give information, answer questions, and render assistance. The design was highly commended by all who came in contact with the machine. While this designer held an important position in the organization, he was not an executive. Often engineers have been handicapped by failure to recognize who should be consulted and who should receive information.

In most technical schools there are a certain number of shop courses and at least some of these should be retained. The young engineer should receive a training in manual dexterity and this can best be accomplished prior to the age of 23. The theoretical and the concrete should always be associated.

If such work is to be retained, should not the electrical department specify shop courses directly related to electrical equipment; and should not each electrical engineer in training experience the thrill of designing, building, and testing at least one piece of such equipment? While the primary purpose of the technical school is a thorough grounding in the technical fundamentals, and with only 4 years allowed for all that is to be accomplished, will not a certain number of shop courses, properly co-ordinated, so effectively supplement the technical training as to give a better result with the same effort?

The training of the young engineer by the company which he has selected or which has selected him, is equal in importance to the training while in the technical school. The course should be carefully planned before the young men enter the company's employ and in consultation with the professors in the technical schools who have given their lives to the training of engineers. The course should be open to modification upon suggestions from the young men in training. There should be a sympathetic interest on the part of all departments in the organization and to this end each department should clearly understand the value of such training in the interest of the company. Lectures should be given by the company executives acquainting the group in training with the history, the development, the ideals, and the policies of the company. Some time should be spent in every department and, in addition, there should be visits to the plants of customers where the young engineers can see machines which the company makes, in actual operation. In this way they can be acquainted with the customer's viewpoint.

Both the company and the group in training should recognize the necessary compromise between the theoretical and the practical. The primary purpose of the organization is the manufacture of the product and while the training of the young engineer is of importance there are times when this training must suffer if it conflicts with the primary purpose of the organization.

The American Institute of Electrical Engineers effectively participates in the training of young engineers. In most of the engineering schools, student branches have been established and these are an integral part of the AIEE. Within the branch, technical papers are prepared and presented, affording valuable training to the young engineer who prepares the paper and valuable information to the other young engineers who hear the paper presented. To the student member of the Institute is available ELECTRICAL ENGINEERING, containing the technical articles presented before the national and regional conventions.

Engineers are sometimes prone to neglect the human relationships and one of the most delightful privileges of an engineer is to meet with other engineers and exchange ideas. This opportunity is offered through the AIEE Branches while the young man is in technical school and through the local Sections after he has joined a commercial organization.

The most effective agency in the training of a young engineer is the engineer himself. He has more to do with his training than the technical school or the company with which he is associated. The arranged training courses simply present an opportunity that he may effectively train himself. In accomplishing this training he should grasp every opportunity to become acquainted with all others in the organization, including the workman in the plant. He should accept every opportunity for contact with outside engineers. He should join in the work of the professional organization of engineers in the city where he lives. He should seek to find answers to every unanswered question in his mind about the company's product. He should never walk through the plant without asking himself what he can find out that he has not previously known. What a man learns for himself, he seldom forgets. He should feel free to make suggestions, for he has a fresh and uninfluenced viewpoint.

One of the most important phases in the training of an engineer is to develop in him the probability of being right. While no engineer will always be right, this is the limit which he should strive to approach. The closer he comes to the limit, the more successful he will be. In his endeavor to approach this limit, he should remember that there is a great value in what may be termed "secondary checks." A secondary check may be defined as a brief comparison between the solution of a new problem and the proved solution of a somewhat similar problem, which comparison will indicate, at least qualitatively if not always exactly quantitatively, whether the new solution is correct. Through a free use of secondary checks costly errors will often be avoided.

The young engineer should be trained in how to accept and delegate responsibility. He should recognize that no man should be asked to accept responsibility without having a corresponding authority. A leading executive once said, "the president of a company is responsible for the work of every man in that company; the chief engineer for the work of every engineer who reports to him; each engineering division head is responsible for the work of everyone in his division; the chief draftsman responsible for all the work of the drafting room including that of the blueprint boy." This is the correct theory of responsibility looking from above downward. On the other hand, looking from below upward, the blueprint boy should be trained to feel responsible for everything regarding blueprints and blueprint service. The draftsman should feel responsible for every drawing he makes. While he should accept advice from the man above him, he should consider the finished drawing as his, for which he alone is responsible. Accepting responsibility for the work of all the men under him, the chief engineer should not feel that the president above him is responsible for the engineering. He should ask for and welcome advice and guidance, but the finished engineering he should consider as his, and his responsibility. A close following of this theory of responsibility should produce unusually good results.

In any course of training there should be sincere co-operation between the technical school, the company, and the young engineer himself. From Columbus comes a story illustrating an unusual degree of co-operation between engineers.

A Columbus engineer was called to Dayton and advised his wife he would return about 11 o'clock that evening. His wife was tired and retired at 9 p.m. When she awoke in the morning she found that her husband had not returned. She became deeply concerned, made a list of 20 engineers living in Dayton and who were acquainted with her husband and to each one of the 20, sent an identical telegram, "Do you know where John is, or what he is doing?" John's automobile had given trouble so that he had stopped in Dayton for the night and had the necessary repairs made the following morning. He reached home about 10 a.m. About 11 o'clock the wife began to receive replies to her telegrams. Exactly 20 replies were received and each reply said the same thing, "Do not worry; John is spending the night with me."

A company which selects a young electrical engineer owes to him a deep responsibility for they are influencing his whole life. They should not invite him to join the organization unless there is likely to be an opportunity such as he deserves. They should never forget their responsibility to him. They should offer to him every opportunity for development and should endeavor to give him a suitable financial return. On the other hand, the young electrical engineer should also fully realize the measure of his responsibility. He has accepted a position which would have been offered to another man. He should use every effort to fully measure up to the opportunities which are open to him. He should recognize that no company should offer an opportunity to a man who does not earn it and that his opportunity with the company largely depends upon his contribution to the company. If there is the fullest co-operation between the company, the technical school, and the young engineer himself, the engineering profession should be carried to far greater heights than have ever been reached in the past.

Engineering Income and Earnings, 1929–34

In this, the fifth article[1] of a series reporting a survey of employment in the engineering profession by the United States Bureau of Labor Statistics, income data for engineers of various professional classes and ages are presented, based on information given for the years 1929, 1932, and 1934. Comparisons may be drawn among professional classes, ages, and educational backgrounds.

REPORTS from 52,589 professional engineers to the Bureau of Labor Statistics, in its survey of the engineering profession,[1] undertaken at the request of the American Engineering Council, make it possible to relate the income data to many aspects of professional engineering activity. These income data are probably the most comprehensive ever presented in regard to a professional group. The following general findings appear from analysis of the reports.

In 1929, 50 per cent of the 30,032 reporting engineers earned more than $3,412, and 50 per cent earned less than that amount. Twenty-five per cent earned more than $5,012, but only 10 per cent had incomes in excess of $7,466 per annum. On the contrary, 25 and 10 per cent of the engineers earned, respectively, less than $2,509 and $1,878 per year.

Comparison of the incomes for 1932 and 1934 with those reported for 1929 shows that from 1929 to 1934, the sharpest absolute declines occurred in the 2 higher income groups. Their percentage decreases were least, however, averaging 31.2 and 31.6 per cent, whereas the middle values of income declined by 33 per cent, and the 2 lower levels by 41.3 and 53.6 per cent, respectively.

Almost two-thirds of these decreases in earned annual income occurred between 1929 and 1932. There were further decreases from 1932 to 1934.

Among the several professional classes, the divergences in earning capacities were most marked in the higher levels of income. Furthermore, the ranking of the profession on the basis of earnings opportunity was the same in the highest 10 and 25 per cent only.

In 1929, without regard to the age distributions of the different classes, 10 per cent of the mining and metallurgical engineers earned more than $9,912 per year, chemical and ceramic engineers ranked second with 10 per cent earning more than $9,103, and were followed in order by mechanical and industrial engineers ($8,508), electrical engineers ($7,185), and civil engineers ($6,507). At the upper 25 per cent level, mining and metallurgical engineers reported earnings of $6,301 per year, and those of the other professional classes ranged from 4 per cent lower for chemical and ceramic engineers, to 28 per cent lower for civil engineers. This order of professional classes was also maintained in 1932 and 1934.

In 1929, the earnings of the mining and metallurgical engineers were highest in the middle and 2 lower levels of income, while those of the electrical engineers were lowest. At these 3 levels, the earnings of the former were $4,010, $2,839, and $1,985; for the latter they were $3,277, $2,339, and $1,662, respectively. Over these 3 levels, the relative positions of the 3 remaining professional classes changed. There were further shifts in 1932 and 1934.

The data also show that, over the period 1929 to 1934, relatively the smallest shrinkages in earnings were reported by the civil engineers, while the chemical and ceramic engineers suffered the greatest reductions. Over the 5 income levels, the former ranged from 29.0 to 47.2 per cent, the latter from 35.6 to 63.8 per cent.

Analysis of the income data reported by all engineers in 1929, 1932, and 1934 shows that earnings advanced with age in 3 distinct phases, showing initial periods of exceptionally rapid rise, followed by 2 others of slower rates of increase. The age spans of these phases varied with the income level.

With advancing age, the spread in earnings became most accentuated beyond the age of 38. Thus, at the ages of 44 and 60, the incomes of the upper 25 per cent differed from the median or middle value by 41.0 and 51.0 per cent. The corresponding incomes of the upper 10 per cent at these ages were greater than the median by 116.0 and 157.0 per cent. Even in 1932 and 1934, this advantage in earning capacity was maintained.

The earnings of the engineers in the lower income levels ceased to increase at a relatively early age. These showed a level period before beginning to decline, whereas the higher levels of income showed continuous increases for a considerably longer period, after which, however, they declined at a much greater rate.

Over the period 1929 to 1934, a rise in earnings occurred for the youngest engineers who were in the profession in 1929 and who were 30 years of age or less in 1934. Thereafter there were progressively larger declines for the older engineers.

Comparison of the earnings of engineers of identical ages in 1929 and 1934 shows that the average income of engineers who had been out of college for 2 years declined 43 per cent. The income of those who had been out 5 years declined 35 per cent. For older engineers the decline approximated 30 per cent.

Consideration of the incomes reported by engineers of different educational backgrounds shows that those with a formal engineering education did receive a higher income. The differentials in earnings, however, did not accrue in equal measure for all 5 professional classes.

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[1] An article prepared by Andrew Fraser, Jr., of the Division of Hours, Wages, and Working Conditions, Bureau of Labor Statistics, United States Department of Labor, which article was published in the August 1937 issue of *Monthly Labor Review*. Articles reporting other phases of this survey were published in ELECTRICAL ENGINEERING as follows: "Professional Aspects of Engineering Education," August 1936, pages 863–7; "Unemployment in the Engineering Profession," February 1937, pages 216–23; "Employment in the Engineering Profession," May 1937, pages 524–31; "Security of Engineering Employment," June 1937, pages 655–61. A detailed report of the survey will be published later in bulletin form by the Bureau of Labor Statistics.

Table I. Comparison of Gross and Net Numbers of Older* and Younger** Graduate and "Other" Engineers Reporting Annual Earnings in 1934

Without regard to employment status reported

Professional Class	Graduate Engineers				Other Engineers			
	Total	Older Group	Younger Group Graduated in—		Total	Older Group	Younger Group Born in—	
			1930–32	1933–34			1907–09	1910–14
	Number							
All classes:								
Number reporting in survey	43,288	24,837	11,039	7,412	9,301	8,440	575	286
Gross number reporting income	38,513	22,178	10,201	6,134	8,277	7,513	523	241
Net number reporting income	33,720	20,376	7,210	6,134	7,569	6,871	457	241
Chemical and ceramic:								
Number reporting in survey	3,697	1,453	1,257	987	203	158	27	18
Gross number reporting income	3,270	1,285	1,169	816	177	137	27	13
Net number reporting income	2,749	1,150	783	816	156	124	19	13
Civil, agricultural, and architectural:								
Number reporting in survey	16,114	10,318	3,598	2,198	4,712	4,341	256	115
Gross number reporting income	14,562	9,344	3,369	1,849	4,275	3,941	233	101
Net number reporting income	12,984	8,596	2,539	1,849	3,915	3,603	211	101
Electrical:								
Number reporting in survey	9,924	5,184	2,947	1,793	1,519	1,285	156	78
Gross number reporting income	8,792	4,688	2,686	1,418	1,345	1,140	140	65
Net number reporting income	7,627	4,368	1,841	1,418	1,256	1,067	124	65
Mechanical and industrial:								
Number reporting in survey	11,643	6,599	2,851	2,193	2,590	2,407	124	59
Gross number reporting income	10,236	5,785	2,620	1,831	2,246	2,082	113	51
Net number reporting income	8,923	5,271	1,821	1,831	2,035	1,889	95	51
Mining and metallurgical:								
Number reporting in survey	1,910	1,283	386	241	277	249	12	16
Gross number reporting income	1,653	1,076	357	220	234	213	10	11
Net number reporting income	1,437	991	226	220	207	188	8	11
	Per Cent of Number Reporting in Survey							
All classes:								
Gross number reporting income	89.0	89.3	92.4	82.8	89.0	89.0	91.0	84.3
Net number reporting income	77.9	82.0	65.3	82.8	81.4	81.4	79.5	84.3
Chemical and ceramic:								
Gross number reporting income	88.5	88.4	93.0	82.7	87.2	86.7	100.0	72.2
Net number reporting income	74.4	79.1	62.3	82.7	76.8	78.5	70.4	72.2
Civil, agricultural, and architectural:								
Gross number reporting income	90.4	90.6	93.6	84.1	90.7	90.8	91.0	87.8
Net number reporting income	80.6	83.3	70.6	84.1	83.1	83.0	82.4	87.8
Electrical:								
Gross number reporting income	88.6	90.4	91.1	79.1	88.5	88.7	89.7	83.3
Net number reporting income	76.9	84.3	62.5	79.1	82.7	83.0	79.5	83.3
Mechanical and industrial:								
Gross number reporting income	87.9	87.7	91.9	83.5	86.7	86.5	91.1	86.4
Net number reporting income	76.6	79.9	63.9	83.5	78.6	78.5	76.6	86.4
Mining and metallurgical:								
Gross number reporting income	86.5	83.9	92.5	91.3	84.5	85.5	83.3	68.8
Net number reporting income	75.2	77.2	58.5	91.3	74.7	75.5	66.7	68.8

* Includes all engineers who were professionally active prior to 1930.

** Includes all engineers who entered the profession in the years 1930–32 and 1933–34.

At about 28 years of age the "other" engineers had lost an initial advantage in earning capacity. At that point, the 1929 earnings of the graduates ranged from $2,725 to $3,000 per year, those of the corresponding "other" engineers from $2,430 to $2,650.

With advancing age, the spreads in earnings in favor of the graduates became very marked indeed. For example, at 5, 20, and 37 years after graduation, the earnings of first-degree mechanical and industrial engineers exceeded by $175, $925, and $1,322 per year those of the engineers of the same professional class whose college course was incomplete, and surpassed by $225, $1,160, and $1,815 per year those of engineers with a noncollegiate technical school education.

Even among the graduate groups there was variation in the increase in earning capacity between the several professional classes. Thus, the 1929 earnings of first-degree civil engineers who had been out of college for 5 years were only $2,050 less than the earnings of those who had graduated 30 years before, whereas the corresponding difference for chemical and ceramic engineers was $3,600. The ranges in earnings of the remaining graduate groups fell between those reported by the civil and chemical and ceramic engineers.

In general, the earnings of the "other" engineers ceased to increase at an earlier age than those of the graduates.

Scope and Method of Study

Analysis was made to determine the changes in the income and earnings of professional engineers during the period 1929 to 1934. The reports covered 2 distinct features of income for the years 1929, 1932, and 1934: (1) earned annual income received from all personal services and (2) average monthly compensation from engineering work only. The annual figures are a measure of what en-

Table II. Comparison of Gross and Net Numbers of Older* and Younger** Graduate and "Other" Engineers Reporting Monthly Engineering Income† in 1934

Without regard to employment status reported

Professional Class	Graduate Engineers				Other Engineers			
			Younger Group Graduated in—				Younger Group Born in—	
	Total	Older Group	1930–32	1933–34	Total	Older Group	1907–09	1910–14
	Number							
All classes:								
Number reporting in survey	43,288	24,837	11,039	7,412	9,301	8,440	575	286
Gross number reporting income	31,574	19,814	7,405	4,355	7,504	6,860	447	197
Net number reporting income	26,680	17,686	4,639	4,355	6,639	6,070	372	197
Chemical and ceramic:								
Number reporting in survey	3,697	1,453	1,257	987	203	158	27	18
Gross number reporting income	2,568	1,151	842	575	141	116	16	9
Net number reporting income	2,071	1,004	492	575	118	99	10	9
Civil, agricultural, and architectural:								
Number reporting in survey	16,114	10,318	3,598	2,198	4,712	4,341	256	115
Gross number reporting income	12,929	8,593	2,817	1,519	3,996	3,691	212	93
Net number reporting income	11,073	7,647	1,907	1,519	3,538	3,260	185	93
Electrical:								
Number reporting in survey	9,924	5,184	2,947	1,793	1,519	1,285	156	78
Gross number reporting income	6,307	4,003	1,524	780	1,138	980	109	49
Net number reporting income	5,376	3,678	918	780	1,041	898	94	49
Mechanical and industrial:								
Number reporting in survey	11,643	6,599	2,851	2,193	2,590	2,407	124	59
Gross number reporting income	8,350	5,105	1,938	1,307	2,022	1,882	102	38
Net number reporting income	6,994	4,511	1,176	1,307	1,765	1,6[illegible]8	79	38
Mining and metallurgical:								
Number reporting in survey	1,910	1,283	386	241	277	2[illegible]	12	16
Gross number reporting income	1,420	962	284	174	207	191	8	8
Net number reporting income	1,166	846	146	174	177	165	4	8
	Per Cent of Number Reporting in Survey							
All classes:								
Gross number reporting income	72.9	79.8	67.1	58.8	80.7	81.3	77.7	68.9
Net number reporting income	61.6	71.2	42.0	58.8	71.4	71.9	64.7	68.9
Chemical and ceramic:								
Gross number reporting income	69.5	79.2	67.0	58.3	69.5	73.4	59.3	50.0
Net number reporting income	56.0	69.1	39.1	58.3	58.1	62.7	37.0	50.0
Civil, agricultural, and architectural:								
Gross number reporting income	80.2	83.3	78.3	69.1	84.8	85.0	82.8	80.9
Net number reporting income	68.7	74.1	53.0	69.1	75.1	75.1	72.3	80.9
Electrical:								
Gross number reporting income	63.6	77.2	51.7	43.5	74.9	76.3	69.9	62.8
Net number reporting income	54.2	70.9	31.2	43.5	68.5	69.9	60.3	62.8
Mechanical and industrial:								
Gross number reporting income	71.7	77.4	68.0	59.6	78.1	78.2	82.3	64.4
Net number reporting income	60.1	68.4	41.2	59.6	68.1	68.5	63.7	64.4
Mining and metallurgical:								
Gross number reporting income	74.3	75.0	73.6	72.2	74.7	76.7	66.7	50.0
Net number reporting income	61.0	65.9	37.8	72.2	63.9	66.3	33.3	50.0

* Includes all engineers who were professionally active prior to 1930.
** Includes all engineers who entered the profession in the years 1930–32 and 1933–34.
† In the text, "monthly engineering income" and "engineering income" are used interchangeably.

gineers were able to earn during each of the 3 years. This was determined not only by the rate of earnings but also by the volume of employment. The monthly figures are a measure of the rates at which engineering services were purchased.

Both income and earnings are shown in relation to (1) professional class, (2) age, (3) type of education, and (4) regional location. A discussion of earned annual income from all sources, by age, is included for engineers engaged in engineering work and also for those in nonengineering work, as well as annual income reported by those engineers who were unemployed at the end of 1929, 1932, and 1934. Monthly engineering earnings, while treated similarly to annual income, can only be shown for the various types of engineering employment. The earnings of those engaged in engineering work are also shown in relation to size of city.

From a consideration of certain pertinent aspects affecting the returns on income and earnings for 1929, 1932, and 1934, it is believed that these data are representative of the engineering profession as a whole. They have been shown to be representative as regards age, if allowance is made for the larger representation of engineers graduating in 1930 or later years. In analyzing the income data, as in the analysis of employment status, the samples of older and younger engineers[2] were combined on an adjusted basis to secure a cross section of the entire profession in 1932 and 1934.[3]

Without exception the several distributions of income follow a consistent pattern even when analyzed in detail.

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[2] Older engineers comprised 15 per cent of all those who were professionally active prior to 1930. The age composition of this group was consistently uniform with the corresponding data derived from the 1930 census on professional engineers. This was so for minor groupings of professional classes by 10 geographical regions and for the country as a whole. The younger engineers included the information furnished by all those who entered the profession in the years 1930–34 inclusive. The returns for each of these years covered the same proportions (32.0 per cent) of the total number of graduations reported by the Office of Education.

Table III. Comparison of 5 Levels of Annual Earnings for All Professional Engineers Reporting in 1929, 1932, and 1934

Figures derived from adjusted data as explained[3] and without regard to employment status reported or type of education

Per Cent at Specified Income Level	Annual Earnings of More Than Specified Amount			Increase or Decrease			Per Cent of Change		
	1929	1932	1934	1929–34	1929–32	1932–34	1929–34	1929–32	1932–34
10	$7,466	$5,605	$5,138	−$2,328	−$1,861	−$467	−31.2	−24.9	−8.3
25	5,012	3,827	3,429	− 1,583	− 1,185	− 398	−31.6	−23.6	−10.4
50	3,412	2,574	2,286	− 1,126	− 838	− 288	−33.0	−24.6	−11.2
75	2,509	1,698	1,473	− 1,036	− 811	− 225	−41.3	−32.3	−13.3
90	1,878	889	872	− 1,006	− 989	− 17	−53.6	−52.7	−1.9

Thus, the same type of distribution emerges when the data are analyzed by ages, by professional classes, or even on a regional basis. This is true of both annual income and monthly engineering income.

The averages proved to be essentially the same, whether derived from a "gross" or "net" number reporting. The "gross" number reporting income is the total number of engineers who reported income in any one year, irrespective of whether or not they reported in other years. The "net" number reporting income is the number who furnished information for all 3 years—1929, 1932, and 1934.[4] In the income tables averages are derived from the "gross" numbers reporting.

The "gross" numbers reporting earned annual income for 1934 and the "net" numbers are shown compared in table I. These numbers are the totals of those returning questionnaires before any adjustment is made in the number of the younger men.

In each case, the base for computing percentages was the number reporting in the survey, that is, those engineers who reported a type of education.[5] For all age groups, it will be noted that there were distinct differences between the "gross" and "net" percentages reporting income. The greatest divergences occurred among the 1930–32 graduate and the 1907–09 "other" engineers. For the country as a whole, these were respectively 27.1 and 11.5 per cent. On a national basis, older graduates differed by 7.3 per cent, older "other" engineers by 7.6 per cent. Despite these variations, a comparison of the corresponding measures of levels of annual income derived from the "gross" and "net" returns showed no significant differences. A similar situation was noted for the "gross" and "net" returns on monthly engineering income.

In the ensuing discussion, the middle values of income were computed for groups with at least 10 engineers reporting. For the upper and lower 25 per cent groups or levels, the measures were based on not less than 50, while the upper and lower 10 per cent groups embraced not less than 100 engineers.

Earned Annual Income

The income data for 1929 were furnished by 30,032 engineers, or slightly over 90 per cent of all reporting engineers who were professionally active prior to 1930. These data, together with the adjusted figures on earned annual income, without regard to employment status reported, for 1932 and 1934, are shown in table III.

In 1929 the range in earned annual incomes among professional engineers was great. Some 60 reported incomes less than $800 per year, while 162 earned more than $19,000 a year. In 1929, without regard to their age distribution, half of the engineers had annual incomes greater than $3,412, while half earned less than that figure. However, 25 per cent earned more than $5,012 per annum. Only 10 per cent of the 30,032 reporting engineers had incomes in excess of $7,466 per annum. On the contrary, one-quarter of all engineers reporting earned less than $2,509 per year, and one-tenth earned less than $1,878 per year.

From 1929 to 1934, marked decreases took place in the

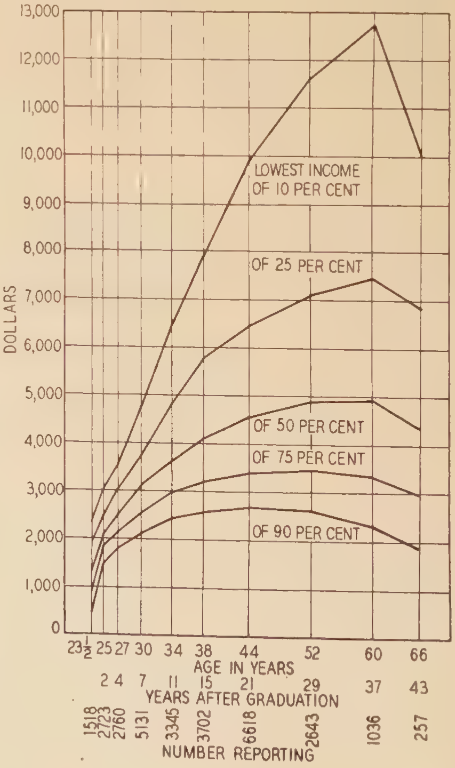

Figure 1. Earned annual income of engineers according to age, 1929

Income from engineering and nonengineering services, including full and part-time employment, all types of education

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[3] The procedure of adjustment adopted was similar to that outlined in the discussion of the distributions of employment status. The numbers of younger engineers reporting in each income class for 1932 and 1934 were reduced in the ratio of 15 to 32, and added to the corresponding figures reported by the older engineers in 1932 and 1934.

[4] In the case of the 1930–32 graduates and "other" engineers born in 1907–09, the net number reporting income is the number furnishing data for the 2 years, 1932 and 1934. In the case of the 1933–34 graduates and 1910–14 "other" engineers, income data for 1934 alone were relevant. Hence, there is no difference between "gross" and "net" data.

[5] The use of type of education as a base was felt to be justified because only 104 of the 52,589 professional engineers reporting in this survey did not report their type of education. The 104 engineers however, did report their professional class; and either year of graduation or year of birth. They were, therefore, used for all purposes, except those dealing with type of education.

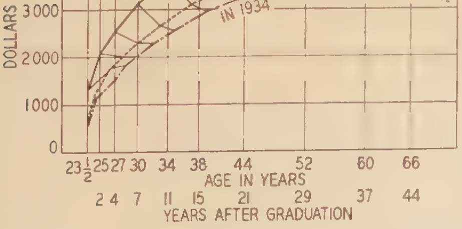
Table IV. Comparison of 5 Levels of Annual Earnings for All Engineers Reporting in Each Professional Class in 1929, 1932, and 1934

Figures derived from adjusted data as explained[3] and without regard to employment status reported or type of education

Per Cent of Professional Class at Specified Income Level*	Annual Earnings of More Than Specified Amount			Increase or Decrease			Per Cent of Change		
	1929	1932	1934	1929–34	1929–32	1932–34	1929–34	1929–32	1932–34
10 per cent:									
Mining and metallurgical	$9,912	$7,011	$6,486	−$3,426	−$2,901	−$525	−34.6	−29.3	− 7.5
Chemical and ceramic	9,103	6,525	5,860	− 3,243	− 2,578	− 665	−35.6	−28.3	−10.2
Mechanical and industrial	8,508	6,220	5,572	− 2,936	− 2,288	− 648	−34.5	−26.9	−10.4
Electrical	7,185	5,785	5,220	− 1,965	− 1,400	− 565	−27.3	−19.5	− 9.8
Civil, agricultural, and architectural	6,507	5,086	4,561	− 1,946	− 1,421	− 525	−29.9	−21.8	−10.3
25 per cent:									
Mining and metallurgical	6,301	4,698	4,328	− 1,973	− 1,603	− 370	−31.3	−25.4	− 7.9
Chemical and ceramic	6,043	4,425	3,703	− 2,340	− 1,618	− 722	−38.7	−26.8	−16.3
Mechanical and industrial	5,582	4,123	3,662	− 1,920	− 1,459	− 461	−34.4	−26.1	−11.2
Electrical	4,806	3,770	3,410	− 1,396	− 1,036	− 360	−29.0	−21.6	− 9.5
Civil, agricultural, and architectural	4,508	3,579	3,266	− 1,242	− 929	− 313	−27.6	−20.6	− 8.7
50 per cent:									
Mining and metallurgical	4,010	3,061	2,626	− 1,384	− 949	− 435	−34.5	−23.7	−14.2
Chemical and ceramic	3,803	2,625	2,047	− 1,756	− 1,178	− 578	−46.2	−31.0	−22.0
Mechanical and industrial	3,699	2,681	2,324	− 1,375	− 1,018	− 357	−37.2	−27.5	−13.3
Civil, agricultural, and architectural	3,291	2,545	2,297	− 994	− 746	− 248	−30.2	−22.7	− 9.7
Electrical	3,277	2,509	2,218	− 1,059	− 768	− 291	−32.3	−23.4	−11.6
75 per cent:									
Mining and metallurgical	2,839	1,788	1,512	− 1,327	− 1,051	− 276	−46.7	−37.0	−15.4
Mechanical and industrial	2,626	1,676	1,424	− 1,202	− 950	− 252	−45.8	−36.2	−15.0
Chemical and ceramic	2,538	1,556	1,213	− 1,325	− 982	− 343	−52.2	−38.7	−22.0
Civil, agricultural, and architectural	2,499	1,770	1,596	− 903	− 729	− 174	−36.1	−29.2	− 9.8
Electrical	2,339	1,634	1,343	− 996	− 705	− 291	−42.6	−30.1	−17.8
90 per cent:									
Mining and metallurgical	1,985	773	893	− 1,092	− 1,212	+ 120	−55.0	−61.1	+15.5
Mechanical and industrial	1,956	919	853	− 1,103	− 1,037	− 66	−56.4	−53.0	− 7.2
Civil, agricultural, and architectural	1,926	909	1,016	− 910	− 1,017	+ 107	−47.2	−52.8	+11.8
Chemical and ceramic	1,686	732	610	− 1,076	− 954	− 122	−63.8	−56.6	−16.7
Electrical	1,662	873	731	− 931	− 789	− 142	−56.0	−47.5	−16.3

* Arranged in ascending order of earned annual income for 1929.

earned annual incomes of professional engineers. The sharpest absolute declines occurred in the higher income levels. Thus, in 1929, while the highest one-tenth of the engineers had earned more than $7,466, in 1934 the earnings of this highest-paid tenth ranged down to $5,138. The middle income declined from $3,412 in 1929 to $2,286 in 1934. However, the percentages of decrease for the highest 10 and 25 per cent of reporting engineers were approximately the same, namely, 31.2 and 31.6 per cent, respectively. The middle or average values of all incomes declined by 33.0 per cent. At the lower income levels, the absolute declines were almost as great as was the decline of the median earnings; but the percentage decreases were greater. This decrease at the lower levels reflects not only salary cuts, but also the low earnings of those with long periods of unemployment. In 1929, while the yearly earnings reported by the lowest 25 and 10 per cent of the engineers were less than $2,509 and $1,878, respectively, by 1934 the former had decreased to $1,473, the latter to $872. The relative percentage decreases were greatest for these 2 lower income levels, namely, 41.3 and 53.6.[6]



Income from engineering and non-engineering services including full and part-time employment, all types of education

Figure 2. Medians of earned annual income according to age, 1929, 1932, and 1934

Almost two-thirds of the decrease in earned annual incomes occurred between 1929 and 1932. This, it will be recalled, was coincident with the greatest declines in employment. There were further declines in the period 1932 to 1934. In general, the order of the absolute decreases and the percentages of change followed those which took place in the period 1929 to 1934, the one exception being that, between 1932 and 1934, there was only a 1.9 per cent decline in the lowest income levels. The corresponding absolute decrease was $17.

When these adjusted data on annual income were compared for all engineers reporting in the 5 professional classes, marked divergences in their respective earning capacities were revealed.

These differences in earning capacity within the several professional classes were greatest at the higher income levels. Thus, in 1929, nine-tenths of all electrical engineers earned $1,662 or more, while a similar proportion of the mining and metallurgical engineers earned $1,985. The range was $323. For the middle values of income, a difference of $733 was noted between the extremes of

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[6] It must be noted that errors of reporting account for part of the decline, at least in the lowest brackets for which comparison is made. The questionnaire called for earnings during the calendar years 1929 and 1934. An engineer graduating from college in either year would usually have had only a half year in which he earned. In both years, there is some evidence that annual rates were occasionally reported. Such overreporting was more common for 1929 than 1934.

$4,010 for mining and metallurgical and $3,277 for electrical engineers. However, at the highest level, one-tenth of the mining and metallurgical engineers earned $9,912 or more, whereas a corresponding proportion of civil engineers earned only $6,507 or more. Clearly, it was in the higher income levels that the earning capacities of the several professional classes diverged the most, even in terms of percentages. Furthermore, this characteristic of greater variability among the professional classes at higher levels persisted in 1929, 1932, and 1934.

However, when the question is asked as to which professional class offers the greatest earnings, or which the lowest, the answer must be carefully qualified. In the first place, the averages shown in table IV relate to the total number in the professional class. For example, 1,319 mining and metallurgical engineers reported incomes for 1929, and of these one-tenth, or 132, had incomes of $10,000 or more. There were 13,424 civil, agricultural, and architectural engineers, of whom one-tenth, or 1,342, had incomes in excess of $6,507. Therefore, it may be concluded that of every 1,000 engineers in either professional class, a larger proportion will earn $10,000 in mining and metallurgical engineering than in civil engineering. But the total number of opportunities to earn $10,000 were greater in civil engineering, much the larger of the 2 professional classes. Among the civil engineers reporting to the Bureau for 1929, there were 469 who indicated incomes of $10,000 or more in 1929.

Again, the ranking of the profession on the basis of earnings opportunity relative to the number of engineers in the professional class was not the same at all levels and in all 3 periods. As regards the relative level of income for the highest 10 per cent, and also the highest 25 per cent, in each professional class, there was a constant relationship. Thus, in 1929, 10 per cent of the mining and metallurgical engineers earned more than $9,912. Chemical and ceramic engineers ranked second with 10 per cent earning more than $9,103, and were followed in order by mechanical and industrial engineers ($8,508), electrical engineers ($7,185), and civil engineers ($6,507). Relative to the mining and metallurgical engineers, the divergence was 8, 14, 27, and 33 per cent. A similar divergence was noted between the lower limit of $6,301 reported by the upper fourth of the mining and metallurgical engineers and those of the other professional classes. In 1929, these ranged from 4 per cent in the case of chemical and ceramic

Table V. Comparison of 5 Levels of Annual Earnings on Age Basis for All Engineers Reporting in 1929, 1932, and 1934

Without regard to employment status reported or type of education

			Proportion With Annual Earnings of More Than Specified Amount (Per Cent)				
Age (Years)	Year of Graduation	Years After Graduation	10	25	50	75	90
					1929		
64 and over	Prior to 1889	41 and over	$ 9,999	$6,942	$4,427	$3,005	$1,904
56—63	1889–96	33–40	12,749	7,493	4,968	3,378	2,328
48–55	1897–1904	25–32	11,701	7,129	4,918	3,471	2,621
40–47	1905–12	17–24	9,913	6,473	4,588	3,403	2,683
36–39	1913–16	13–16	7,936	5,802	4,121	3,210	2,563
32–35	1917–20	9–12	6,520	4,850	3,674	3,004	2,448
28–31	1921–24	5–8	4,797	3,786	3,145	2,567	2,130
26–27	1925–26	3–4	3,621	3,099	2,550	2,149	1,827
24–25	1927–28	1–2	3,049	2,497	2,098	1,822	1,462
23	1929	0	2,330	1,922	1,313	882	478
					1932		
67 and over	Prior to 1889	44 and over	8,940	5,931	3,650	2,104	953
59–66	1889–96	36–43	9,318	6,167	3,959	2,462	964
51–58	1897–1904	28–35	8,350	5,753	3,832	2,525	1,219
43–50	1905–12	20–27	7,570	5,192	3,619	2,520	1,410
39–42	1913–16	16–19	6,395	4,592	3,385	2,475	1,478
35–38	1917–20	12–15	5,519	4,115	3,135	2,307	1,355
31–34	1921–24	8–11	4,287	3,386	2,677	2,025	1,229
29–30	1925–26	6–7	3,428	2,862	2,314	1,810	1,110
27–28	1927–28	4–5	2,970	2,452	2,020	1,504	876
26	1929	3	2,451	2,075	1,772	1,271	735
25	1930	2	2,139	1,898	1,540	1,007	577
24	1931	1	1,960	1,605	1,224	733	293
23	1932	0	1,673	1,097	645	322	129
					1934		
69 and over	Prior to 1889	46 and over	7,360	5,182	3,138	1,470	787
61–68	1889–96	38–45	8,280	5,391	3,497	1,800	770
53–60	1897–1904	30–37	7,720	5,264	3,502	2,165	1,157
45–52	1905–12	22–29	7,226	4,907	3,380	2,364	1,441
41–44	1913–16	18–21	6,204	4,441	3,211	2,256	1,491
37–40	1917–20	14–17	5,336	3,953	2,977	2,173	1,463
33–36	1921–24	10–13	4,259	3,334	2,569	1,958	1,358
31–32	1925–26	8–9	3,496	2,840	2,294	1,817	1,276
29–30	1927–28	6–7	3,004	2,467	2,023	1,568	1,061
28	1929	5	2,567	2,162	1,858	1,431	998
27	1930	4	2,318	1,987	1,666	1,256	865
26	1931	3	2,100	1,816	1,441	1,067	631
25	1932	2	1,952	1,581	1,275	945	495
24	1933	1	1,801	1,454	1,139	813	333
23	1934	0	1,310	927	598	299	120

Table VI. Comparison of 5 Levels of Annual Earnings for 5 Age Groups of Older* Engineers Reporting in 1929, 1932, and 1934

Without regard to employment status reported or type of education

Per Cent at Specified Income Level	Engineers, With Annual Earnings of More Than Specified Amount, Whose Age Was—														
	60 in 1929	63 in 1923	65 in 1934	38 in 1929	41 in 1932	43 in 1934	30 in 1929	33 in 1932	35 in 1934	25 in 1929	28 in 1932	30 in 1934	231/2 in 1929	26 1/2 in 1932	28 1/2 in 1934
10	$12,749	$9,318	$8,280	$7,936	$6,395	$6,204	$4,797	$4,287	$4,259	$3,049	$2,970	$3,004	$2,330	$2,451	$2,567
20	7,493	6,167	5,391	5,802	4,592	4,441	3,786	3,386	3,334	2,497	2,452	2,467	1,922	2,075	2,162
50	4,968	3,959	3,497	4,121	3,385	3,211	3,145	2,677	2,569	2,098	2,020	2,023	1,313	1,772	1,858
75	3,378	2,462	1,800	3,210	2,475	2,256	2,567	2,025	1,958	1,822	1,504	1,568	882	1,271	1,431
90	2,328	964	770	2,563	1,478	1,491	2,130	1,229	1,358	1,462	876	1,061	478	735	998
	Per Cent of Increase or Decrease														
	1929–34	1929–32	1932–34	1929–34	1929–32	1932–34	1929–34	1929–32	1932–34	1929–34	1929–32	1932–34	1929–34	1929–32	1932–34
10	−35	−27	−11	−22	−19	−3	−11	−11	− 1	− 1	− 3	+ 1	+ 10	+ 5	+ 5
25	−28	−18	−13	−23	−21	−3	−12	−11	− 2	− 1	− 2	+ 1	+ 12	+ 8	+ 4
50	−30	−20	−12	−22	−18	−5	−18	−15	− 4	− 4	− 4	0	+ 42	+35	+ 5
75	−47	−27	−27	−30	−23	−9	−24	−21	− 3	−14	−17	+ 4	+ 62	+44	+13
90	−67	−59	−20	−42	−42	+1	−36	−42	+10	−27	−40	+21	+109	+54	+36

* Includes those engineers who were professionally active prior to 1930.

engineers ($6,034) to 28 per cent for the civil engineers ($4,508). This order of professional classes was also maintained in 1932 and 1934.

In 1929, the relative order of the median professional incomes was the same as that just described, as regards mining, chemical, and mechanical engineers. But whereas the upper limit of the earnings of the lowest 25 per cent of the electrical engineers exceeded that of a similar proportion of the civil engineers by 6.6 per cent, at the median level the situation was reversed—half the civil engineers earned $3,291 or more, whereas half the electrical engineers earned $3,277 or more. This change in order persisted in 1932 and 1934.

In 1932 and 1934, the median earnings of mining and mechanical engineers led all the others. Chemical engineers were below mechanical engineers in 1932. The median earnings of chemical engineers were lower than the median of every other professional class in 1934.

At the lower levels the most marked shift in rank was that of civil engineers and chemical engineers. In 1929, one-quarter of the civil engineers earned less than $2,499, exceeding only the comparable earnings of electrical engineers. But in 1932, civil engineers were in second place as regards the level of earnings of the lowest quarter of the profession, and in 1934 they were in first place. Even in 1929, the lowest 10 per cent of the civil engineers had earned almost as much as the lowest 10 per cent of the mining and mechanical engineers, and substantially exceeded the level of the lowest 10 per cent of the chemical and electrical engineers. With reference to both the lowest 25 and 10 per cent groups, chemical engineers' earnings came in last place in 1932 and 1934.

Further examination of the adjusted data in table IV, given without regard to age, demonstrates not only differences in earning capacity but also variations in the decreases in earned annual income over the period 1929 to 1934. Without exception, the greater part of all contractions in income reported occurred between 1929 and 1932, though there were further declines in the period 1932 and 1934.

Annual Income Related to Age

The effects of age upon earned annual incomes, for all professional engineers combined are presented in table V.

This table makes it plain that the engineers' earnings advanced with age. In 1929, the income level of half of the engineers who had graduated in 1927 or 1928, or were from 24 to 25 years of age, exceeded $2,098, whereas that of half of the engineers in the age group 56–63 exceeded $4,968. Similarly, in 1932 and 1934, there was a continuous advance. But apparently age 60 represented a turning point in the average earnings of professional engineers. Furthermore, the increase in earnings with age comprised 3 distinct phases: Initial periods of exceptionally rapid rise which contain the maximum average yearly increase, followed by 2 others in which the rates of increase were progressively slower up to the respective maxima of the 5 income groups or levels. The age spans of these phases differed with the income level. Thus, for the middle and 2 lower levels of income in 1929, the maximum yearly increase of $450 was reached at the age of 25. On the other hand, the initial periods of rapid rise for the upper 25 and 10 per cent levels were not reached until the ages of 27 and 34, respectively. The subsequent periods of increase for

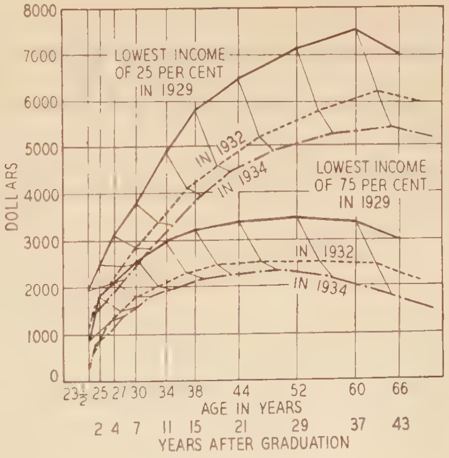

Figure 3. Upper and lower quartiles of earned annual income according to age, 1929, 1932, and 1934

Income from engineering and non-engineering services including full and part-time employment, all types of education the upper 10 per cent embraced the age spans of 34 to 44, and 44 to 60, whereas those for the next highest income level extended from 27 to 38, and 38 to 60 years of age. The 3 remaining income levels increased at practically the same rate between 25 and 34 years. But while the age span for the third phase of increase was from 34 to 52 for the middle level, it only extended from 34 to 44 in the case of the 2 lower levels of income.

There was also an increased spread of earnings with advancing age. Even in the early ages there was a fairly considerable range. Thus, in 1929, among those who graduated in 1927 or 1928, the lowest quarter earned less than $1,822, whereas the earnings of the highest quarter were about 1/3 higher than this amount, or $2,497 per annum. It is also true that the highest 10 per cent of these ages earned at least twice as much as the lowest 10 per cent. But among engineers who were about 52 in 1929, the highest-paid quarter earned at least twice as much as the lowest-paid quarter, and the highest 10 per cent earned more than 4 times as much as the lowest 10 per cent. Similar relationships existed in 1932 and 1934, although the range of increases was due in large part to the influence of unemployment on the earnings in the lower levels of income.

The spread in earnings was accentuated beyond the age of 38. At that point, the income curve of the upper 10 per cent diverged upward and continued thus to the age of 60. On the other hand, the remaining curves ran consistently parallel up to their respective maxima. For example, at the ages of 25, 44, 52, and 60 the incomes of the upper 25 per cent differed from the median by 19, 41, 45, and 51 per cent, while the order of differences for the lower 25 per cent was 13, 26, 29, and 32 per cent. By contrast, the corresponding incomes of the upper 10 per cent at these ages were greater than the median by 45, 116, 138, and 157 per cent. Clearly, beyond the age of 44, the earned annual incomes reported in 1929 by the upper 10 per cent of all engineers differed very greatly from those in the other income levels, and this advantage in earning capacity was maintained in the years 1932 and 1934 also.

The earnings of engineers in the lower income brackets ceased to increase at a relatively early age. It is only approximately correct to assume that ability and income are in direct proportion. By and large, however, it is perhaps safe to assume that the engineers at any age in the lowest 10 per cent income group are less able than the average at that age; and that the highest 10 per cent are substantially more able than the average. The Bureau's data reveal that the maximum earnings of the lowest 10 and 25 per cent were reached in 1929 at about 44 years of age, and the average engineer reached his maximum earnings at about 52. The same relationship held in 1932 and 1934.

In the 2 later years the maximum for the average engineer occurred at about 60, but it is evident that earning power for engineers above the average continued to rise beyond this age. Thus, the average earnings of engineers 53 to 60 years of age in 1934 exceeded $3,502, whereas the average earnings of those 61 to 68 years were more than $3,497. But the highest 10 per cent of the first group earned over $7,720, whereas the same class of the second group earned more than $8,280.

Table V discloses differences in the age-income cycles beyond the points of maximum earnings, that is, at the end of the third phases of increase. In any case of the 2 higher income levels, the fourth and last phase in 1929 was one of decrease. This was not so for the 3 lower income levels. For them, there was a fourth phase of no change in income which covered a span of approximately 8 years: 52 to 60 for the middle group, and from 44 to 52 in the case of the 2 lower levels. The fifth phases were ones of decrease. Relatively, however, the steepest declines occurred in the 2 higher income levels.

Changes in income brought about by the depression in various years may be considered from the point of view of particular individuals whose age and experience was increasing, or from the point of view of the expectations of men with comparable periods of experience. For example, the average earnings of engineers who graduated in

Table VII. Comparison of 5 Levels of Annual Earnings for Corresponding Years After Graduation in 1929, 1932, and 1934

Without regard to employment status reported or type of education

Age of Engineers (Years)	Years After Graduation	Proportion With Annual Earnings of More Than Specified Amount (Per Cent)														
		10			25			50			75			90		
		1929	1932	1934	1929	1932	1934	1929	1932	1934	1929	1932	1934	1929	1932	1934
23½	½	$ 2,330	$1,673	$1,310	$1,922	$1,097	$ 927	$1,313	$ 645	$ 598	$ 882	$ 322	$ 299	$ 478	$ 129	$ 120
25	2	3,049	2,150	1,880	2,497	1,750	1,500	2,098	1,399	1,190	1,822	840	860	1,462	420	410
28	5	3,980	2,970	2,410	3,300	2,452	2,080	2,710	2,020	1,750	2,275	1,504	1,340	1,925	876	940
33	10	6,000	4,287	3,700	4,560	3,386	2,990	3,500	2,677	2,380	2,890	2,025	1,850	2,375	1,229	1,299
43	20	9,450	6,680	6,204	6,350	4,750	4,441	4,450	3,420	3,211	3,380	2,490	2,256	2,650	1,460	1,491
53	30	11,700	8,000	7,450	7,190	5,580	5,050	4,900	3,780	3,420	3,430	2,520	2,240	2,590	1,285	1,286
60	37	12,749	8,900	7,900	7,493	5,960	5,250	4,968	3,900	3,500	3,378	2,490	2,010	2,328	1,050	980
		Per Cent of Increase or Decrease														
		1929–34	1929–32	1932–34	1929–34	1929–32	1932–34	1929–34	1929–32	1932–34	1929–34	1929–32	1932–34	1929–34	1929–32	1932–34
23½	½	−44	−28	−22	−52	− 43	−15	−54	−51	− 7	−66	−63	− 1	−75	−73	−1
25	2	−38	−29	−13	−40	−30	−14	−43	−33	−13	−53	−54	+ 2	−72	−71	−2
28	5	−39	−25	−19	−37	−26	−15	−35	−25	−13	−41	−34	−11	−51	−54	+7
33	10	−38	−29	−14	−34	−26	−12	−32	−24	−11	−36	−30	− 9	−45	−48	+6
43	20	−34	−29	− 7	−30	−25	− 7	−28	−23	− 6	−33	−26	− 9	−44	−45	+2
53	30	−36	−32	− 7	−30	−22	− 9	−30	−23	−10	−35	−27	−11	−50	−50	0
60	37	−38	−30	−11	−30	−20	−12	−30	−21	−10	−40	−26	−19	−58	−55	−7

Table VIII. Comparison of 5 Levels of Earned Annual Income for Older,* Graduate,** and "Other"† Engineers Reporting in Each Professional Class at End of 1929, 1932, and 1934

Without regard to employment status reported

Per Cent of Professional Class‡ at Specified Income Level	Annual Earnings of More Than Specified Amount: Graduate Engineers**			Annual Earnings of More Than Specified Amount: "Other" Engineers†			Per Cent Income of "Other" Engineers Formed of That of Graduate Engineers—		
	1929	1932	1934	1929	1932	1934	1929	1932	1934
10 per cent:									
Mining and metallurgical	$10,015	$7,413	$7,530	$8,940	$6,370	$6,263	89	86	83
Chemical and ceramic	9,173	7,432	7,414	8,100	6,435	6,795	88	87	92
Mechanical and industrial	8,715	6,481	6,269	8,161	6,273	5,997	94	97	96
Electrical	7,301	6,263	6,084	6,654	5,943	5,670	91	95	93
Civil, agricultural, and architectural	6,853	5,478	5,133	5,745	4,470	4,222	84	82	82
25 per cent:									
Mining and metallurgical	6,349	4,973	4,833	6,025	4,640	4,587	95	93	95
Chemical and ceramic	6,111	5,116	4,976	5,210	4,540	4,335	85	89	87
Mechanical and industrial	5,603	4,414	4,262	5,536	4,269	3,972	99	97	93
Electrical	4,886	4,175	4,062	4,497	3,762	3,654	92	90	90
Civil, agricultural, and architectural	4,732	3,923	3,671	4,036	3,349	3,163	85	85	86
50 per cent:									
Mining and metallurgical	4,053	3,299	3,227	3,762	3,082	3,072	93	93	95
Chemical and ceramic	3,839	3,368	3,308	3,525	3,050	2,857	92	91	86
Mechanical and industrial	3,663	3,025	2,848	3,777	2,929	2,645	103	97	93
Civil, agricultural, and architectural	3,375	2,834	2,636	3,099	2,530	2,362	92	89	90
Electrical	3,296	2,863	2,821	3,213	2,627	2,517	97	92	89
75 per cent:									
Mining and metallurgical	2,841	2,013	2,029	2,829	2,013	1,947	99	100	96
Mechanical and industrial	2,556	1,981	1,952	2,902	1,930	1,837	114	97	94
Civil, agricultural, and architectural	2,541	2,009	1,956	2,429	1,860	1,747	96	93	89
Chemical and ceramic	2,538	2,178	2,195	2,540	1,885	1,834	100	87	84
Electrical	2,317	1,997	2,003	2,400	1,883	1,792	104	94	89
90 per cent:									
Mining and metallurgical	1,977	1,052	1,279	2,030	1,210	1,283	103	115	100
Civil, agricultural, and architectural	1,935	1,136	1,361	1,910	1,028	1,152	99	90	85
Mechanical and industrial	1,888	1,212	1,245	2,170	1,040	1,169	115	86	94
Chemical and ceramic	1,657	1,443	1,458	1,820	699	783	110	48	54
Electrical	1,609	1,311	1,308	1,860	1,185	1,088	116	90	83

* Includes all engineers who were professionally active prior to 1930. ** Graduate engineers embrace all postgraduates, nonengineering graduates, and first degree engineering graduates. † "Other" engineers embrace all engineers with college course incomplete, noncollegiate technical school course, and secondary school education. ‡ Arranged in ascension of graduate earned annual income for 1929.

1927–28 declined only from $2,098 in 1929 to $2,020 in 1932, and increased slightly to $2,023 in 1934. On the other hand, those who graduated in 1897–1904 averaged $4,918 in 1929, $3,832 in 1932, and $3,502 in 1934. Such comparisons for each group of engineers over the period 1929–34 may be made from table V. They are, more conveniently arranged for direct comparison in table VI.

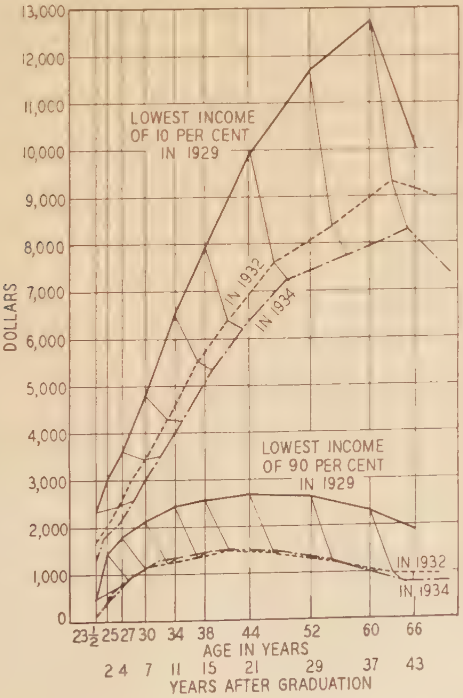

Figure 4. Upper and lower deciles of earned annual income according to age, 1929, 1932, and 1934

Income from engineering and nonengineering services including full and part-time employment, all types of education

This table shows a rise in earnings for the youngest engineers in the profession in 1929, who were near to 30 years of age in 1934. The advance in earning capacity in the first 5 years of engineering experience was so great that it offset the influence of the depression in the case of the youngest engineers. At higher ages, when an added year's experience influenced income less, the incomes of particular engineers declined by as much as 30 per cent for the average graduate of the classes of 1889–96.

In order to trace the influence of the depression on professional opportunity and on the normal expectations of members of the profession, table VII compares the earnings for identical ages in each of the 3 years.[7]

For each level of experience, incomes declined from 1929 to 1934, and two-thirds to three-quarters of this decline occurred from 1929 to 1932. Over the entire period the average income of those who had been out of college for 2 years declined 43 per cent. Those who had been out 5 years had, on the average, 35 per cent less income in 1934 than the corresponding group in 1929. For older engineers the decline approximated 30 per cent, being slightly more than this for those with 10 years' experience, and slightly less for those with 20 years' experience.

The most significant differences brought about by the depression were in the spread of incomes at various ages.

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[7] The figures are not derived from direct tabulations which were made on the basis of the same combinations of years of birth or graduation in each of the 3 years, but the movements of earnings proved to be sufficiently regular to justify reading the values for particular ages from the chart.

In all cases, primarily because of the influence of unemployment, annual income for the lowest 25 per cent and the lowest 10 per cent declined more than the average income at a given age. Thus, 2 years after graduation, 10 per cent of the engineers earned less than $1,462 in 1929 as compared with a corresponding group earning less than $410 in 1934. This was a decline of 72 per cent as compared with a 43 per cent decline for the average of this age. Similarly, the average for engineers 30 years after graduation declined 30 per cent from 1929 to 1934, while the level below which the earnings of 10 per cent of such engineers were found fell by 50 per cent.

There was comparatively little difference for the various ages in the decline of the median income or in the decline of the level of income above which only 25 per cent of the engineers of corresponding ages were found. The highest level of income (that achieved by only 10 per cent at each age) declined more than the average in the case of all groups of engineers with 5 years' or more experience. Thus this select group among engineers 60 years of age lost 38 per cent from 1929 to 1934, whereas the average decline at that age was 30 per cent.

Annual Income and Education

The foregoing analysis of annual incomes reported by professional engineers for 1929, 1932, and 1934 took no account of differences in educational background. In 1929, some 22,386, or 86 per cent, of the 24,837 older graduates in engineering courses in the sample reported income and 7,646, or 90 per cent, of the 8,440 older "other" engineers who were in the profession in 1929 reported. The 5 income levels for these 2 groups are shown in table VIII.

Consideration of the ratios derived from the 1929 earnings reported by the "other" and graduate engineers clearly indicates that those with a formal engineering education had higher incomes. Thus, in the highest 10 and 25 per cent income groups, the earnings of the graduates in each professional class exceeded those of the "other" engineers. It will, however, be noted that there were wide variations in the ratios in the earnings of the 2 groups, indicating that the differentials in earnings do not accrue in equal measure for all 5 professional classes. In the upper 25 per cent level, the order of yearly differences in favor of the graduates was $901, $696, $389, $324, and $67, the 3 smallest of these being in the electrical, mining and metallurgical, and mechanical and industrial classes. In the highest income level, differences of $647 and $554 per year were reported by the graduate electrical and mechanical and industrial engineers; in the 3 remaining professional classes none of the graduate groups reported differentials of less than $1,000 per year. In general, from 1929 to 1934, the earnings of the upper 25 per cent of the older graduates and of the "other" engineers declined by similar amounts.

The advantage enjoyed by engineers with college degrees is less marked at the lower levels. The average income of mechanical engineers in the profession in 1929 reporting income for 1929 was actually higher for noncollege graduates than for graduates. This was the only professional class of which this was true for the average, but, in general, the lowest 25 per cent and the lowest 10 per cent among the "other" engineers had higher incomes than among the graduates. But even at these levels, it will be seen that an advantage accrued over the period 1929 to 1934 to the college graduates. The income of the lowest 25 per cent of those graduating prior to 1929 declined less than the income of the lowest 25 per cent of the corresponding group of "other" engineers and by 1934, college graduates had the higher incomes even at this level.

The apparent advantage enjoyed in 1929 by the "other" engineers at the lower levels of income may best be explained in conjunction with the data in tables IX and X. The data for the median values[8] only of earned annual income of all engineers classified by age in the 3 years, 1929, 1932, and 1934 are presented in table IX.[9]

The first observation to be made has regard to the fact that the "other" engineers, at a very early age, lose the advantage in earning capacity, arising from practical experience gained, while the college graduate is in school. This loss of advantage is best exemplified by considering the ranking order of the 12 groups of engineers at corresponding years after graduation (table X).

At the end of 2 years after graduation[10] in 1929, the "other" engineers held ranks 1, 2, 4, 5, and 6, the first-degree civil graduates were third, and the remaining graduate groups occupied positions 7 to 12, inclusive. Five years after graduation, however, there was a complete reversal of this situation, which placed all graduates ahead of the "other" engineers.

Relatively, the greatest shifts in position occurred between the second and fifth years after graduation. Secondary-school engineers dropped from sixth to twelfth place. The noncollegiate civil engineers declined from second to eleventh place, while the civil engineers whose college course was incomplete declined from fifth to tenth place. Positions 8 and 9, respectively, were occupied by the mechanical engineers whose college course was incomplete and those who were graduates of noncollegiate technical schools; 2 years after graduation they had held first and fourth places. Among the engineering graduates, the greatest shift in position occurred among the first-degree chemical and ceramic engineers, who moved from tenth to first place at 5 years after graduation, and maintained that position throughout the 1929 age cycle. It will also be noted that the 1929 earnings reported by the first-degree civil engineers were greater at 5 years after graduation than those of both postgraduates and first-degree electrical engineers. On the other hand, the earnings of these 3 groups were less than those reported by first-degree mining and metallurgical, and mechanical and industrial engineers, and nonengineering graduates.

Thus, it follows that the greater decline from 1929 to 1934 in the incomes of the lower 10 and 25 per cent of the "other" engineers reflects in part the greater advantage of

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[8] I.e., the middle value of earned annual income—50 per cent earning more and 50 per cent earning less than the figure shown.

[9] Insufficient data were furnished for the 2 lower and higher income levels to enable a complete comparison for all age groups and all types of education.

[10] The incomes reported just after graduation are not compared. These are invalidated by the fact that, while the graduates could have reported incomes for 6 months only, it was possible for the "other" engineers to have reported an income for a full year or more.

Table IX. Median Annual Earnings in 1929, 1932, and 1934, for All Engineers Reporting by Age and Type of Education

Without regard to employment status reported

Age (Years)	Year of Graduation	Years After Graduation	Postgraduates	Nonengineering Graduates	First-Degree Engineering Graduates					Others With—				Secondary School Education
										College Course Incomplete		Noncollegiate Technical Course		
					Chemical and Ceramic	Civil, Agricultural, and Architectural	Electrical	Mechanical and Industrial	Mining and Metallurgical	Civil, Agricultural, and Architectural	Mechanical and Others	Civil, agricultural, and architectural	Mechanical and Others	
			1929 Income (Dollars)											
64 and over	*	41+	**	6,000	**	4,460	**	5,100	4,800	3,400	5,700	**	**	3,800
56-63	1889-96	33-40	6,030	5,500	6,450	4,981	5,580	6,255	6,180	3,733	4,933	3,657	4,440	4,120
48-55	1897-1904	25-32	5,320	5,663	6,600	4,804	5,460	5,793	5,200	3,774	5,179	3,514	4,714	4,333
40-47	1905-12	17-24	5,000	5,000	6,429	4,411	5,239	5,434	5,400	3,565	4,579	3,500	4,246	3,708
36-39	1913-16	13-16	4,286	4,933	6,021	3,964	4,480	4,878	4,825	3,328	4,031	3,168	3,889	3,625
32-35	1917-20	9-12	4,608	4,086	4,523	3,616	4,048	4,057	4,600	3,036	3,793	3,121	3,448	3,267
28-31	1921-24	5-8	3,164	3,072	3,544	3,122	3,190	3,354	3,306	2,614	3,200	2,574	2,925	2,814
26-27	1925-26	3-4	2,565	2,675	2,729	2,579	2,505	2,616	2,840	2,410	2,468	2,400	2,482	2,250
24-25	1927-28	1-2	2,008	1,978	2,007	2,150	1,977	2,099	2,086	2,127	2,232	2,200	2,144	2,100
23	1929	0	1,040	1,500	1,200	1,256	996	1,146	1,175	2,062	2,129	1,780	1,950	1,900
			1932 Income (Dollars)											
67 and over	*	44+	**	3,000	**	3,600	**	5,200	1,100	2,600	4,600	**	**	3,467
59-66	1889-96	36-43	5,160	4,600	**	4,090	4,733	4,700	5,000	2,950	3,600	2,933	3,400	3,433
51-58	1897-1904	28-35	4,757	5,000	5,200	3,761	4,677	4,189	3,940	2,969	3,688	2,846	3,550	3,650
43-50	1905-12	20-27	4,155	4,475	5,320	3,533	4,214	4,070	4,222	3,038	3,631	2,777	3,241	3,178
39-42	1913-16	16-19	3,745	4,222	4,657	3,318	3,877	3,800	3,700	2,702	3,162	2,709	2,968	3,173
35-38	1917-20	12-15	3,377	3,600	3,773	3,048	3,509	3,470	3,444	2,518	3,029	2,475	2,682	2,720
31-34	1921-24	8-11	2,882	2,945	3,167	2,628	2,807	2,861	2,744	2,406	2,563	2,173	2,310	2,488
29-30	1925-26	6-7	2,472	2,655	2,580	2,343	2,309	2,403	2,200	2,090	2,105	1,971	2,000	2,057
27-28	1927-28	4-5	2,029	2,167	2,064	2,124	1,992	1,985	1,940	1,932	1,906	1,836	1,686	1,840
26	1929	3	1,794	1,800	1,773	1,896	1,660	1,716	1,560	1,657	1,638	1,900	1,600	1,350
25	1930	2	1,378	1,575	1,628	1,709	1,451	1,495	1,492	1,709	1,373	1,700	1,450	1,633
24	1931	1	948	1,367	1,208	1,349	1,032	1,247	1,147	1,620	1,390	**	1,600	1,700
23	1932	0	687	1,800	606	629	586	645	480	1,433	1,183	**	1,400	**
			1934 Income (Dollars)											
69 and over	*	46+	**	3,100	**	3,200	**	3,933	1,200	2,000	3,400	**	2,000	3,100
61-68	1889-96	38-45	4,333	4,400	**	3,567	4,143	4,100	4,333	2,800	2,900	2,600	2,460	3,133
53-60	1897-1904	30-37	4,488	4,400	5,100	3,337	4,400	3,731	3,900	2,787	3,493	2,429	3,364	3,508
45-52	1905-12	22-29	3,923	4,088	5,000	3,311	4,089	3,875	4,086	2,780	3,497	2,533	2,974	2,940
41-44	1913-16	18-21	3,517	4,467	4,111	3,129	3,917	3,538	3,588	2,496	2,990	2,420	2,713	3,031
37-40	1917-20	14-19	3,250	3,575	3,850	2,826	3,393	3,278	3,567	2,354	2,890	2,286	2,543	2,711
33-36	1921-24	10-13	2,864	2,857	3,244	2,516	2,801	2,704	2,811	2,147	2,432	2,036	2,161	2,356
31-32	1925-26	8-9	2,513	2,475	2,586	2,263	2,364	2,378	2,463	1,974	2,093	1,863	1,957	1,900
29-30	1927-28	6-7	2,072	2,200	2,225	2,069	2,021	2,058	2,100	1,875	1,911	1,567	1,738	1,860
28	1929	5	1,888	1,871	1,982	1,892	1,759	1,908	1,809	1,678	1,808	1,867	1,525	1,575
27	1930	4	1,563	1,800	1,795	1,762	1,527	1,719	1,542	1,518	1,500	1,800	1,433	1,660
26	1931	3	1,437	1,457	1,426	1,545	1,263	1,433	1,479	1,567	1,633	1,400	1,467	1,800
25	1932	2	1,255	1,233	1,286	1,384	1,138	1,261	1,241	1,388	1,400	**	1,367	**
24	1933	1	840	1,314	1,162	1,249	1,047	1,126	1,159	1,283	1,267	**	1,600	**
23	1934	0	612	1,000	567	638	540	560	564	1,200	1,089	**	1,263	1,325

* Prior to 1889. ** Less than 10 persons reported.

5 years additional experience among the younger college graduates.

Between 5 and 10 years after graduation, there were but slight changes in relative position. At the latter period, however, all first-degree and nonengineering graduates were ahead of the postgraduate engineers. The secondary-school engineers reported earnings greater than either of the 2 groups of "other" civil engineers. The "other" mechanical engineers trained in noncollegiate technical schools followed ninth in order, after the postgraduates.

At 20 years after graduation, the first-degree civil engineers ranked below both the "other" mechanical engineers whose college course was incomplete and the postgraduate engineers; between 20 and 37 years after graduation, the relative standing of the several groups remained comparatively stable. It will also be noted that even in 1934 the order of the groups shows no marked departure from the situation which prevailed in 1929.

A further explanation may be given of the apparent advantage of "other" engineers in the lower income levels, as shown in table VIII. There are too few cases to warrant showing text tables of income for the lowest 10 and 25 per cent of the engineers classified simultaneously by professional classifications, age, and type of education.

Table X. Rank of Engineers of Specified Education According to Median Annual Earnings in 1929 and 1934, for Corresponding Years After Graduation

Without regard to employment status reported

	Years After Graduation											
Type of Education	2	5	10	20	30	37	2	5	10	20	30	37
	Rank in 1929						Annual Earnings in 1929					
College course incomplete: Mechanical and others*	1	8	6	7	7	8	$2,232	$2,650	$3,580	$4,425	$5,100	$4,933
Noncollegiate technical course: Civil, agricultural, and architectural	2	11	11	12	12	12	2,200	2,450	2,950	3,425	3,550	3,657
First-degree engineering graduates: Civil, agricultural, and architectural	3	5	7	8	8	7	2,150	2,750	3,490	4,300	4,800	4,981
Noncollegiate technical course: Mechanical and others*	4	9	9	9	9	9	2,144	2,600	3,300	4,190	4,650	4,440
College course incomplete: Civil, agricultural, and architectural	5	10	12	11	11	11	2,127	2,475	2,900	3,500	3,750	3,733
Secondary school education	6	12	10	10	10	10	2,100	2,430	3,150	3,700	4,275	4,120
First-degree engineering graduates:												
Mechanical and industrial	7	3	3	2	2	2	2,099	2,825	3,875	5,350	5,800	6,255
Mining and metallurgical	8	2	2	3	6	3	2,086	2,975	4,200	5,300	5,350	6,180
Postgraduates	9	6	8	6	5	4	2,008	2,750	3,475	4,850	5,375	6,030
First-degree engineering graduates: Chemical and ceramic	10	1	1	1	1	1	2,007	3,000	4,250	6,350	6,600	6,450
Nonengineering graduates	11	4	5	5	3	6	1,978	2,800	3,800	4,990	5,600	5,500
First-degree engineering graduates: Electrical	12	7	4	4	4	5	1,977	2,725	3,800	5,075	5,450	5,580
	Rank in 1934						Annual Earnings in 1934					
Secondary school education	1	9	9	8	9	8	$1,550	$1,625	$2,025	$3,031	$3,200	$3,350
Noncollegiate technical course: Mechanical and others*	2	12	10	10	10	10	1,475	1,475	2,010	2,713	3,150	2,980
College course incomplete: Mechanical and others*	3	10	8	9	7	9	1,350	1,600	2,200	2,990	3,490	3,200
Noncollegiate technical course: Civil, agricultural, and architectural	4	2	12	12	12	12	**	1,830	1,900	2,420	2,490	2,490
First-degree engineering graduates: Civil, agricultural, and architectural	5	5	7	7	8	7	1,325	1,800	2,350	3,129	3,300	3,400
College course incomplete: Civil, agricultural, and architectural	6	11	11	11	11	11	1,300	1,600	2,000	2,496	2,780	2,790
Nonengineering graduates	7	3	3	1	2	3	1,275	1,825	2,600	4,467	4,250	4,400
First-degree engineering graduates:												
Chemical and ceramic	8	1	1	2	1	1	1,250	1,875	2,750	4,111	5,050	**
Mining and metallurgical	9	7	4	4	5	5	1,200	1,700	2,550	3,588	3,980	4,050
Mechanical and industrial	10	4	6	5	6	6	1,180	1,825	2,490	3,538	3,780	3,880
Electrical	11	8	5	3	3	4	1,080	1,650	2,500	3,917	4,250	4,325
Postgraduates	12	6	2	6	4	2	940	1,750	2,610	3,517	4,175	4,425

* Includes chemical, ceramic, electrical, industrial, mining, and metallurgical engineers.
** Less than 10 engineers reported.

But such values have been computed for every such classification embracing more than 100 reports and the results may be summarized. In 1929, at every age the lowest 10 per cent of the civil engineers with incomplete college courses had lower incomes than the lowest 10 per cent of those with completed college courses. This continued to be true in 1934 for engineers who graduated prior to 1928, the last year with an adequate number of reports to warrant this particular comparison. For the 5 age groups from 28 to 47, for which comparisons can be made, the lowest 10 per cent of civil engineers with noncollegiate technical school courses had lower incomes than those with incomplete college courses. These statements hold, not only for the lowest 10 per cent of the civil engineers but also for the lowest 25 per cent. There are not enough cases of engineers in the professional classes, other than civil engineers, to warrant a detailed analysis of nongraduates on an age basis for the separate classes. But the income of the lowest 10 and 25 per cent of the nongraduates of the 4 classes combined is less than the corresponding level of income for graduates in any of the professional classes at almost all ages for which comparisons can be made. It seems highly probable, in the light of these facts, that part of the advantage shown by "other" engineers in table VIII is due to a higher age among the nongraduates in the groups compared.

Table X also shows that, although the earnings reported in 1929 by the "other" engineers at 2 years after graduation were higher than those of the graduates, the differentials were not great. For the former, the 1929 median earnings ranged from $2,100 in the case of secondary-school engineers to $2,232 for mechanical engineers who had not completed their college course, while the range for the latter was from $1,977 for electrical to $2,099 for mechanical and industrial engineers. At 5 years after graduation, when the positions were reversed, the differentials were still slight. The earnings of the graduates ranged from $2,725 to $3,000 per year and those of the "other" engineers from $2,430 to $2,650. With advancing age, however, the spreads in earnings in favor of the graduates became very marked. In the case of mechanical engineers, for instance, the difference in earnings between the first-degree engineers and those who did not complete their college course was $175, $295, $925, $700, and $1,322 per year, and between first-degree and noncollegiate mechanical engineers it was $225, $575, $1,160 $1,150, and $1,815 per annum. Similarly, for the civil engineers, the difference in earnings between those with first-degrees and whose college course was incomplete was $275, $590, $800, $1,050, and $1,248 per year, while between first-degree civil and noncollegiate technical school engineers, the order was $300, $540, $875, $1,250, and $1,324 per annum.

Even among the graduate groups there was variation in earning capacity. Thus, while the earnings of first-degree civil engineers ranged from $2,750 to $4,800 per year between 5 and 30 years after graduation, the range for the first-degree chemical and ceramic engineers was from

$3,000 to $6,600 per year. In other words, over a period of 25 years, the civil engineers' earnings increased by only $2,050, whereas those of engineers in the chemical and ceramic field increased by $3,600. The ranges in earnings of the remaining graduate groups fell between those reported by the civil and the chemical and ceramic engineers.

Earnings of the "other" engineers cease to increase several years before those of the graduates. Their earnings began to decline after 55 years of age in 1929, whereas the earnings of the graduates continued to increase even at 64 years of age and over (table IX).

When consideration is given to the changes in income status between 1929 and 1934 of selected age groups of engineers in each type of education, it appears, again, that the depression bore hardest upon the older engineers.

As indicated in table XI, over the period 1929–34 the decreases in earnings of engineers who were 60 years of age in 1929 and 65 in 1934 ranged from 20 to 45 per cent. The smallest range, however, occurred among the graduate engineers. This is explained by the fact that for the "other" engineers, the earnings reported at the age of 60 in 1929 were those for the period of decline, since they were less than those reported for engineers who were 52 in 1929. When, however, comparison is made of the earnings for the 2 age groups of engineers who were 52 and 44, respectively, in 1929 and 5 years older in 1934, it will be noted that the decreases in their earnings over the period 1929–34 show little variation. The important thing to note is that the effect of the depression was approximately the same on both graduates and "other" engineers. On the other hand, for the 2 younger groups shown in table XI, the graduate engineers who were 25 in 1929, practically all showed increases in their earnings by 1934, whereas the "other" engineers showed further decreases. This situation was even more pronounced in favor of the graduate engineers who were $23^{1}/_{2}$ years of age in 1929.

Throughout the whole of this analysis of differences in earnings by type of education, there have only been incidental references to the changes which occurred over the period 1929–34 and in the intervening periods, 1929–32 and 1932–34. It will be recalled, however, that the percentage decreases in the incomes for all engineers at corresponding years after graduation were practically the same. This was also the case for the 12 groups of engineers when segregated by type of education, as is evidenced by a consideration of the data shown in table XII.

Table XI. Comparison of Median Annual Earnings in 1929, 1932, and 1934 of Selected Age Groups of Engineers Reporting by Type of Education

Without regard to employment status reported

	Engineers Whose Age Was—														
Type of Education	60 in 1929	63 in 1932	65 in 1934	52 in 1929	55 in 1932	57 in 1934	44 in 1929	47 in 1932	49 in 1934	25 in 1929	28 in 1932	30 in 1934	$23^{1}/_{2}$ in 1929	$26^{1}/_{2}$ in 1932	$28^{1}/_{2}$ in 1934
	Median Annual Earnings														
Postgraduates	$6,030	$5,160	$4,333	$5,320	$4,757	$4,488	$5,000	$4,155	$3,923	$2,008	$2,029	$2,072	$1,040	$1,794	$1,888
Nonengineering graduates	5,500	4,600	4,400	5,663	5,000	4,400	5,000	4,475	4,088	1,978	2,167	2,200	1,500	1,800	1,871
First-degree engineering graduates:															
Chemical and ceramic	6,450	**	**	6,600	5,200	5,100	6,429	5,320	5,000	2,007	2,064	2,225	1,200	1,773	1,982
Civil, agricultural and architectural	4,981	4,090	3,567	4,804	3,761	3,337	4,411	3,533	3,311	2,150	2,124	2,069	1,256	1,896	1,892
Electrical	5,580	4,733	4,143	5,460	4,677	4,400	5,239	4,214	4,089	1,977	1,992	2,021	996	1,660	1,759
Mechanical and industrial	6,255	4,700	4,100	5,793	4,189	3,731	5,434	4,070	3,875	2,099	1,985	2,058	1,146	1,716	1,908
Mining and metallurgical	6,180	5,000	4,333	5,200	3,940	3,900	5,400	4,222	4,086	2,086	1,940	2,100	1,175	1,560	1,809
College course incomplete:															
Civil, agricultural, and architectural	3,733	2,950	2,800	3,774	2,969	2,787	3,565	3,038	2,780	2,127	1,932	1,875	2,062	1,657	1,678
Mechanical and others*	4,933	3,600	2,900	5,179	3,688	3,493	4,579	3,631	3,497	2,232	1,906	1,911	2,129	1,638	1,808
Noncollegiate technical course:															
Civil, agricultural, and architectural	3,657	2,933	2,600	3,514	2,846	2,429	3,500	2,777	2,533	2,200	1,836	1,567	1,780	1,900	1,867
Mechanical and others*	4,440	3,400	2,460	4,714	3,550	3,364	4,246	3,241	2,974	2,144	1,686	1,738	1,950	1,600	1,525
Secondary school education	4,120	3,433	3,133	4,333	3,650	3,508	3,708	3,178	2,940	2,100	1,840	1,860	1,900	1,350	1,575
	Per Cent of Change														
	1929–34	1929–32	1932–34	1929–34	1929–32	1932–34	1929–34	1929–32	1932–34	1929–34	1929–32	1932–34	1929–34	1929–32	1932–34
Postgraduates	−28	−14	−16	−16	−11	− 6	−22	−17	−6	+ 3	+ 1	+ 2	+82	+73	+ 5
Nonengineering graduates	−20	−16	− 4	−22	−12	−12	−18	−11	−9	+11	+10	+ 2	+25	+20	+ 4
First-degree engineering graduates:															
Chemical and ceramic	**	**	**	−23	−21	− 2	−22	−17	−6	+11	+ 3	+ 8	+65	+43	+12
Civil, agricultural, and architectural	−28	−18	−13	−31	−22	−11	−25	−20	−6	− 4	− 1	− 3	+51	+51	0
Electrical	−26	−15	−12	−19	−14	− 6	−22	−20	−3	+ 2	+ 1	+ 1	+77	+67	+ 6
Mechanical and industrial	−34	−25	−13	−36	−28	−11	−29	−25	−5	− 2	− 5	+ 4	+66	+50	+11
Mining and metallurgical	−30	−19	−13	−25	−24	− 1	−24	−22	−3	+ 1	− 7	+ 8	+54	+33	+16
College course incomplete:															
Civil, agricultural, and architectural	−25	−21	− 5	−26	−21	− 6	−22	−15	−8	−12	− 9	− 3	−19	−20	+ 1
Mechanical and others*	−41	−27	−19	−33	−29	− 5	−24	−21	−4	−14	−15	0	−15	−23	+10
Noncollegiate technical course:															
Civil, agricultural, and architectural	−29	−20	−11	−31	−19	−15	−28	−21	−9	−29	−17	−15	+ 5	+ 7	− 2
Mechanical and others*	−45	−23	−28	−29	−25	− 5	−30	−24	−8	−19	−21	+ 3	−22	−18	− 5
Secondary school education	−24	−17	− 9	−19	−16	− 4	−21	−14	−7	−11	−12	+ 1	−17	−29	+17

* Includes chemical, ceramic, electrical, industrial, mining, and metallurgical engineers.

** Less than 10 engineers reported.

At 5 years after graduation the range of decreases of earnings over the period 1929–34 was from 25 to 43 per cent. At 10 years after graduation, the range was from 25 to 39 per cent, while even at 30 years after graduation the percentage decreases ranged only from 22 to 35 per cent. It will, however, be noted that the extremes of the ranges are the exception, indicating that regardless of type of education, the incomes of engineers of identical ages in 1929 and 1934 suffered about the same from the depression.

It cannot be too strongly emphasized that these data relate only to what engineers of various ages were earning in 1929, 1932, and 1934. Since there are no better data available it is almost inevitable that the figures will be used to predict what young engineers may expect to earn 10 or 15 years hence. For that reason, it is important to emphasize the severe limitations which attach to the data in this connection.

The first point—that the general level of engineering income fluctuates from year to year—needs merely to be mentioned in passing. Therefore, the absolute level of incomes for engineers with any given amount of experience cannot be forecast for any future year.

The chief danger to be guarded against is the assumption that the income relationships for 1934 will hold in some future year. A cautious use of such information may add to the value of the advice of those who are directing young men into the various fields of specialization. A careless assumption that this same relationship will hold 20 years hence for future graduates will make the resulting advice dangerous.

Table XII. Percentage Decreases in Median Annual Earnings Over Period 1929–34, for Corresponding Years After Graduation Among Engineers Reporting by Type of Education

Without regard to employment status reported

Type of Education	Per Cent of Decrease in Income, at End of Specified Period After Graduation					
	2 Years	5 Years	10 Years	20 Years	30 Years	37 Years
Postgraduates	—53	—36	—25	—27	—22	—27
Nonengineering graduates	—36	—35	—32	—10	—24	—20
First-degree engineering graduates:						
Chemical and ceramic	—38	—38	—35	—35	—23	**
Civil, agricultural, and architectural	—38	—35	—33	—27	—31	—32
Electrical	—45	—39	—34	—23	—22	—22
Mechanical and industrial	—44	—35	—36	—34	—35	—38
Mining and metallurgical	—42	—43	—39	—32	—26	—34
College course incomplete:						
Civil, agricultural, and architectural	—39	—35	—31	—29	—26	—25
Mechanical and others*	—40	—40	—39	—32	—32	—35
Noncollegiate technical course:						
Civil, agricultural, and architectural	**	—25	—36	—29	—30	—32
Mechanical and others*	—31	—43	—39	—35	—32	—33
Secondary-school education	—26	—33	—36	—18	—25	—19

* Includes chemical, ceramic, electrical, industrial, mining, and metallurgical engineers.

** Less than 10 engineers reported in 1934.

The tabulations show merely the facts of the income distribution in 1934 or some other particular year. For example, chemical and ceramic engineers 20 years after graduation averaged $4,100 and one-quarter of such engineers earned more than $6,000. On the contrary, civil engineers 20 years after graduation averaged $3,100 and the best-paid quarter averaged only $4,100 or more. It is evident, therefore, that men graduating from college in 1914 advanced on the whole to higher levels of income if their college work had been in the field of chemical or ceramic engineering rather than if it had been in the field of civil engineering. This much is fact.

But it must not be concluded from these data on income alone that it is wise to encourage men entering college in 1937 to specialize in chemical and ceramic engineering and to discourage their entering the field of civil engineering. Such advice will be sound only if the conditions surrounding the two fields of engineering and their prospects for the next 20 years are similar to the conditions of 1914 to 1934. Years of experience are themselves a factor in determining what kind of engineers are available. It is quite possible that there may be a relative scarcity of engineers with a given type of academic background and with 20 years of experience, while at the same time the supply of younger engineers with that same type of formal education may have become excessive. It is quite easy to see that a special scarcity value may have attached in 1934 to chemical engineers that did not accrue to civil engineers graduating in 1914. There was a tremendous expansion of the chemical industry in the United States during and following the war. It is a matter of common knowledge that such enterprises had the greatest difficulty in finding sufficient men with the requisite education and experience. On the other hand, while there are more civil engineers and more jobs for civil engineers than in 1914, a decrease in certain types of civil engineering work—as, for example, the construction of new railroads—has acted to restrain the development of relative scarcity values such as may have attached to chemical engineers.

At the same time it is impossible to make a comparison of the earnings of the younger engineers in the several professional classes and to conclude that the relationship between the professional groups will hold when they have had 20 years' experience. Thus in 1934 we find that chemical and ceramic engineers who graduated in 1932 averaged $1,286, whereas civil and mechanical engineers averaged $1,384. The statistics do not preclude the possibility that there is a longer period of apprenticeship for some types of engineering work than for others and that, following such a period of apprenticeship, there may be a more rapid advancement in the one line than in the other, ultimately to a higher level of income.

Conclusions as to the relative desirability of entering one type of engineering rather than another should be drawn only by those with an intimate acquaintance with all fields of engineering. Available statistics are probably a less satisfactory basis for advice than would be the pooled nonstatistical judgments of a number of people with a wide knowledge of the engineering profession and its opportunities. The statistical materials of this study can be used safely in projection only to fortify the judgments and forecasts of such individuals. To those who know not only the present situation of the various professional groups, but also the changing background of those professions over the past few decades, the income data will have particular significance. One who knows how the supply of and demand for particular types of engineering training has changed can make allowances and may attempt to estimate the most probable changes in future relationships. He will be helped to appraise the state of the current market for engineering services by earlier articles in which has been indicated the extent to which recent engineering graduates have been able to find engineering jobs.

The statistics which are presented here have a value that varies with the richness of the background of the user. Educational advisers and directors are performing specialized work and by the very nature of their work must guess what the future holds in store for various professions. Their prognosis becomes better if it is based on an extensive and accurate knowledge of existing conditions. This basis of knowledge the Bureau supplies on a hitherto unprecedented scale. The Bureau has attempted as comprehensive an analysis of recently existing relationships as its resources permit, and will welcome further critical analysis from any source of the detailed materials.

The Bureau has carefully refrained in this article from describing differences in the average incomes of graduate and nongraduate engineers as a measure of the value of a completed college course in engineering. From the data in hand, it is impossible to determine whether the differences are due to the fact that given individuals have received a college education or to other factors. It is possible that the differences in income arise from an initial process of selection. It is also possible that a prejudice in favor of the college graduate affords him better opportunities to acquire valuable experience than are given to the noncollege graduate. In such case, the advantages would arise not from formal education which the man had received but from his status as a college graduate.

It is a matter of common belief that college training has economic value for the prospective engineer. The figures in this study support this belief but cannot be taken as conclusive proof. Rather more conclusively they prove the great importance of other factors in addition to formal education. If formal education is an asset, the young graduate engineer should advance more rapidly than the nongraduate of corresponding age. This does happen. The data thus furnish supporting evidence as to the value of a formal education. But were formal education an all-important element in determining income in these early years, there should come a point at which no further relative spread developed between the average earnings of college graduates and nongraduates.

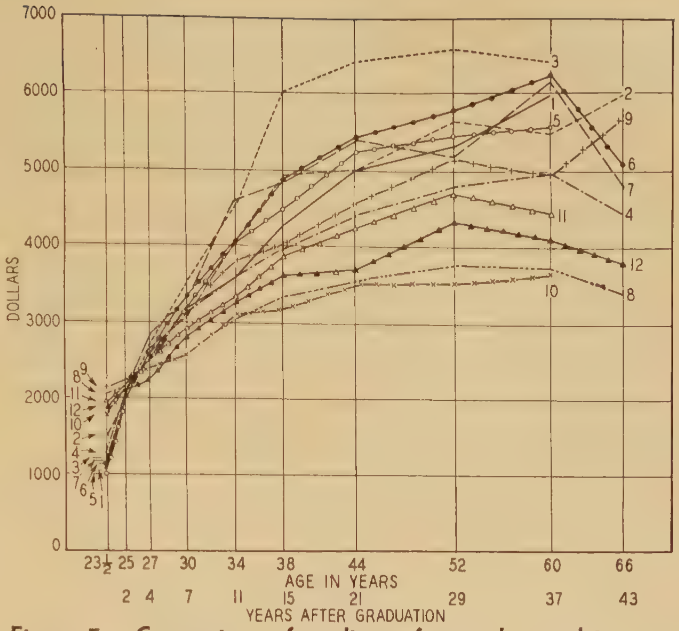

Figure 5. Comparison of medians of earned annual income according to age, 1929

Income from engineering and nonengineering services, including full and part-time employment for specific types of education

1—Postgraduates 2—Nonengineering graduates

First-degree engineering graduates

3—Chemical and ceramic 5—Electrical
4—Civil, agricultural, and architectural 6—Mechanical and industrial
7—Mining and metallurgical

College course incomplete

8—Civil, agricultural, and architectural 9—Mechanical and others

Noncollegiate technical course

10—Civil, agricultural, and architectural 11—Mechanical and others
12—Secondary school education

The facts show that the spread does increase. The difference is greater both in absolute terms and in relative terms after 30 or 40 years of experience than it is after 10 or 15 years of experience. This increased spread is noted not only with reference to the average of the 2 classes but applies at the 5 levels that have been studied. It was found not only in 1929 but also in 1932 and 1934.

It can hardly be argued that the scholastic background of engineers who entered the profession in 1900 is a controlling factor with reference to their earnings in 1929 and 1934. Certainly, the value of their services is no longer primarily dependent upon the odds and ends of information which they acquired in college, although it is possible that habits of thinking and study which the engineer received in his college days constitute a permanent legacy. By and large the factors controlling the value of a man's engineering services after 30 years or more of experience must be primarily his native capacity and the training which he has received on the various jobs that he has performed.

As regards native capacity, there is reason to believe that, on the average, better material will be found among college graduates than among those who failed to complete a college course. There are, of course, many individuals who are unable to complete an engineering course for financial reasons. There are also many individuals of limited capacity who receive degrees. But there is also a wholesale process of weeding out that goes on in the engineering schools. Thus even the differences in income shown in the earliest years of experience may reflect differences in capacity rather than differences arising from the value of the formal training.

Whether ability is more important than the differences in the kinds of experience that are open to the college graduate and the nongraduate, it is impossible to tell. It is a matter of common knowledge that for a number of years college education has been thought of as a normal prerequisite to engineering work. Some large employers of engineers deliberately differentiate between the college graduate and the nongraduate, offering the young engineer with a college education opportunities for training on the job which either are not available to the nongraduate or which are open to him after special consideration rather than as a matter of routine.

It must be noted that this latter type of advantage will tend increasingly to accrue to the status of the graduate as opposed to the nongraduate. In this sense status is gained by graduation, to some extent no longer with regard to the value of the formal education as such. The more common a college education becomes, the more widespread the assumption of a difference in capacity between the college graduate and the nongraduate becomes, the more certain it is that employers will discriminate in favor of the college graduate. Such discrimination means that the college graduate will generally be given more favorable opportunites for training on the job than the nongraduate of equal capacity.

It is known, however, that many large employers of young engineers have already developed a highly selective process of employment in interviewing candidates from engineering colleges. They may assume that in general college graduates are more promising material than nongraduates. They no longer recognize the mere fact of graduation as evidence of employability and give special status only to those who graduate with a standing substantially better than the average of the class. Thus the advantage of status which may have accrued a number of years ago through the fact of college graduation alone now accrues in equal measure only to graduation with exceptional standing.

The candidate for a position in the engineering profession should study carefully the tables which will be presented in the final bulletin[1] not only for average earnings (as in table IX) but also for the upper and lower 10 and 25 per cent of the engineers. Table IX showed clearly enough that a young man is well advised to enter the profession after successfully completing a period of formal education beyond the high-school level. An earlier chapter on education indicated that college training was coming to be a prerequisite for entrance to the profession. But it is unwise to rely heavily on the value of a college education per se.

The present study gives conclusive demonstration of differences of income which generally reflect different degrees of capacity. The lot of the college graduate may be somewhat easier than that of the nongraduate. But even in 1929 the lowest 10 per cent of the college graduates in civil engineering were earning less than $2,500 after 10 years of experience and $2,700 after 20 years of experience. These figures are to be compared with the average earnings of $3,600 and $4,400 for these respective groups. On the other hand, the upper 10 per cent of the civil engineers whose college course was incomplete earned $5,200 or more after 10 years of experience and $7,600 or more after 20 years of experience. The upper 10 per cent of the engineers with only a secondary-school education similarly show earnings substantially above those of the average college graduate. In other words, graduation from an engineering school is no guaranty of a satisfactory income, while there is still apparently an opportunity for a man of outstanding capacity to secure far better than an average engineering income even though he has not attended college.

The most significant differences in income revealed by the present survey are not the differences in the average income received by individuals who have received a college degree and those who have not. Nor are they the differences as between individuals who have entered one professional class rather than another. These differences on the whole are moderate, though they are large enough to prove the desirability of choosing well both the field of endeavor and the type of training best adapted to advancement in that field. The most striking differences are those which exist within each profession and within each group classified on the basis of its educational background. One out of 10 of the engineers in each such group secures an income several times as great as the average for the group as a whole. At least one out of 10 at the bottom of each such group, whether a college graduate or not, whether a chemical engineer or a civil engineer, whether a man with many years of service or freshly out of college, is hardly to be distinguished as regards income from a skilled wage earner.

We [of the Bureau] do not feel ourselves competent to project from these data what the future holds in store for the high-school boy who must be advised as to what type of training will be most advantageous. We hope he may be fortunate enough to encounter wise advice. We dare do no more than point out that in 1929 the average income of graduate engineers with 10 years' experience ranged from $3,600 to $4,600 in the various professional classes. In 1929 only 6.4 per cent of the incomes in the United States exceeded $4,000. Furthermore, engineering is a profession in which earning capacity advances and is sustained until late in life. But if in these respects the profession appears attractive on the average, its rewards are not particularly attractive to the poorer or less fortunate engineers. Even in 1929 the lowest-paid 10 per cent of the engineers could hope for no more than $2,500 to $3,000, though they might stay in the profession for 40 years. In 1934, exposed as the profession was to the risks of unemployment, the lowest-paid 10 per cent of the engineers with less than 5 years' experience after graduation earned less than $1,000. Even with 10 to 30 years' experience they earned no more than $1,000 to $1,500. Judged from the basis of money income, there can be no question but that the best of a group of skilled wage earners are in better economic position than those who struggle to maintain a position on the fringes of the engineering profession.

Electrical Applications in Bethlehem's 72-Inch Strip Mill

By F. D. EGAN
MEMBER AIEE

THE RAPIDLY increasing demand for flat-rolled sheet steel and strip products has made necessary the application of electrical apparatus to modern developments in rolling-mill machinery and caused the rapid construction of many wide continuous hot- and cold-strip mills in every section of the country. A comprehensive picture of a strip mill, and the application of electricity to the production of sheet steel can be seen in Bethlehem Steel Company's hot- and cold-strip mill.

This mill is built on a plot of 67 acres adjacent to, and across the highway from the main plant, acquired for the purpose of providing ample space for the best arrangement of buildings, in order that the most economic routing of materials and product through its 22 acres of floor space could be obtained.

This mill, which has an annual hot-rolled capacity of strip, sheet, and light plate of 800,000 tons, of which 360,000 tons can be converted into cold-rolled finished product, is capable of producing wide strip up to 60 inches in width by 0.0625 inch in thickness, and 72 inches in width by 0.078 inch in thickness, with a minimum width of 18 inches by 0.05 inch thick; as well as light plates up to $^1/_2$ inch in thickness. Cold-rolled strip is produced ranging from $18^1/_2$ inches to 72 inches wide and from 0.0125 inch in thickness up to 0.109 inch in thickness, and by cross rolling in a 93-inch 4-high skin pass mill, sheets may be rolled up to 84 inches in width.

All possible refinements for hot-rolled products, as well as for cold-rolled sheet and strip for which there is a commercial demand, are provided in the finishing departments.

The mill's location gives it a nearby source of its basic material; namely, the slab, the steel made at the open hearth furnaces, and the ingots rolled to required sizes in the main plant. Its location enables it to use electricity, water, and gas from the main plant; also permits service, controlled to meet its needs by the South Buffalo Railroad, connecting with 13 trunk line railroads entering Buffalo, provides the advantages of shipping facilities on the Great Lakes for raw and finished materials, as well as the advantage of a location, central in relation to the consumers of sheet product.

The slab storage building (figure 1) comes first in the sequence of rolling operation; this building is 925 feet long and 85 feet wide, and has a capacity of approximately 10,000 tons of slabs. The stock of slabs varies in widths from 20 inches to 48 inches, in thicknesses from 3 inches to 6 inches, and in lengths from 5 feet to 15 feet. These slabs, of various chemical composition, are ready for heating and rolling as needed.

The slabs are removed from storage by an overhead traveling crane and piled on a magazine pusher, which is driven by a 35-horsepower motor through a rack and pinion arrangement. Each operation of the machine lifts the magazine so that a chain drive can carry but one slab at a time, onto the furnace charging table. This 3-section roller table carries the slab to a position in front of one of 3 heating furnaces, from where a pusher forces it into the furnace. These furnaces are of the recuperative type, 3 zone and triple fired. They have an effective heating length of 80 feet and can be either gas or oil fired. Each is capable of heating 55 gross tons of slabs per hour, giving a total furnace hourly capacity of 165 gross tons. At a signal from the head roller, the pulpit *A* operator, located in the slab yard, opens the furnace door, and then pushes a slab into the rear of the furnace; this causes one of the heated slabs to drop out of the furnace at the other end onto the 3-section furnace delivery table. These tables are duplicates of the charging tables.

The slab leaves the furnaces at approximately 2,250 degrees Fahrenheit and proceeds down the roller table to number 1 scale breaker, which precedes a series of 4 mill stands in the roughing train. The scale breaker is a 2-high mill, having 24-inch diameter by 76-inch long rolls, through which the slab passes and is given a slight reduction in thickness to break up any scale that may have formed on the surface of the slab as it is leaving the furnace. As the slab leaves the scale breaker, water at 1,000 pounds pressure is sprayed upon it to remove the loosened scale.

The slab now enters a 4-high broadside mill, which has 2 36-inch diameter working rolls, and 2 49-inch diameter backup rolls, all 96 inches long. Slabs which are to be finished as strip less than 48 inches, pass straight through this broadside mill; but, for a strip width of greater than 48 inches, the slab is turned crosswise on the turntable and is cross rolled to the desired strip width, after which it is turned back 90 degrees for rolling lengthwise in the remaining mill stands. On cross roll passes, a 150-horsepower motor-driven pusher is provided to force the slab into the stand.

Before the slab enters the remaining mills of the roughing group, a slab squeezer, driven by a 275-horsepower motor, squeezes the slab sideways to true and square its edges. The remaining 3 roughing stands are 4-high universal mills, each has $24^1/_2$-inch diameter working rolls and 49-inch diameter backup rolls 79 inches long, as well as 2 vertical edging rolls, the latter driven by a 150-horsepower motor. These rolls are used to maintain the required slab width.

After leaving the last universal mill, the slab emerges onto a long, 2-section approach table. Each section is

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Figure 1. Slab storage building

driven by 2 35-horsepower motors operated in either series or parallel, to give 2 table speeds. Since the finishing temperature of the hot strip is a prime factor in the quality of the product produced, this roller line serves as a special cooling place between the roughing and finishing stands. When the thin slab has reached the proper temperature, which is automatically indicated by a photoelectric pyrometer, it enters the second scale breaker, which breaks up the secondary scale, formed on the cooling table. As the slab leaves the scale breaker, water at 1,000 pounds pressure removes the scale.

The cleaned slab now enters the 6-stand finishing train (figure 2). However, unlike the roughing mill, the strip is in all of these stands at the same time; thus, the speed of successive mill stands must be increased to compensate for the elongation in the preceding stand. This, together with the loopers, provide a means of keeping the slack out of the sheet throughout the finishing mill.

As in the roughing mills, so in the finishing mill, every precaution is taken during the rolling to eliminate scale and dirt. Here, steam instead of water is sprayed on the rapidly thinning strip just after it passes through each set of rolls. As it leaves the last finishing stand, the rapidly moving strip may be cut to any predetermined length from 11 to 22 feet, by a flying shear, synchronized with the speed of the strip, or the shear may be used to cut off the front and rear crop of the strip to be coiled.

On the front of each mill, there is a screwdown dial resembling a clock face, the hands of which indicate the movement of each end of the top roll. The rolls are roughly set up with a feeler gauge to produce a definite gauge of product. Then, to obtain perfectly accurate adjustments, either one or both ends of the top roll can be moved as small as $^{1}/_{1,000}$ of an inch by the screwdown motors, these movements being indicated by the hands on the screwdown dial. These screws are driven by 70-horsepower motors, which are coupled by a magnetic clutch when the screws are moved together.

Throughout the whole series of mill stands, grease is automatically pumped into the roll neck bearings at intervals of a few seconds, and a constant flow of oil is maintained to all motor bearings, gear and pinion housings, and backup roll bearings. These automatic lubrication systems help to insure a smooth operating mill.

In the production of hot-rolled strip it is important to secure the desired grain structure, which is controlled by the temperature of the strip as it leaves the finishing mill train. The finishing temperature is regulated by the length of time that the plate is held on the cooling tables ahead of the finishing stands. To indicate the finishing temperature, a recording Speedomax has been installed and placed within view of the finishing mill operator.

The finishing end of the hot-strip mill, designed to dispose of the various products rolled by the hot mill proper, consists of runout tables, cooling beds, trimming line, and other processing equipment. The runout table equipment serves to transport the steel from the mill to the coilers in a continuous strip, and to transport sheets, cut to length, by the flying shear, to the pilers.

That part of the runout table *A* between number 10 stand of the mill and the coilers is made up of 125 16-inch-diameter rollers, direct connected to gear head motors through flexible couplings. The motors are 2.25/7.5-horsepower 600/1,800-rpm 147/440-volt 20/60-cycle squirrel-cage induction motors, with a 5 to 1 gear ratio output shaft. The remainder of *A* table run, from the coilers to the piler, is made up of 216 rollers, 12 inches in diameter, arranged for flange-mounted motors with one motor driving 2 rollers. These motors are 1.37/5-horsepower 560/1,770-rpm 147/440-volt 3-phase 20/60-cycle induction motors.

As the continuous strip approaches the coiler, a door is opened in the table roller line, to direct the strip down into the coiler, where it is wound around a mandrel to a compact coil by individual-motor-driven revolving rolls. The finished coil is deposited on a chain conveyor and either carried through an underground tunnel to storage space in the continuous-pickler building of the cold mill, or taken from the conveyor in the hot-mill shipping building to be delivered as hot-rolled coils, or further processed in shearing, trimming, and leveling lines.

These continuous trimming lines consist of a coil box, a trimmer, a leveler, and a shear with interconnecting roller lines. The strip passes through these machines and emerges as cut-to-length flat sheets, ready for shipment or storage. The cut-to-length strip from the flying shear is collected by automatic pilers and transported to temporary storage for self-annealing. Part of these sheets are delivered as self-annealed black sheets; however, if deep drawing qualities are required by the customer, the sheets are normalized in a 150-foot-long gas-fired furnace.

The normalized sheets may be further processed by passing through a roller leveler and a skin pass mill. However, other finished product specifications may require that the normalized sheets be first taken through a sheet pickling process. This pickling process consists primarily of dipping the sheets into a sulphuric-acid bath and a hot-water rinse and then a cold-water rinse. The sheet next passes through a scrubber and dryer for final cleaning.

For handling and processing plates, 3 additional roller tables, designated as *B*, *C*, and *D* lines, have been provided south of, and parallel to, *A* table line. When plate is being rolled it is routed from *A* to *B* table line by a chain transfer, driven by 3 70-horsepower motors. The *B* table run is made up of 434 12-inch-diameter rollers, arranged for flange-mounted 0.66/2-horsepower 147/440-volt 3-phase 20/60-cycle motors, with each motor driving 2 rollers. For flexibility of control, this *B* table run is divided into 3 major divisions. The first section, consisting of 174 rollers, delivers the plate to a leveler. The second section of 172 rollers carries the plate from the leveler and feeds it into a shear, which cuts it to desired lengths. The third section of 88 rollers carries the plate from the shear to the *B* to *C* table transfer.

C table, used entirely as a transfer and cooling table, is made up of 436 12-inch-diameter rollers, arranged for flange-mounted 0.5/1.5 horsepower 240/720-rpm 147/440-volt 3-phase 20/60-cycle motors, with each motor driving 2 rollers.

For convenience in plate handling, this *C* table is divided into 5 separately controlled sections. The last section of this table delivers plate to the *C* to *D* table transfer. The *D* table run, utilized as a $^1/_2$-inch-plate processing line, consists of 480 12-inch-diameter rollers, geared to flange-mounted motors duplicate of those driving *C* table. This table is divided into 7 sections. The first 2 sections are used either to spot material already processed on *B* table for removal by plate handling equipment, or to deliver material requiring further processing to the third section of this line which serves as an approach to a rotary side-trimming shear. Section number 4 is used to convey the material from this shear to a leveler from where it is

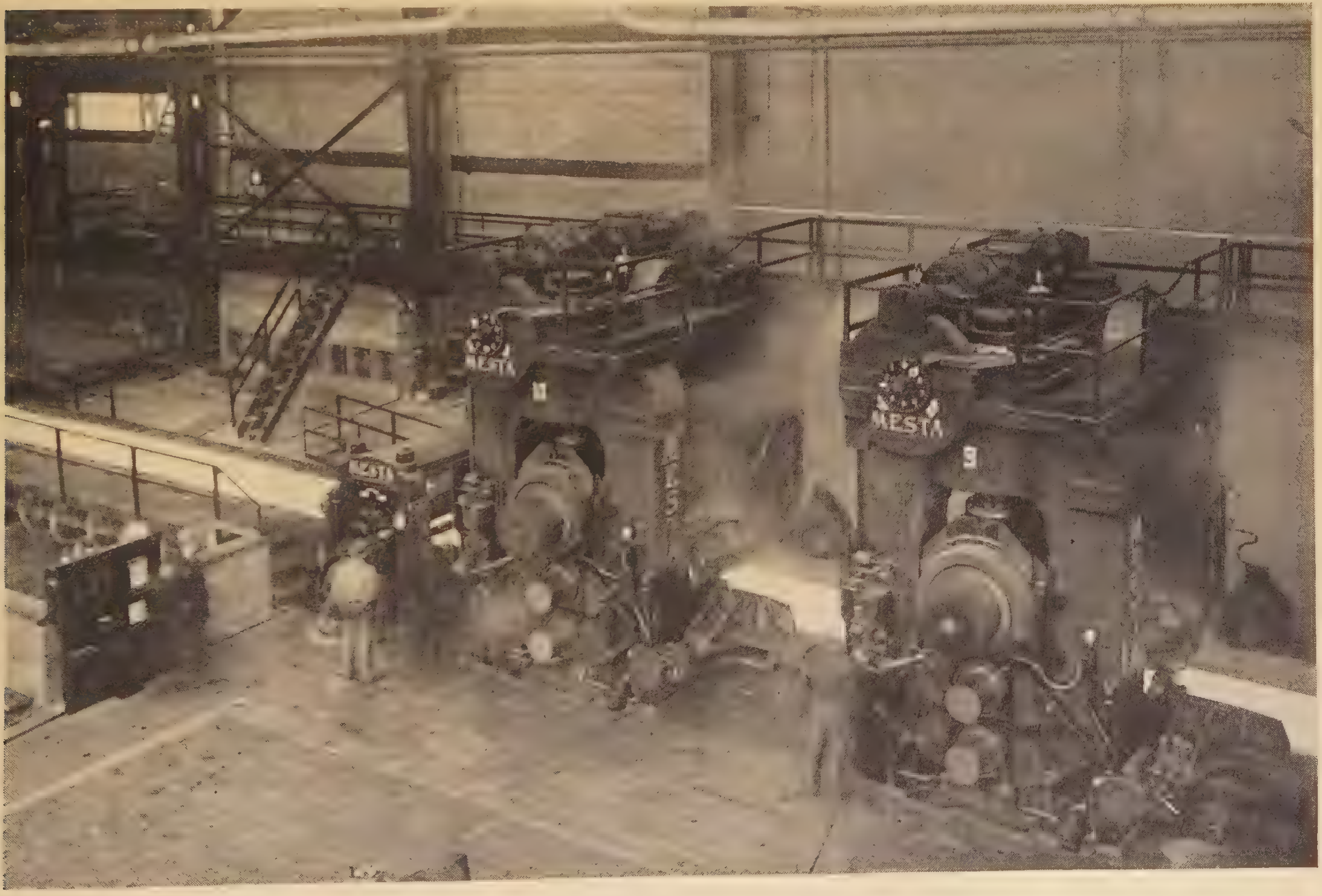

Figure 2. Side view of number 9 and number 10 stands rolling wide strip

delivered by section number 5 to an up-cut shear. The sheared plate is carried by sections number 6 and number 7 to piling skids, for transfer to storage and shipment or transferred to the $^5/_{16}$-inch trimmer line, or $^1/_2$-inch re-squaring shears for further processing.

All of the motors (figure 3), driving *A* table rollers and both coilers, the motors driving the second section of *B* table, and the motors driving the fourth and fifth sections of *D* table, receive their power from 6 166/500-kva 147/-440-volt 3-phase 20/60-cycle synchronous generators, each driven by 2 125/250-horsepower 300-volt 400/1,200-rpm shunt-wound, separately excited at 250 volts, adjustable-speed d-c motors connected in series to operate from the 600-volt d-c hot-mill bus. The table sections and coilers are arranged to be driven from the 6 alternator sets as follows:

Number 1 motor-alternator drives section number 1 of *A* table.
Number 2 motor-alternator drives section number 2 of *A* table.
Number 3 motor-alternator drives either number 1 coiler or section number 3 of *A* table.
Number 4 motor-alternator drives either number 2 coiler or section number 4 of *A* table.
Number 5 motor-alternator drives section number 2 of *B* table.
Number 6 motor-alternator drives section number 4 and number 5 of *D* table.

The motors driven from these alternators are started at base speed, 20 cycles at 147 volts on the line and increased to full speed, 60 cycles at 440 volts by field weakening of the d-c motors driving the alternators. In reverse manner the table drives are decelerated to base frequency and voltage by field strengthening of the driving motors, which pump regenerated energy into the 600-volt system. At base speed the motors are cut free from the alternators and brought to rest by d-c dynamic braking. This direct current is supplied by a 200-kw 75-volt separately excited shunt-wound generator driven by a 300-horsepower 440-volt motor. Excitation for this dynamic braking machine and the motor-alternator sets is supplied by a 100-kw self-excited 250-volt generator driven by a 150-horsepower 440-volt motor.

The motors driving the remaining sections of *B*, *C*, and *D* tables are operated at a fixed frequency of 25 cycles at about 187 volts, supplied by 2 banks of 3 100-kva single-phase, 440-volt primary, 170/200-volt secondary, transformers.

Coils of strip rolled in the hot mill are carried underground by 3 motor-driven coil conveyors to coil storage in the continuous pickler building. From here the hot-rolled coils definitely start on their journey through the cold mill for the process of becoming full-finished cold-rolled sheets.

The coils are taken from storage with a lifting magnet suspended from a traveling crane, to one of the continuous pickling lines, in vertical position. An up-ender tilts this coil to a horizontal position, from where it is rolled down an incline to the feed reel. The coil is lifted into position in this feed reel by an electric hoist, where motor-adjusted heads clamp it into place. The strip is threaded into a combination processor and straightener, which unwinds the coil and levels the sheet. The end of the strip continues to a shear where it is cropped square, so that it can be fastened to the squared end of the preceding strip by a stitcher to form a continuous ribbon of steel.

A looping pit is provided between the stitcher and the acid tanks, where sufficient slack can be kept in the strip to allow time for the stitching and shearing processes and still maintain a continuous passage of steel through the pickle tanks. The strip is then pulled by pinch rolls, out of the looping pit and through 4 60-foot acid tanks, a 25-foot cold-water-rinse tank, and a 25-foot hot-water-rinse tank, which helps to dry it. Emerging from the latter it moves over a roller table, under which is a hot-air dryer, designed to remove all traces of moisture. The pickling solution normally consists of a sulphuric-acid bath, kept at a temperature of 180 degrees Fahrenheit by pyrometrically controlled steam heat.

As the sheet is pulled from the drying table by the third pinch roll, it is recut to original coil lengths by a double-cut shear which removes the stitch. A recoiler located at the end of the line, coils the strip, from which it is discharged to a gravity conveyor, weighed, and delivered to the storage space adjacent to the tandem cold-rolling mills.

The next step after pickling is to reduce the strips' thickness to such gauges as may have been ordered by customers. This is accomplished by rolling the metal cold in the 3-stand 4-high tandem mills, there being 2 sets. One set of mills is designed to roll coils up to 48 inches maximum width, while the other will roll them up to a width of 72 inches.

The tandem arrangement of 3 mills in series makes possible 3 cumulative reductions at one pass of the strip through the mill train. Synchronizing the series of mills to the proper speed and reduction is accomplished by an elaborate electrical control system. The gauge or thickness of the strip on its passage through the mill stands is indicated to the operator by a flying micrometer which automatically indicates on a dial any variations in the thickness in increments of 0.0002 of an inch. The dial is located within view of the operator, and enables him quickly to adjust the pressure on the rolls to maintain uniform gauge.

Each mill stand is driven by a 1,250-horsepower motor, sufficient power being available to give a delivery speed at the last stand of 660 feet per minute. From the stand (figure 4), the strip is wound by a reel with a collapsible core, which maintains a predetermined strip tension during recoiling. After the strip is coiled the reel is stopped, then a pneumatic pusher discharges the coil from the reel, and the coil is either sent to storage for shipment, or subsequent treatment.

Preparatory to annealing, the coils from the tandem cold mills are cut into sheets, or especially processed in coil form. If the steel is to be annealed in sheet form, the coils are taken to a group of 3 cutting up lines. Each of these lines consists of a feed reel, a side trimming shear, roller leveler, and flying shear, arranged in tandem. From these lines the strip, now flattened sheets, are delivered in packs by gravity conveyors to the annealing department. Strip, to be annealed in coil form, is taken to a loose

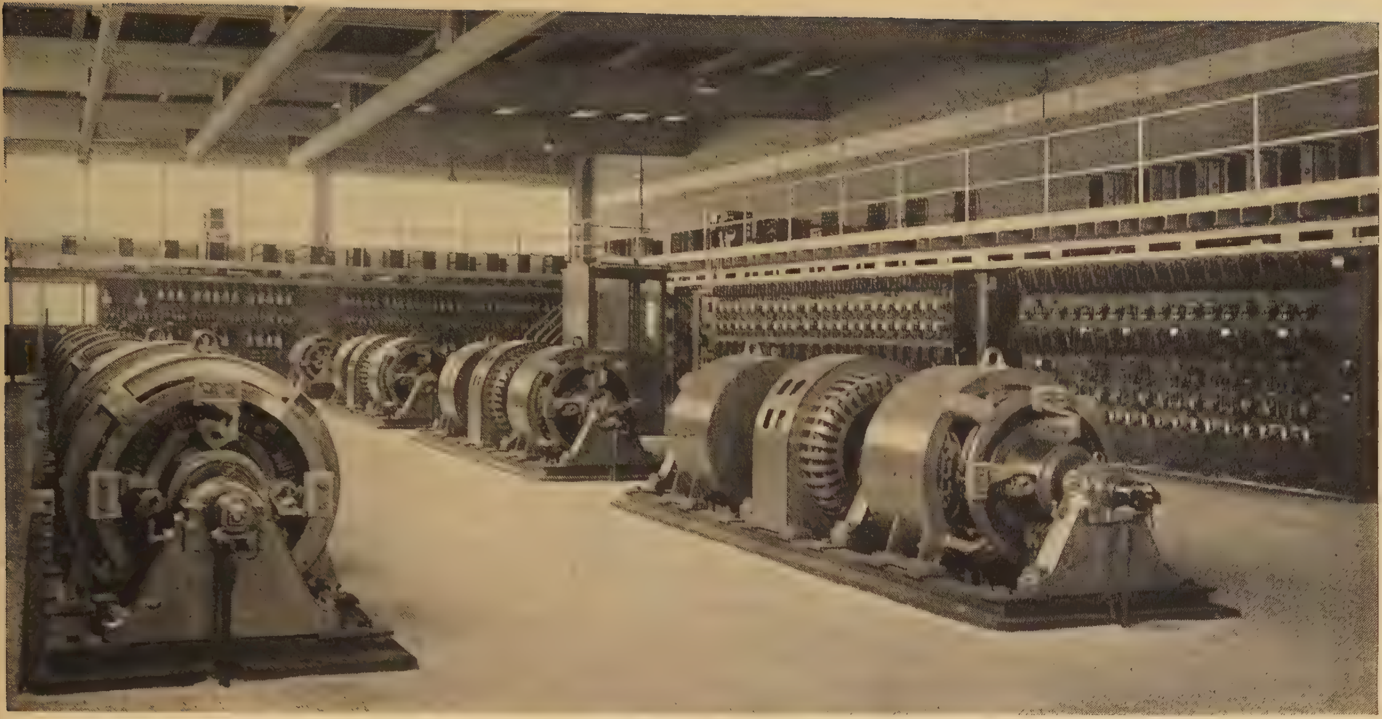

Figure 3. Variable-frequency-motor room

winding machine, which rewinds the tightly coiled strip as it comes from the tension reel of the tandem mills. This is done to permit a more thorough annealing and to eliminate the possibility of the smooth surface of the coils adhering to each other under the high annealing temperature.

The annealing equipment consists of 44 gas-fired radiant-tube furnaces. Thirty-one of these furnaces, provided with 91 bases and inner covers, will handle sheets up to 75 inches in width by 220 inches in length or coils up to 59 inches in width; 5 furnaces, provided with 14 bases and inner covers, are capable of taking sheets up to 84 inches by 186 inches, or coils up to 72 inches in width; and 8 furnaces, provided with 28 bases and inner covers, size 90 inches by 250 inches, are now being installed. The use of several bases to one furnace has secured maximum efficiency of furnace capacity.

In the sequence of the annealing operation, sheets are piled on a base, the inner cover placed over the pile, and the furnace lowered over this cover. The lower edges of both the inner cover and the furnace are sand sealed. During the annealing operation, an atmosphere of deoxidized gas is introduced under the inside of the inner cover at a slight positive pressure. The temperature in the furnace is slowly brought up, until it is a little above the predetermined annealing point. To reach this temperature requires from 30 to 40 hours on sheets, and about 20 hours on coils. The furnace temperature is maintained at this level by automatic control, while the pack of sheets is soaked for about 22 hours, or in the case of coils, 6 hours. The furnace is then removed by an overhead crane to another base that has been charged in a manner similar to the first. The annealed pile, enclosed in its inner cover, is allowed to cool to about 200 degrees Fahrenheit. The inner cover is then removed and after the pile has cooled to atmospheric temperature, it is removed. The base is now ready for another cycle of annealing operations.

Strip steel, when annealed in coils, is placed upright on the bases and covered by the inner cover, and furnace. The same method of annealing processes pertains to coils as to sheets, except that, due to the separation between coil turns, the soaking period can be reduced, making possible a shorter annealing cycle.

Four individual mills have been provided to skin pass the annealed sheets, namely: 2 4-high 76-inch mills; one 4-high 90-inch mill; and one 2-high 54-inch mill. In addition to skin rolling single sheets, one of the 76-inch 4-high mills and the 2-high 54-inch mill are equipped with feed and tension reels for skin passing strip in coil form. The slight reduction of one to 3 per cent, not only smoothens and polishes the sheet but also affects the hardness and ductility of the sheet. These qualities can be modified by varying the amount of reduction, depending upon the draw required in the sheet.

Strip, in coil form, can be delivered in widths, ranging from one inch to 72 inches, and in varying thicknesses from 0.0125 inch to 0.125 inch. To supply the narrow widths of 18 inches and under, the hot- and cold-rolled coils are slit to required widths by especially designed slitting units. One of these machines is capable of handling strip up to 36 inches in width and the other up to 72 inches in width. The machines may also be used to trim wide strip in coil or sheet form to accurate widths.

Typical uses of sheet and strip products, rolled by this mill, are as follows:

Hot-rolled sheets are used in the manufacture of tacks, large steel tanks, concrete forms, brick pallets, perforated screens, range boilers, and similar articles.

Hot-rolled annealed sheets are used in the manufacture of steel drums, barrels, metal flasks, coiling stock, stove parts, low price cabinets, lockers, and shelving, and other products, which do not have very rigid forming or surface requirements.

Blued sheets are used for the manufacture of stove pipe, stove parts, and other articles that are finished with a smooth unpainted surface.

Extra-smooth hot-rolled annealed sheets, also called japanning steel, are used principally for steel cabinets, shelving, stove parts, and various metal novelties that require a smooth surface suitable for lacquer or enamel finish and which do not have severe forming requirements.

Hot-rolled annealed sheets-pickled, have a clean, porous surface and are suitable for products that have drawing or electric welding requirements, but they are not recommended for articles on which a smooth finished surface is required. They are used by the automobile industry for inside body and chassis parts, such as: inside door panels, seat backs and bottoms, floor pans, and pillar posts.

Cold-rolled sheets are used in the automobile industry for turret tops, doors, panels, hoods, fenders, radiator shells, and side aprons; in the container industry for milk, paint, ink, grease, and oil cans; in the refrigerator industry for panels and doors; in the radio industry for chassis parts; and in the furniture industry for filing cabinets, desks, chairs, partitions, hot-air registor and radiator covers; in the electrical industry, cold-rolled stretcher-leveled sheets are used for control panels, metal-clad switchgear, instrument panels, breaker panels, and for other products which require perfectly flat, nonresilient steel.

Due to the distance between the main plant and the strip mill, it became necessary to erect an independent boiler plant to provide steam for processing and heating. This station is equipped with 4 500-horsepower boilers, 150 pounds pressure, 200 degrees superheat, capable of operating at 200 per cent rating. The units are designed to use coke breeze for fuel; this fuel is handled from cars to bunkers over the boilers by an automatic skip hoist and belt conveyor. The fuel is fed into the boilers by chaingrate stokers, and the ash removed by dumping directly into hopper cars. Three of the 4 boilers are equipped with steam-driven auxiliaries, the fourth being equipped with motor-driven auxiliaries. The boiler feed pumps, water-softening plant, and 3 2,000-cubic-feet-per-minute air compressors for general plant service, are located in a room adjoining the boiler house proper.

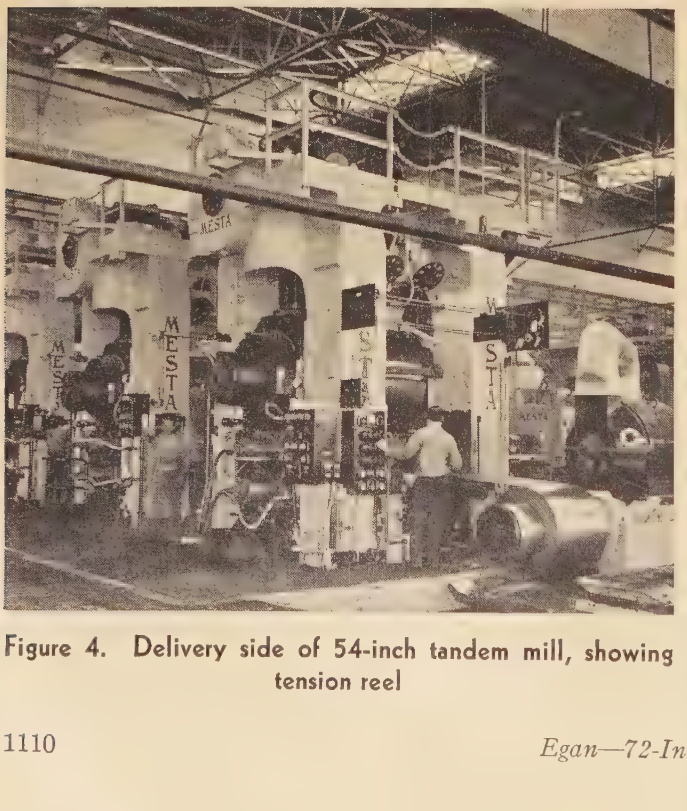

Figure 4. Delivery side of 54-inch tandem mill, showing tension reel

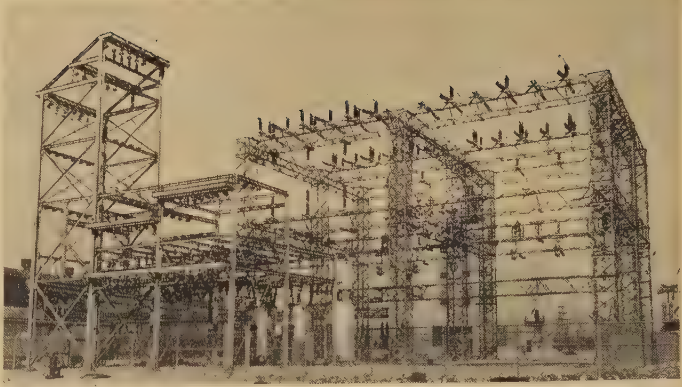

Figure 5. Number 5 substation

A roll shop is provided as a continuation of the cold-mill building, for grinding both the hot- and cold-mill rolls. This shop is equipped with 2 53-inch by 20-foot and 2 36-inch by 18-foot grinders, together with bearing changing blocks, roll racks, and sandblasting equipment for surfacing the cold-mill rolls.

A service building located central to the hot and cold mill, contains an electric shop, machine shop, and storehouse for small machines and tools required for general plant maintenance.

For the convenience of the large number of workmen a welfare building was built between the hot and cold mills to provide shower baths, washing basins, and individual lockers for the workmen. This building is steel brick construction, with glazed tile interior; covered walkways have been provided between this building and the various departments for the convenience of the workmen.

As the steel industry has developed, electricity has played an increasingly important part in the production of steel. Thus, with the large investment in the modern strip mill, it becomes most important that provision be made to supply adequate power with minimum interruption to service.

The power for this strip mill is supplied from 3 different sources over 6 lines from 2 different directions. The main source of supply is number 5 substation (figure 5). This station is located approximately 600 feet east of the hot-mill motor room, and consists of a 2-bay outdoor high-voltage switching structure, 56 feet by 80 feet. This station is fed from the power company over 2 3-phase 25-cycle 60,000-volt lines, each rated at 40,000 kva. These lines pass through this structure and continue to another substation feeding the main plant.

Each of the power company's lines occupy one bay of the high-voltage structure and with the use of a cross bus and tie switch, it is possible to feed this station from either or both of these lines. The structure is designed to accommodate 2 15,000-kva 60,000/6,600-volt 25-cycle 3-

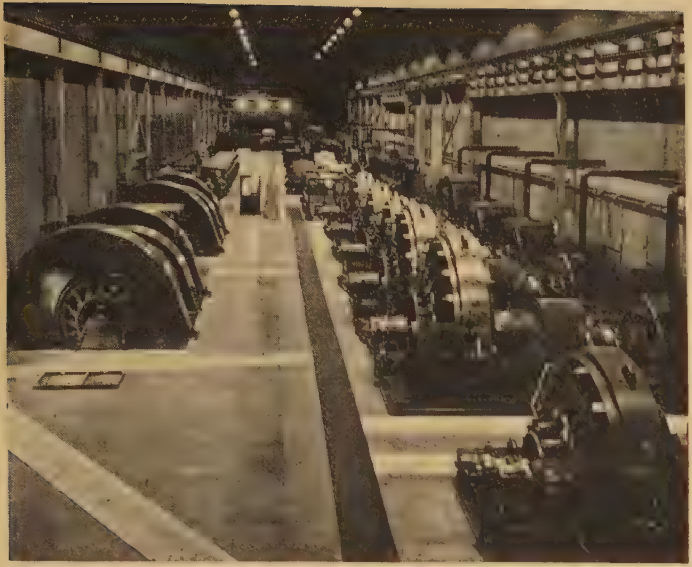

Figure 6. General view of motor room

phase water-cooled banks of transformers, one of which is installed at present.

A secondary bus structure, 20 feet deep by 40 feet long, is provided to accommodate the bus work, oil switch on the secondary side of the transformer, and the disconnect switches on the outgoing lines. Provision is made to add a duplicate secondary structure for the future bank of transformers.

The secondary bus structure and the towers between the substation and the motor room are designed for 6 500,000-circular-mil circuits, 4 of which are installed at present. These lines are tied to the secondary bus of the transformers through 1,200-ampere high-pressure-contact outdoor disconnect switches and 2 circuits are paralleled on the other end to feed the motor-room bus through a 1,200-ampere 500,000-kva metal-clad oil circuit breaker.

The 2 power company lines that pass through station number 5, continue to the main plant to feed station number 3 through 2 400-ampere 1,000,000-kva oil circuit breakers. This station is very similar in design to station number 5, including the cross bus and transformer layout for 2 15,000-kva 60,000/6,600-volt 3-phase 25-cycle air-cooled banks of transformers, both of which are installed and operating. Space is available at this station for the installation of a third 15,000-kva bank, duplicate of above. This station is located about one mile north of the strip mill, with a 6,600-volt tower line connecting the substation with the mill, designed for 4 500,000-circular-mil circuits, 2 of which are installed at the present time. Each line is tied to the substation bus through a 1,200-ampere 25,000-volt outdoor-type oil circuit breaker and each is tied to the motor-room bus through a 1,200-ampere 500,000-kva metal-clad oil circuit breaker.

(Figure 6) this is a general view of the motor room with the d-c motors driving the finishing mill in the right foreground, the 2 1,500-kw auxiliary motor-generator sets in the right center, and the 6,600-volt a-c roughing-mill induction motors in the right background. The 2 6,000-kw motor-generator sets driving the finishing-mill motors are shown in the left foreground and the 6,600-volt metal-clad bus structure in the left center.

This metal-clad bus structure consists of 22 high-speed 1,200-ampere 500,000-kva oil switches, complete with sealed-in bus compartment. A 250-volt storage battery is located in the basement to provide closing and tripping power for these switches.

The main bus in this metal-clad structure is separated into 2 parts which are tied together through a 2,000-ampere nonautomatic 500,000-kva oil circuit breaker. The incoming lines from number 3 substation feed into the west end of this bus and the incoming lines from number 5 substation feed into the east end of this bus. The roughing-mill induction motors, the 3 cold-mill motor-generator sets, and one of the auxiliary motor-generator sets feed off the west bus and the finishing-mill motor-generator sets, auxiliary transformers, spray pumps, and one of the auxiliary motor-generator sets feed off the east bus. The normal operation of the power system is to have the east and west busses tied together with the lines from both number 3 and number 5 substations operating in parallel.

The bus at number 3 substation that feeds the 2 500,000-circular-mil lines to the strip mill is tied in with a 25,000-kva generating station through the plant network.

The roughing-mill motors driving number 1 scale breaker and stands numbers 1 to 4, inclusive, are 6,600-volt 3-phase 25-cycle induction motors equipped with automatic liquid slip regulators for starting and regulating the load, with the exception of the 1,000-horsepower squirrel-cage motor driving number 1 scale breaker, which starts across the line. Each motor has 6 leads brought out of the stator, with current transformers ahead of the motor and in the neutral for differential protection.

These motors are stopped by applying direct current to the motor windings, which results in a modified form of

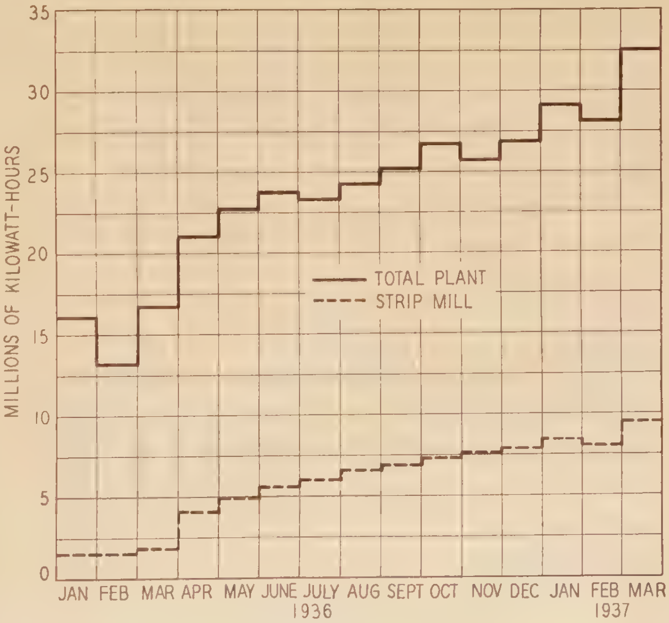

Figure 7. Kilowatt-hours per month

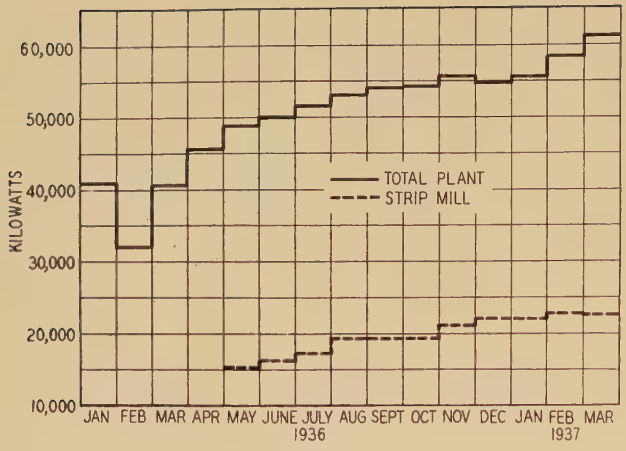

Figure 8. Maximum 15-minute demands

dynamic braking. The ratings of these motors are as follows:

Number 1 scale breaker	1,000 horsepower	Squirrel cage	375 rpm
Number 1 stand	3,000 horsepower	Wound rotor	150 rpm
Number 2 stand	3,000 horsepower	Wound rotor	500 rpm
Number 3 stand	3,000 horsepower	Wound rotor	500 rpm
Number 4 stand	3,000 horsepower	Wound rotor	500 rpm

Number 1 stand has a flywheel mounted on the motor shaft and numbers 2, 3, and 4 stands have 2 flywheels mounted on the pinion shaft of the gear unit.

The 600-volt direct current, to operate the finishing-mill motors, is supplied by 2 6,000-kw motor-generator sets, each set consisting of one motor and 2 generators. The motor driving these sets is a 7,200-kva 6,600-volt 3-phase 25-cycle 375-rpm synchronous motor, connected for differential protection, the same as roughing-mill induction motors. The motors are started through the 1,200-ampere metal-clad switchgear and are brought up to speed as an induction motor, the current being limited by the starting reactor to 165 per cent in terms of normal rating. At approximately 90 per cent speed, a light field is applied to the motor to pull the machine in step with proper polarity on field poles. After the machine pulls into step, the reactor is shorted out of the circuit, placing the machine across the line, and at the same time full field is applied. The d-c generators are rated 3,000-kw 600-volt d-c continuous rated, open, compensated, shunt wound, separately excited with 250 volts, direct current.

The 4 generators feed into a common bus, from which, the 7 d-c finishing-mill motors and the variable-frequency motor-generator sets obtain their power. The ratings of these motors are as follows:

Number 2 scale breaker	1	500 horsepower	150/600 rpm
Numbers 5, 6, 7 stands	3	3,500 horsepower	175/300 rpm
Numbers 8, 9 stands	2	4,500 horsepower	125/250 rpm
Number 10 stand	1	2,500 horsepower	175/350 rpm

making a total connected load of 22,500 horsepower for finishing stands.

These 22,500 horsepower of d-c motors, together with the 4 3,000-kw generators, form an extremely heavy concentration of d-c power on the 600-volt d-c bus. In the case of a short circuit on this bus, there would be developed a current in the nature of 300,000 amperes, which would produce an estimated separating force of several tons per running foot for ordinary bus spacing. Since the force is inversely proportional to the distance between the busses, this installation was made with busses placed at several times normal spacing. The positive and negative busses were placed on opposite sides of the bus room, to give this wide spacing. Aluminum channel, placed back to back, was used to increase the lateral stiffness and in addition, the bus was firmly braced against the walls of the room. Each motor and generator and the 600-volt bus to the variable-frequency room are tied to the main bus through 2 single-pole high-speed d-c circuit breakers, incorporating the "rate of rise" feature of instantaneous trip on direct short circuits. These breakers are located in both the positive and negative legs of each machine and spaced on opposite sides of the room each being firmly braced to the back walls. This entire construction of bus, circuit breakers, and feeder bus was designed to withstand heavy short-circuit stresses.

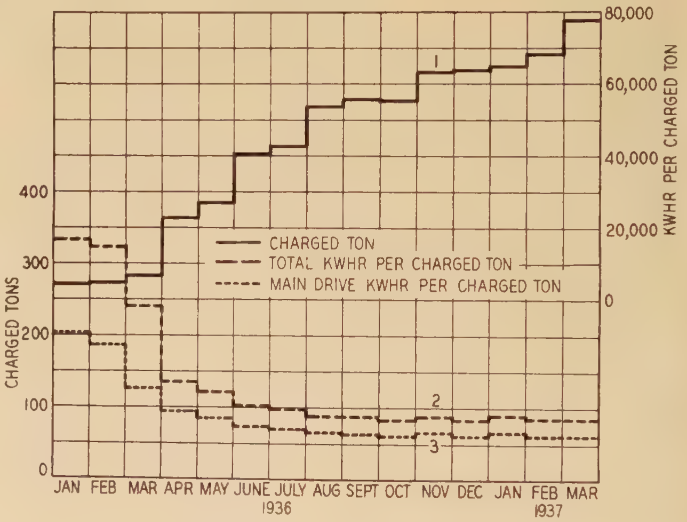

Figure 9. Hot-strip mill

The d-c motors and generators are connected to the bus at zero voltage and motors brought up to speed by building up the generator voltage by the use of a motor-operated potentiometer rheostat. Each individual motor is adjusted to the required speed by weakening or strengthening the field of the motor with the use of a motor-operated coarse rheostat and a hand-operated vernier rheostat, both located in the mill pulpit *C*, from which point the finishing-mill motors are controlled. Six meters, calibrated in horsepower, are located in the face of *C* pulpit, in view of the roller, to indicate load on the 6 finishing stands.

A great deal of heat is generated through the heavy loads imposed on the main drive equipment. To dissipate this heat a recirculating system of ventilation is provided. The finishing-mill motor foundations are so arranged that all 6 motor pits are connected with each other forming a tunnel, closed on each side and open at the ends. The 2 6,000-kw motor-generator sets, located directly across the room have a similar layout in the foundation arrangement. The rotating motors and generators pick up cool, clean air in the motor room by their natural fan action, which ventilates the machines and is then discharged into the foundation tunnel. At each end of these 2 foundation tunnels is a surface cooler through which the heated air from the machines must pass, to be recooled to normal temperature. A blower is located near the discharge of each of these coolers, the sucking action of which draws the air through the coolers and discharges it into the motor room for recirculation. As the 4 blowers are not connected direct to the coolers, any one of the units can be shut down without affecting the coolers. Under normal operation the blowers operating depends upon the temperature in the motor room. To offset air losses, as well as to maintain a slight positive pressure in the motor room, makeup fans are provided at each end of the room. These fans draw fresh air from the outside, through a filter cleaning equipment and discharge it into the motor-room basement.

The 2 hydraulic spray pumps are driven by 2 1,000-horsepower 6,600-volt wound-rotor induction motors, the pumps deliver 1,000 gallons per minute at 1,000 pounds pressure to the spray nozzles.

The cold-mill substation contains the motor-generator sets and auxiliary units for driving the tandem and skin pass mills. A 3-unit 6,000-kw motor-generator set, duplicate of the hot-mill main drive sets, is provided to furnish 600-volt direct current to the 76-inch tandem mill from one generator to the 54-inch tandem mill from the other generator. A 2,500-kw motor-generator set driven by a 3,750-horsepower 6,600-volt synchronous motor, direct connected to 2 1,250-kw 600-volt generators, furnishes power to the 2 76-inch skin pass mills. A 1,300-kw motor-generator set driven by a 2,000-horsepower 6,600-volt synchronous motor, is direct connected to 3 generators. Two of these generators are 500-kw 300-volt capacity and one generator is 300-kw 250-volt capacity. The 2 500-kw generators are normally connected in series to provide 600 volts to drive the 90-inch skin pass mill. However, they can be connected in parallel to provide emergency 250-volt direct current for plant auxiliaries at reduced capacity. The 300-kw generator furnishes power to drive the 54-inch skin pass mill. The ventilating scheme of the substation is duplicate of the hot-mill motor room, described earlier in this paper.

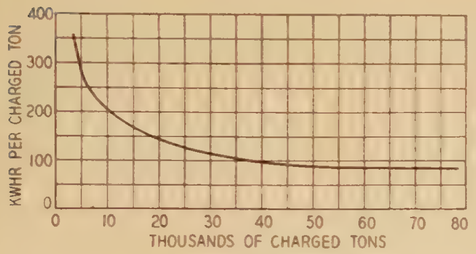

Figure 10. Hot-strip mill

A bus tunnel is provided to connect the cold mill substation with the various cold-mill control rooms, which are located in the basement. The top of this tunnel houses the bus runs between the generators and the tandem and skin pass mill motors and the tunnel proper is used to provide makeup air for the ventilation of the cold-mill drive motors. The makeup air equipment consists of an air washer and blower located at the end of the tunnel, the blower discharging into the tunnel to maintain a slight positive pressure in the tunnel and control rooms. Each of the cold-mill drive motors has its own recirculating ventilation system, taking air from the tunnel, circulating it through the motors and coolers to discharge into the tunnel again. In this manner, each motor is cooled with fresh clean air.

The d-c auxiliary power for both hot and cold mill is supplied by 2 1,500-kw motor-generator sets, each driven by a 2,100-kva 6,600-volt 500-rpm 80-per-cent-power-factor synchronous motor with 250-volt d-c field excitation from the d-c end. The motor is designed for full-voltage starting, requiring 500 per cent kilovolt-ampere inrush, in terms of normal rating. The generator is rated 1,500 kw at 250 volts, and is self-excited and flat compound wound. The 2 generators are arranged to operate in parallel, but only one set is required to furnish both hot- and cold-mill power.

The 2 1,500-kw generators tie into the 250-volt bus through 2 8,000-ampere single-pole 600-volt air circuit breakers, each equipped with overload, under voltage, and reverse-current features. The d-c power is fed into the various sections of both hot and cold mill over 6 feeder circuits, each connected to the bus with a 4,000-ampere single-pole 600-volt air circuit breaker with inverse time overload protection.

The a-c auxiliary power is supplied by 3 1,000-kva 6,600/440-volts ingle-phase 25-cycle air-cooled transformers connected delta-delta. The power is distributed to the mill over 3 3-phase circuits, each protected with a 1,600-ampere 3-pole 600-volt air circuit breaker.

The entire lighting for the mill is supplied by a bank of 3 500-kva 6,600/440-volt single-phase 25-cycle air-cooled transformers, connected delta-delta. The primary side of this bank is connected in parallel with the primary side of the power transformers, and both are fed through a common oil circuit breaker from the station bus. The

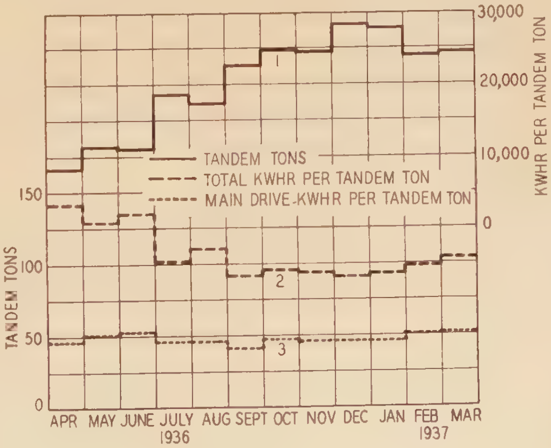

Figure 11. Cold mill

secondary side of this bank feeds through a 4,000-ampere 3-pole 600-volt air circuit breaker, to the various distribution points throughout the mill. At these points, the 440-volt power is stepped down to 3 phase, 110-volts, through $7\frac{1}{2}$- and 10-kva single-phase oil-insulated air-cooled transformer banks, to sectionalized 110-volt feeder circuits.

The overhead lighting consists of 300 porcelain-enamel reflectors, 225 silver-backed glass reflectors with steel covers, and 450 prismatic glass reflectors with spun-on steel covers. All of these units are of the high-bay type of 750 watts capacity. The low-bay lighting, control house and pulpit lighting, and various scattered units, consists of over 700 units, varying in capacity from 50 to 500 watts. The tunnel lighting consists of 220 units, built into the side walls, each of 200 watt capacity.

Summary of 6,600-Volt Connected Load

	Kilovolt-Amperes
Number 1 scale breaker	860
Number 1 stand	2,650
Numbers 2, 3, 4 stands	7,260
Finishing mill—motor-generator sets	14,400
Spray pumps	1,640
Auxiliary motor-generator sets	4,200
Cold-mill motor-generator sets (1)	7,200
(2)	3,110
(3)	1,660
Power transformers	3,000
Lightning transformers	1,500
Total	47,480

Summary of Total Connected Horsepower

	Number	Horsepower
Motors—hot mill	1,407	81,550
Generators—hot mill	24	23,850
Total—hot mill	1,431	105,401
Motors—cold mill	514	35,050
Generators—cold mill	18	13,675
Total—cold mill	532	48,725
Total installed units	1,963	154,126

All of the above units are controlled by 350 2-pole 60-ampere 110-volt circuit breakers with overload protection, enclosed in cast iron case. These breakers are fed from the sectionalized 110-volt lighting busses.

To give a more comprehensive picture of the power requirements of this hot- and cold-strip mill, the following curves have been plotted to graphically show the relation between the kilowatt-hours, the kilowatt demand, and tons charged.

(Figure 7) top curve shows the kilowatt-hours consumed by the total Lackawanna plant for 1936 and the first quarter of 1937, including the strip mill, while the bottom curve shows kilowatt-hour consumption of the strip mill alone for the same period. The top curve shows the gradual increase in kilowatt-hours for the total plant from 13,200,000 kilowatt-hours in February 1936 to 32,578,000 kilowatt-hours in March 1937 and corresponding to this increase in total energy consumed, the strip mill, starting in January 1936, used 1,600,000 kilowatt-hours, gradually increasing to 9,256,000 kilowatt-hours in March 1937, or approximately 28.4 per cent of the total plant power, which averages 1,050,000 kilowatt-hours per day.

Figure 12. Cold mill

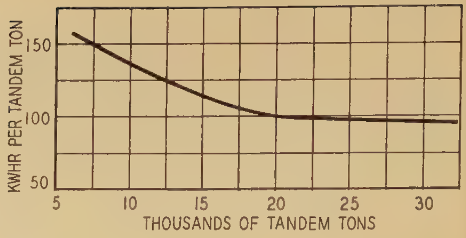

Figure 8 shows the maximum 15-minute monthly demand for the total plant and the maximum 15-minute monthly demand for the hot- and cold-strip mill. The total plant demand has increased from 32,000 kw in February 1936 to 61,140 kw in March 1937. The maximum demand for the strip mill has gradually increased from 15,000 kw in May 1936, the first month that the demand meter was installed, to 22,680 kw in March 1937. This demand has gradually increased with the increase in tonnage rolled.

(Figure 9) curve 1 on this graph represents the charged tons which have gradually increased from 4,000 tons, the first month of rolling, to 77,800 tons in March 1937. Curve 2, showing total kilowatt-hours per charged ton, has gradually decreased from 333 to 85 kilowatt-hours per ton. Curve 3 shows the kilowatt-hours per charged ton for the main drive, and represents that part of the total power used by the main drive motors, and the difference between curves 2 and 3 represent that part of the total power used by the auxiliary equipment. The main drive equipment uses approximately 75 per cent of the total power and the auxiliary equipment uses approximately 25 per cent.

Figure 9 shows that the total kilowatt-hours per ton remains approximately constant above 55,000 tons. However, below 55,000 tons, the curve begins to rise at an increasingly rapid rate. Figure 10 shows a curve plotted between charged tons and kilowatt-hours per charged ton and confirms the above statement.

In analyzing the tandem tons rolled in the cold mill, (figure 11), it will be noted that the total kilowatt-hours per ton, curve 2, has decreased from 143 to 92, as the tandem tons, curve 1, increased from 8,000 to 28,000 tons. Further, it will be noted that the kilowatt-hours per ton for the main drive equipment remained almost constant at 49 kilowatt-hours per ton, while the variation in total kilowatt-hours per ton to the mill occurred in the auxiliary drive equipment. The cold-mill main drive equipment uses approximately 53 per cent of the total energy and the auxiliaries, approximately 47 per cent, and the kilowatt-hours per ton for the cold mill remain almost constant above 22,000 tons, but rises sharply as the tonnage decreases. Also, the curve in figure 12, plotted between tandem tons and kilowatt-hours per tandem ton, flattens out at 22,000 tons, indicating approximately a constant value of kilowatt-hours per ton above this tonnage.

Some High Lights in the Use of Electricity in Steel Mills

By GORDON FOX
MEMBER AIEE

Synopsis

Approximately one tenth of the investment in the American steel industry is in electrical plant. The power consumption of steel mills is increasing. About two thirds of the power used is generated within the plants but there is a trend toward increased purchase of power.

Electricity is gaining a foothold in the field of transportation through the Diesel-electric locomotive.

D-c motors are being used more for main-roll drives. Synchronous motors are also gaining favor. Reversing drives have developed interesting ramifications.

There is a trend toward subdivision of main roll drives. The combination of reduction gears and mill pinions into a single unit is of interest. The use of new types of bearings affects power requirements.

The concentration of d-c power associated with the main-roll drives of wide strip mills has introduced problems concerning interrupting capacity of circuit breakers and strength of bus installations.

In the field of auxiliary drives, the increased application of small individual induction motors is notable.

Electrical precipitation finds greater acceptance in the field of gas cleaning.

Electric melting furnaces supply a small but growing increment of total steel production. High-frequency heating is finding some uses. New types of electric furnaces are being installed for heat treating and annealing operations.

NOT SO MANY years ago electricity played but a minor part in the steel industry. It has gained rapidly and steadily in importance. Today, in a total investment on the order of $5,000,000,000 represented by the American steel industry, more than $500,000,000 is involved in electrical plant. Considering a typical steel plant as a whole, about 10 per cent of the total investment is electrical.

The relationships between electrical plant and total investment are indicated by the approximate figures shown in table I.

The cost of materials is the major element in the cost of producing steel. The cost of electric power is a minor element. In the iron- and steel-producing departments, the cost of electric power is generally not more than 5 per cent of the total conversion cost. In the rolling mills, the cost of electric power commonly amounts to about 10 to 15 per cent of the conversion cost. Although relatively a minor item, in an absolute sense the amount of electric power consumed and the cost thereof are substantial figures.

The installed capacity in main-drive motors in American steel mills is nearly 3,000,000 horsepower. The capacity of main and auxiliary drives combined approaches 8,000,000 horsepower. The installed generating capacity driven by prime movers amounts to about 1,250,000 kw. The present annual power consumption of the industry is about 7,000,000,000 kilowatt-hours. All American public utilities combined generated in 1934 about 95,000,000,000 kilowatt-hours.

The rate of power consumption of American steel mills is increasing for several reasons:

1. Mills are becoming more fully electrified.
2. New applications of electricity are being developed.
3. The character of product is changing.

In 1926, about 20 per cent of the rolled product was represented by heavy sections such as rails and structurals, which involve only a moderate amount of work per ton due to moderate elongation in rolling. The total power represented in a ton of rails is about 100 kilowatt-hours. Today less than 15 per cent of the finished product leaves the mills in these heavy shapes.

Such products as sheets, strip, tin plate, and wire rod require relatively large amounts of power for rolling. A ton of sheets or rods may represent an accumulated expenditure of more than 200 kilowatt-hours of power. In 1926, about one third of the product took these forms. Today about two thirds is so rolled.

Approximately two thirds of the power consumed in our steel plants is generated in their own power plants. About one third is purchased from utilities.

The figures for 1929 are approximately as follows:

Electric power purchased from public utilities	3,500,000,000 kilowatt-hours
Electric power generated by prime movers in steel plants	6,000,000,000 kilowatt-hours
Total use of electric power	9,500,000,000 kilowatt-hours

Table I

Unit or Department	Per Cent Investment Represented by Electrical Plant
Coke ovens, blast furnaces, and steel making (open hearths)	5
Mills, average	15
Power and blowing plant	25
Total plant average	10
Modern wide strip mill (hot)	25
(cold)	10
(total)	20

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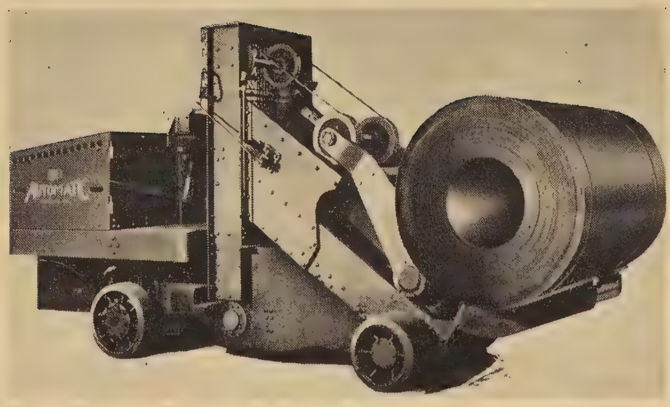

Figure 1. Industrial truck handling 8-ton coil of strip

The steel industry is probably the largest industrial consumer of electric power. However, because of the large amount of power generated in its own plants, it is only on an approximate par with 3 other industries as a purchaser of electric power from utilities. The big four are the steel industry, chemical industry, food products, and printing.

The trend, in the steel industry, is toward purchase of a greater amount of power, both in absolute and in relative sense. Three factors which encourage this trend may be mentioned:

1. Large increases in power demand are generally associated with installation of heavy mill equipment involving large expenditures. Additional simultaneous investment for power generating plant may be undesirable.

2. The increase in power consumption per ton of rolled product results in a decrease in the amount of available by-product fuels relative to the power load. Moreover, by-product fuels are being diverted to process uses where they are more valuable. The availability of by-product fuels for power generation is decreasing both in relative and in absolute sense. Hence, power generation must be based, in increasing degree, upon purchased fuel.

3. The utilities are encouraging power purchase through more reliable supply and through development of contracts suited to the special requirements of the steel mills.

4. The applications of electricity in steel mills are so varied in character and have so many ramifications that it is possible, in a brief discussion, to touch upon only a few features, selected somewhat at random, but with a view to developments of recent years.

Modernization of Generating Stations

A number of steel plants have taken steps toward modernization of their existing power plants, involving the installation of new high-pressure boilers and topping turbine generators exhausting into the existing steam mains. This practice has permitted modernization of boiler plants, increase in generating capacity, and improvement in economy to be accomplished at minimum expense. The resulting plants compare favorably in performance with those of the public utilities.

Steam Drives

Although electrification in steel plants is very general, steam drive is still retained for a few classes of equipment. Most notable are coke-oven exhausters and gas boosters and blast-furnace blowers. Exhausters are steam driven because of the nicety of speed regulation required, and because the exhaust steam is needed for process uses. Gas boosters are quite large units, well suited in speed requirement to turbine drive. Centrifugal blast-furnace blowers continue to be driven by steam turbines because of ease and perfection of speed control and also because they excel, both in over-all economy and in first cost, any method of electric drive yet proposed. Moreover, involving less steps of transformation, they are more reliable than would be motor-driven units supplied by turbine generators.

In the Buffalo district, where power is exceptionally cheap, a motor-driven centrifugal blast furnace blower has been installed. The drive is a 2-pole synchronous motor. Control of pressure and volume is by a device, termed a power wheel, which throttles the inlet pressure by means of a set of adjustable vanes somewhat akin to the vanes of a water wheel. The velocity imparted to the intake air in throttling is partially converted into power in the first stage of the compressor intead of being dissipated in eddies, as by an ordinary damper. Good regulation of volume is obtained. The equipment is quite simple. The efficiency is comparable with that which might be obtained with a wound-rotor motor drive using secondary resistance control.

Transportation

Another field in which electricity has not made great inroads is that of transportation. There are a few individual runs in steel plants where electric locomotives or cars are employed. Both overhead and third-rail systems are represented. But there is, I believe, no steel plant in this country which has a completely electrified transport system. The principal deterrent rests in the

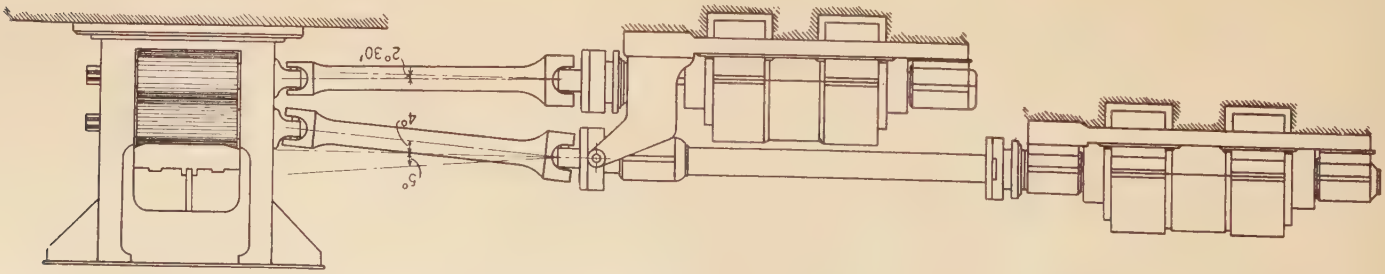

Figure 2. Twin-motor reversing drive

objections to either overhead or third-rail systems of supply.

The Diesel-electric locomotive has made its debut in the steel plant, where it is used for switching purposes. It is relatively high in first cost but it is faster in operation and the operating costs and maintenance costs are substantially lower. For some classes of service its use appears to be well warranted.

There has recently been put into service at one of the plants in the Chicago district a gas-electric locomotive (60 ton, 500 horsepower) using butane as fuel. This unit is broadly comparable in first cost with the gasoline-electric unit and comparable in operating cost with the Diesel-electric type.

Another field of transportation in which electricity is playing an important part is the industrial truck. These trucks are finding considerable use in the finishing departments of strip mills and wire mills, for instance. Storage-battery units and so-called "ready power" units employing a gasoline-engine-driven generator as the power source are both popular. Figure 1 shows a truck handling one 8-ton coil of strip.

Connected Load and Power Consumption

In a typical steel plant the large main-drive motors at the rolling mills represent less than half of the total installed motor capacity. They have a higher load factor than the auxiliary drive motors, however, and are therefore responsible for a greater relative share of the power consumption. Table II shows typical relationships.

D-c auxiliary drives are used mostly on cranes, roller tables, and manipulating devices which are subject to much starting and stopping, but which have substantial idle periods. Their load factor is very low. A-c auxiliaries include many pumps, fans, conveyors, and similar continuously running drives. Their load factor is quite high.

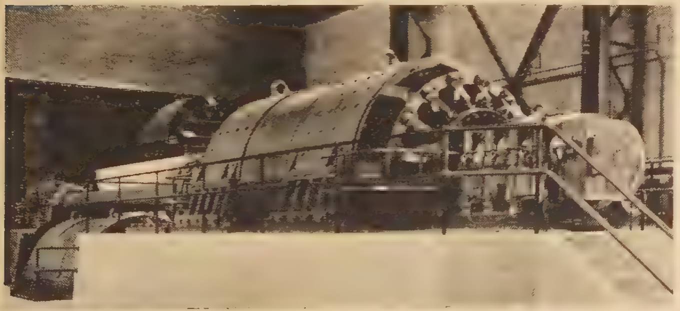

Figure 3. Twin-motor reversing drive

The Trend in Main-Roll Drives

Over a period of years, the application of motors to main-roll drives has changed in a number of respects. The changes are more or less interrelated but may be discussed separately.

Formerly the a-c motor prevailed. Today the d-c motor holds foremost place. In 1928, about 40 per cent of the main roll drive motors sold were for alternating current, 60 per cent were for direct current. In 1936, only 13 per cent were a-c motors and 87 per cent were d-c motors.

D-c main-drive motors are being so generally adopted because of their controllability, as for reversing drives, and because of their ability to meet adjustable-speed requirements. Adjustable speed is more and more commonly demanded, partly for reasons of flexibility and greater precision of rolling, partly because of higher rolling speeds, and partly because more modern continuous and semi-continuous types of mills, with stands in tandem, introduce requirements which were not met in the earlier layouts using Belgian roll trains.

Table II. Motor Capacity and Motor Load—Typical Steel Plant

Class of Motors	Per Cent of Total Installed Capacity	Per Cent of Total Motor Load
Main-roll drives	37	55
D-c auxiliaries	50	20
A-c auxiliaries	13	25

There has been a trend toward individually driven roll stands or subdivision into small groups as contrasted with the older practice in which one or two large motors drove a whole mill. Some of the reasons have been higher speeds, greater capacities, greater operating flexibility, greater flexibility of layout, mechanical simplification.

In a-c drives, there has been a trend away from the induction motor and toward the synchronous motor as will be discussed at greater length a little later.

In general, the trend has been in such direction as to increase greatly the cost of main roll drive equipment.

Reversing Drives

Reversing motors have been used for many years to drive blooming mills, roughing mills, plate mills. Somewhat more recent is their application in tandem on slabbing mills and beam mills. In a slabbing mill, for instance, 2 sets of rolls work on the piece simultaneously. The horizontal rolls reduce its thickness; the vertical rolls restrict or reduce its width. These 2 sets of rolls are driven by separate reversing motors. These motors must accelerate, run, decelerate, and reverse in unison. The relative speeds must be different in the forward and in the reverse direction of travel. This is accomplished by feeding both motors from the same generator or generators, using variable-voltage control. The relative speed of the vertical roll motor is changed, for the 2 directions of travel, through change of the field strength of that motor.

There are installed in one plant in the Chicago district 2 very interesting reversing motor drives designated as double motor drives. Each roll in the 2-high reversing

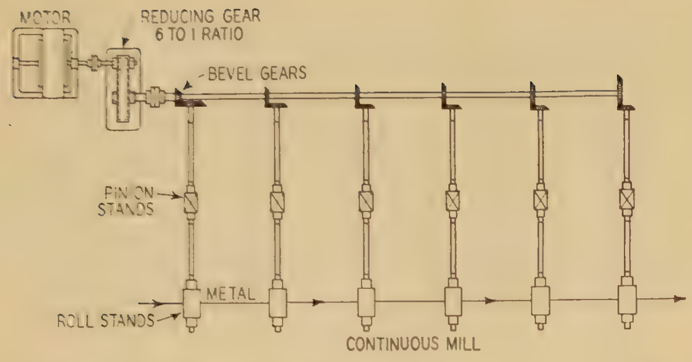

Figure 4. Group drive through lay shaft and bevel gears

mill is driven by a separate 5,000-horsepower motor. The driving spindle for the upper roll passes over the motor which drives the lower roll. The use of the twin motor is dictated, in a large measure, by mechanical considerations. The construction of a 10,000-horsepower motor at 40 rpm base speed within limits of size feasible for transport and erection, would be difficult. The manufacture of suitable mill pinions would also be difficult. The twin motor affords substantial advantage due to its relatively low inertia, resulting in a "lively" mill. It eliminates the necessity of matching roll diameters. Omission of the pinions reduces friction losses and eliminates a source of back-lash and noise.

It has long been customary to employ Ilgner-type motor generator sets for supplying direct current to reversing motors. Flywheels were provided. To drive these sets, wound-rotor induction motors were used, in conjunction with liquid slip regulators. This is still the accepted practice for blooming mills, slabbing mills, sheared plate mills, etc., where the load is heavy and passes are short and of brief duration and the load is consequently very peaked. In a few recent instances a synchronous motor has been substituted for the wound-rotor motor of the set and the flywheel has been omitted. This practice is only feasible for mills having relatively long passes. Even then the power fluctuations are so severe that they can be permitted only on a large power system. Where applicable, the synchronous motor drive has advantage in first cost, efficiency, power factor, and sustained speed as well as in greater simplicity.

A relatively new field for reversing drives is found in both hot and cold strip mills. These mills involve not alone the main reversing motor, but also the drives for the coiling reels upon which the strip is alternately wound and unwound as the mill reverses its direction of rolling. The reel motors must reverse and the speed must increase and decrease in unison with speed changes of the main reversing motor; in addition, they must compensate automatically for the varying diameter of the coil as it builds up or runs off the reel.

Synchronous Motors

Ten years ago there were very few synchronous motors in use in the steel industry. They were applied only to drive motor-generator sets and for occasional pumps or blowers. Today they are widely used for main roll drives. Perhaps more commonly they are applied to groups of stands. In many instances, however, they drive single stands. In very few instances are the starting requirements of main roll drives difficult for synchronous motors to meet. More exacting, perhaps, are the severe peak loads of brief duration sometimes experienced. The synchronous motor is now the normal selection for constant-speed main roll drives except where flywheels are required. Incidentally it may be remarked that flywheels are not now used as generally as they once were.

An interesting application of synchronous motors is that in which stands or pairs of stands in tandem, in continuous relationship, are driven by separate motors. The synchronizing power of the motors causes the rolls in the several stands to retain their speed relationship as effectively as if they were driven from a common line shaft. Figure 4 shows the former prevalent practice, employing a single drive motor with line shaft and bevel gears.

Figure 5 shows the newer practice employing subdivided drive and herringbone gearing.

The number of synchronous motors connected to steel-plant systems is increasing, not alone because of their more frequent use for main roll drives and for driving accessory equipment but also because of the great increase in motor-generator capacity occasioned by the increased adoption of adjustable-speed d-c drives.

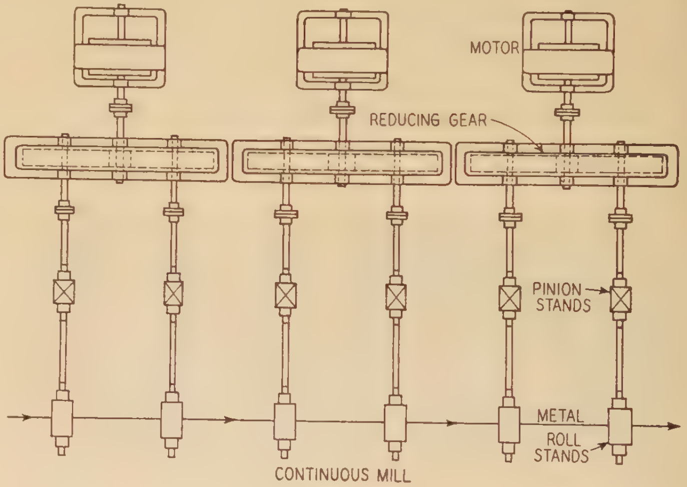

Figure 5. Subdivided drive through helical or herringbone gears

Most of the synchronous motors initially used in steel mills were rated on 80-per-cent-power-factor basis. They were overexcited and were utilized for power-factor correction. Due to the substantial increase in synchronous equipment in use, the need for power-factor correction has diminished. It is not desirable to operate 80-per-cent-power-factor motors with much less than their full excitation lest they be deficient in pull-out torque. Hence, more motors rated on 90- to 100-per-cent-power-factor basis are being selected.

D-C Drives

Adjustable speed is being provided to an ever-increasing extent for main roll stands. The d-c motor at 600 volts is practically standard for this service. Figure 6 shows a 4,500-horsepower 125/250-rpm motor of this type such as is applied to the finishing stands of a hot wide strip mill. This is representative of the maximum size attained by motors of this type. These motors are usually started by connecting them at standstill to the supplying generator or generators at zero voltage, then building up the voltage as the motors accelerate. Because of the large motor capacities involved, it is usually necessary to operate several generators in parallel to supply the direct current. Several schemes have been tried to obtain satisfactory parallel operation of these generators and, at the same time, to secure a closely maintained operating voltage. One of these schemes involved interconnected cumulative and differential series windings. It is now prevailing practice to use simple shunt-wound generators or to provide a slight differential winding to give a drooping voltage characteristic, thereby insuring satisfactory parallel operation. Generators of this type may be paralleled at zero voltage. The voltage of all units may then be built up in unison, thereby starting all the driven motors at once. When the motors attain full speed, all is in readiness for rolling. It is no longer necessary to start the motors by building up the voltage of one generator and then to parallel with the other generators before the load is applied.

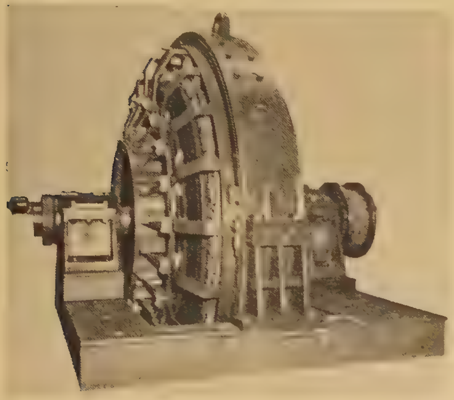

Figure 6. Adjustable-speed d-c motor

When generators with drooping voltage characteristics are employed, a voltage regulator is provided to maintain constant operating voltage. This functions in connection with an exciter which is common to the fields of all of the generators operating in parallel; see figure 7.

D-c reversing motor drives are an exception to this practice. When 2 or 3 generators are paralleled to supply a single reversing motor or combination of motors, interconnected differential and cumulative field windings are generally used to insure division of load among the generators, even under severe load changes; see figure 8.

Cold-Rolling Mills

In final cold rolling of coils of annealed strip in a so-called skin pass, it is advantageous to maintain tension upon the piece while it is being compressed between the main rolls. It is also desirable that different amounts of tension be independently maintained in the process of unwinding the coil from one reel and in winding it upon another reel. In a recent design of skin pass mills, pinch rolls are installed between the unwinding reel and the mill stand being entered; a similar pair of pinch rolls is located on the delivery side of the mill ahead of the reel upon which the coil is wound. These pinch rolls are controlled to maintain the most advantageous drag and pull tensions in the strip on the entering and delivery sides of the main rolls. The unwinding and winding reels, in turn, are controlled to maintain suitable values of drag and pull tension in the strip between these respective reels and the adjacent pinch rolls.

These tensions are governed by field control responsive to current, which determines the tension. The speed of the mill as a whole can be varied, without disturbing the relationships of the individual elements, by variation of the voltage of the generator which supplies all drives in common.

The ability to control the tension as well as the rolling pressure appears to be an important contribution to the art of cold rolling steel.

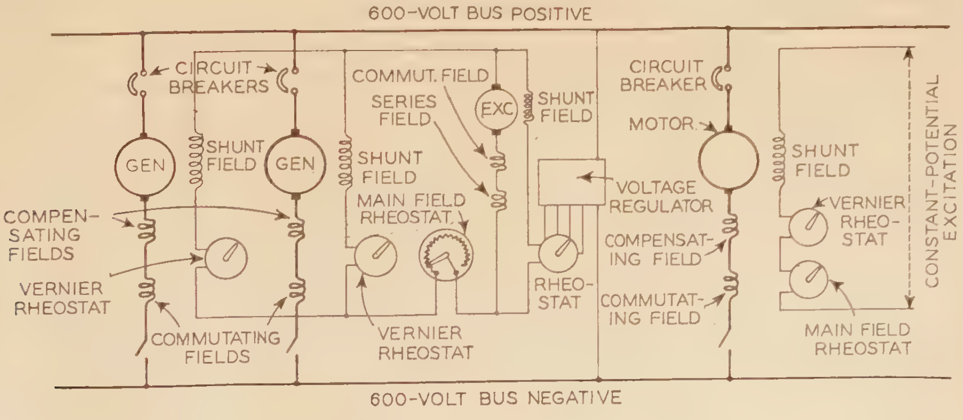

Figure 7. Parallel operation of shunt generators supplying adjustable-speed motors

Combination Gear and Pinion Stands

While discussing main roll drives, mention should be made of recent practice in which the motor speed reducing gear or the train of helical or herringbone gears, through which the motor drives, is combined with the mill pinions in a single unit. Such an arrangement results in compactness and rigidity.

Bearings

As a development affecting motor drive practice, we should mention the increasing use of roller bearings, composition bearings, and precision type oil-lubricated bearings in connection with mill rolls. These bearing types have resulted in reductions of rolling power in amounts ranging from 10 to 50 per cent, depending upon the type of mill, the product being rolled, and the particular conditions of the case.

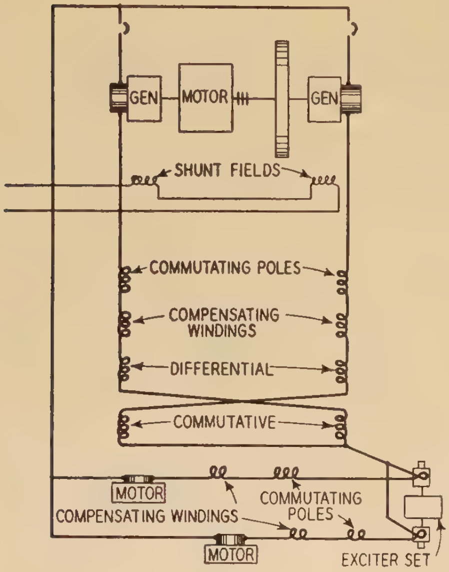

Figure 8. Parallel operation of cumulative - differential compound-wound generators

Concentrations of D-C Power

Wide strip mills usually have about 6 finishing stands, each individually driven by a motor of 2,000- to 4,000-horsepower capacity. This is a substantial aggregation of motors. See figure 9. The combined full-load rated current on a common bus connecting such a group of generators and motors may be on the order of 30,000 to 40,000 amperes. Perhaps 15,000 kw of d-c generators will be connected to such a bus. The motors may total the equivalent of 20,000 kw. Due to their flat regulation characteristics, these motors can and will act as generators almost instantly. In the event of a short circuit on such a system, the amount of fault current which can flow is tremendous. It amounts to several hundred thousand amperes, and experience, in the matter of damage done, indicates that it has plenty of wrecking ability. Very substantial bus construction using structural shapes is essential. The positive and negative busses are well separated to minimize magnetic stresses. Circuit breakers must be reinforced to prevent being bent out of shape by magnetic forces. Circuit breakers have been recently developed for this work which, under test, have ruptured successfully 70,000 amperes of direct current (as much as it is possible to provide for test purposes). These circuit breakers have ruptured successfully under test as much as 400,000 amperes at 60 cycles. The time of interruption is about 3 cycles.

An interesting development in connection with these large circuit breakers is a feature which causes them to trip particularly quickly in case the rate of increase of current is rapid, as when a severe short circuit occurs. This is accomplished in the following manner: The section of bus connecting to the circuit breaker is divided into 2 parallel paths. One path is surrounded by a laminated steel yoke. The series trip coil of the circuit breaker is interlinked with the other path. Normally the current flows equally or proportionally through the 2 paths. Only a portion of the current through the breaker passes through and acts upon its trip coil. However, when a short circuit occurs and the current rises quickly, most of the current flows through the low reactance path which is not surrounded by the steel yoke. As a consequence of this action, most of the current flowing through the breaker passes through and acts on its trip coil and causes it to trip more quickly than it would in the case of gradually applied load of equal value.

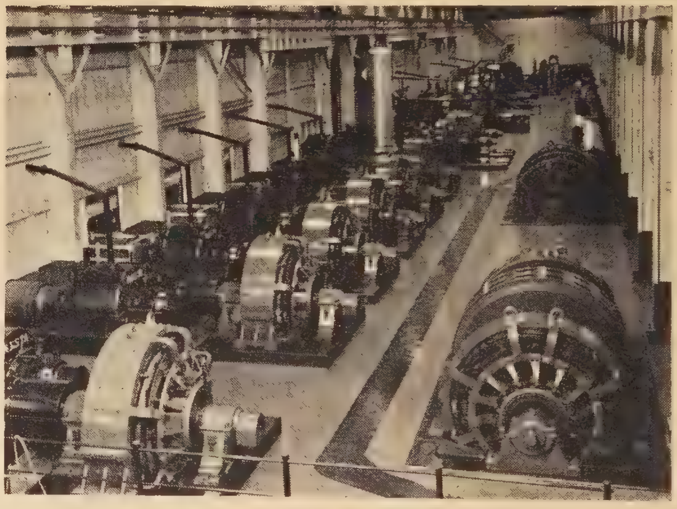

Figure 9. Motor room; wide strip mill, Youngstown Sheet and Tube Company

Auxiliary Drives

It has long been the practice to equip the main tables, side guards, manipulator and screw-down mechanisms of blooming mills with mill-type motors and magnetic controllers supplied from a 230-volt d-c system. In a recent installation, individual generators are provided to supply the respective drives and Ward-Leonard variable voltage control is employed. This arrangement results in somewhat faster action together with closer speed control over a wider range. Since approximately half of the total rolling time of such a mill is consumed in reversing and manipulating the ingot between passes, expedition of these functions may have an appreciable influence upon output. This is increasingly true in view of the higher percentage of slabs now being rolled in many such mills.

A notable recent development in auxiliary drives is the extensive application of individual small squirrel-cage induction motors to table rollers, coilers, sheet mill lifting tables. In a single mill recently constructed there are some 900 of these motors; see figure 11.

For individual table-roller drives, the adjustable-frequency system of speed control is commonly employed. The table drive motors are supplied from a-c generators which, in turn, are driven by adjustable speed d-c motors. A speed range of 3 or 4 to 1 is common.

In some cases it is necessary that these tables reverse frequently. If the table drive motors were plugged from full speed, they would draw very heavy currents from the generators supplying them and they would also overheat due to the large rotor losses incident to plugging. To avoid these difficulties, the table-drive motors are first slowed down by reducing the frequency to a low value. They are then plugged, from low speed in one direction to low speed in the other direction. They are then accelerated by increasing the frequency. By this means most of the deceleration and acceleration of the table drive motors takes place with the motors operating with running characteristics, that is, with small values of slip. Only during a small part of the reversing cycle do the motors operate with starting characteristics, that is, with large values of slip.

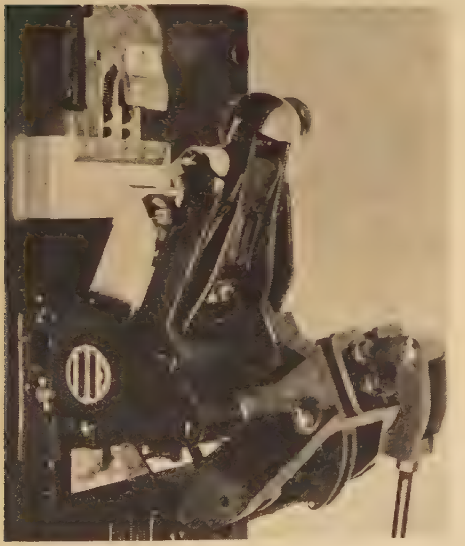

Figure 10. D-c circuit breaker

In the operation of the coilers, the mechanisms which coil the flat product emanating from a wide strip mill, frequent starting and stopping is involved. The coilers are not reversed but it is desirable to stop them quickly and accurately. The simplest possible mechanism involves the use of several separate motors of small horsepower; see figure 12. The most successful control to meet these requirements employs dynamic braking. The motors are of squirrel-cage induction type. They are supplied with power at adjustable frequency in the general manner previously described. To stop them, the frequency is first reduced, then the primary windings are disconnected from the a-c supply, and direct current at low voltage is applied to these windings. This direct current is obtained from a small motor-generator set.

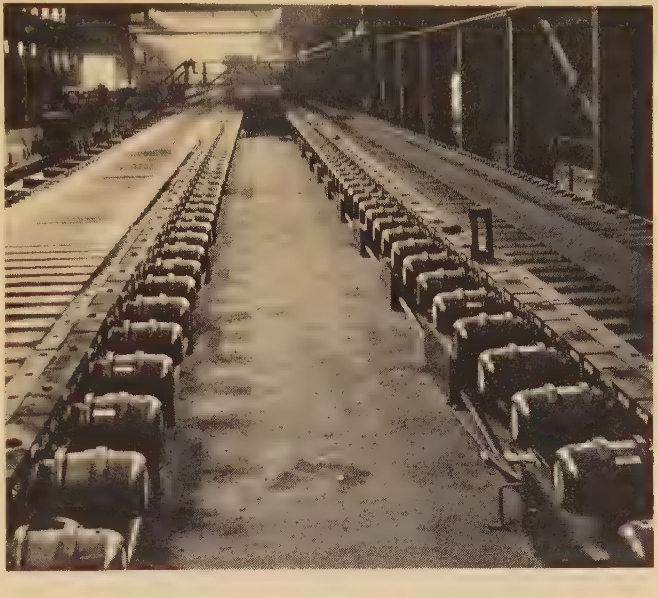

Figure 11. Individual roller-table drives

Water-Cooled Motor

Small induction motors have been applied to a number of drives which involve very frequent starting, stopping, and reversing. The load is comprised largely of inertia. Squirrel-cage motors can be made with very light rotors. When 40 starts per minute, or perhaps more, are involved, it becomes quite a problem to dissipate the heat generated in the motor. To use a larger motor does not help much since the rotor inertia increases and the load increases likewise. For such cases a water-cooled motor has been developed and tried with considerable success; see figure 13.

A-C Switching

Many steel plants have been confronted with problems in distribution and in switching, due to the rapid growth of their a-c systems. In some cases the increased use of synchronous motors has been a factor in increasing the current to be handled in case of short circuit.

In some cases the difficulties have been accentuated by the advent of purchased power from utilities. Because of the substantial amounts of purchased power involved and the desire for economy and reliability of supply, the connection between the utility generating capacity and the steel plant consuming system is usually a fairly direct one. Consequently the flow of power in case of a fault may be great. It then becomes necessary to employ generally switching equipment of high rupturing capacity, or to devise compromises using group breakers or reactors in order that existing equipment may be safely continued in service.

In general, it may be said that the switching now being installed is of much higher rupturing capacity and substantially higher cost than in the past. The scheme of distribution and switching to be adopted is therefore deserving of increased consideration.

The advent of deionizing and oil-expulsion types of circuit breakers has been timely in permitting higher rupturing capacities to be attained within existing limitations of space and reasonable limitations of cost. In this connection, it may be mentioned that the high cost of switching has stimulated the use of fewer relatively larger motor-generator sets and the use of 3-unit sets in which one synchronous motor drives 2 d-c generators.

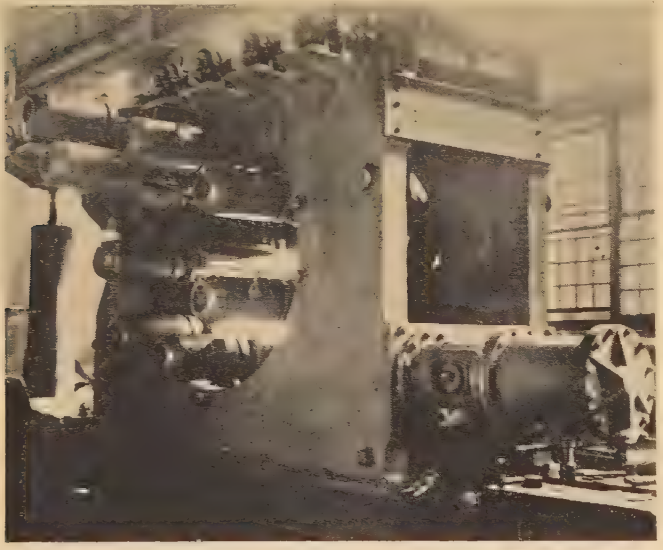

Figure 12. Individual drives for coilers

Mercury-Vapor Rectification

The first installation of a 250-volt rectifier with grid-control of voltage has recently been made in a steel mill in the Chicago district. The output is used for general d-c supply to wire drawing machinery and auxiliaries.

Electrical Precipitation

Among the newer uses of electricity in the steel mill, electrical precipitation deserves mention. This method has been successfully applied to the removal of tar from coke oven gas. There are now a number of precipitators applied to cleaning blast furnace gas. Attempts to clean this gas in the hot, dry state, preserving its sensible heat, have indicated this method to be too expensive and too sensitive to be generally acceptable. Precipitation has proved to be moderately successful as a method of fine cleaning following wet scrubbing. One deterrent to its more general adoption is the rather high cost of necessary plant.

Electric Furnaces

Electric melting furnaces and electric heating furnaces are both gaining in popularity. The amount of steel produced in electric furnaces for ingot production is a small percentage of total production. Nevertheless the percentage tends to show steady growth, as indicated by the following figures showing the relationship of the tonnage of ingots produced by electric furnaces as a per cent of total ingot tonnage:

1922	0.55 per cent
1924	0.61 per cent
1926	0.70 per cent
1928	0.90 per cent
1930	0.78 per cent
1932	1.05 per cent
1934	1.35 per cent

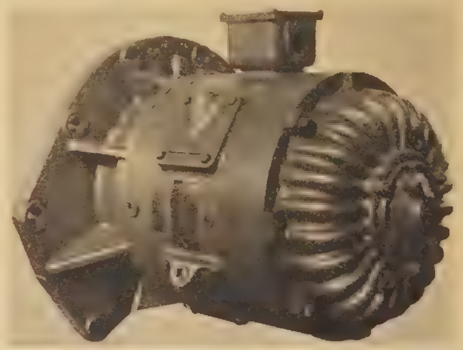

Figure 13. Water-cooled motor

Sizeable installations are now being made of electric arc furnaces of the top-charged type for the melting of scrap for production of carbon steels in small ingot sizes. These furnaces involve a substantial increase over previous practice in the ratio of transformer capacity to furnace volume.

The application of high-voltage air-break (deionizing) circuit breakers for electric arc furnace service, involving frequent circuit interruption, is deserving of mention.

The coreless induction furnace is earning recognition as one of the tools of the industry. This furnace comprises essentially an insulated crucible surrounded by a cylindrical coil of copper tubing. Through this coil, current at high frequency is passed, inducing currents in the charge in the crucible. The primary or inducing coil is water-cooled.

This type of furnace is essentially a single-phase load. The frequencies used range up to 5,000 cycles per second, but 960 cycles is a typical value. The voltage is usually about 1,000 volts. The power factor is inherently very low but is largely corrected by use of capacitors. Due to the high frequency, the cost of these capacitors is not excessive. Power of these characteristics is supplied by special motor-generator sets. The consumption ranges between 600 and 1,000 kilowatt-hours per ton of metal, being comparable with arc furnaces of similar size.

As yet the coreless induction furnace has been built only in small sizes, having capacity of one ton or less. It is probable that larger units will be developed. The principal advantages of this type of furnace are:

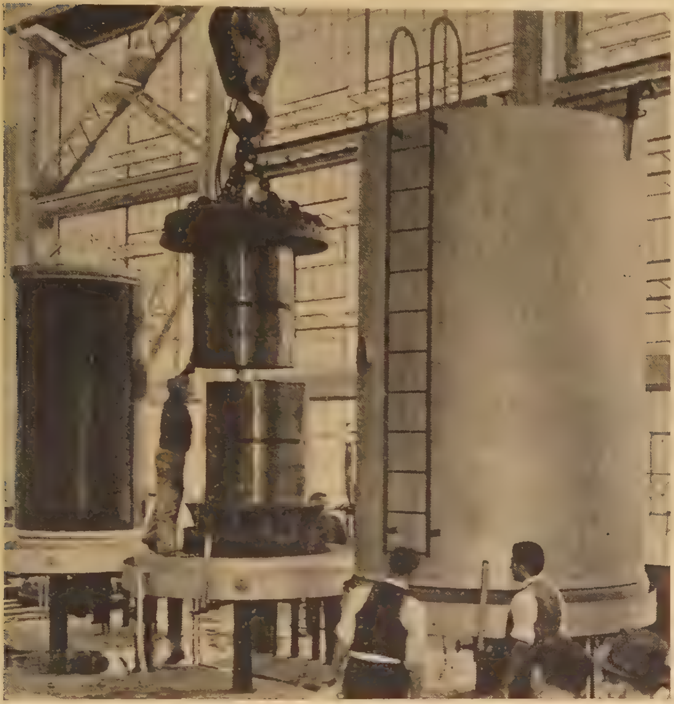

Figure 14. Bell-type cylindrical annealing furnace

1. Freedom of contamination of the charge (absence of electrodes).
2. Very high temperatures attainable since heat is internally generated.
3. Uniformity of analysis and rapid refining due in part to electromagnetic circulation of the charge which brings it into contact with the slag.
4. Ability to melt scraps containing alloys without oxidizing and losing these alloys in the melting process.

The coreless induction furnace is applied principally to production of steels of very low carbon content (such as some magnetic steels); highly refractory alloy melts; tool steels and heat-resisting alloys such as iron, nickel, chromium alloys.

High-frequency heating is being applied to a few lower temperature heating operations, such, for instance, as local annealing at circumferential pipe welds. Development of this field of electrical heating lies ahead of us.

Electric heating for annealing and heat treating operations gains steadily in prominence. Although bulk heat in electrical form is relatively expensive, advantages leading to better product often more than compensate. Better control, better furnace insulation, more advantageous and uniform heat application, less weight of auxiliary equipment heated—these are principal advantages.

Electric heating furnaces take a great variety of forms. Figure 14 shows a recent application of cylindrical bell-shaped furnaces for annealing coiled strip.

Electric heating is being adopted for some controlled-atmosphere furnaces. It is desirable that neither free air nor products of combustion be permitted to contact the metal being heated. The electric resistance furnace is inherently well suited since it is easily made externally gas tight. The direct, unmuffled heat tends to be most uniform and may be distributed to afford favorable temperature gradients for heat flow.

Electronic Control

While the amount of power used in electronic devices is negligible, they are being adapted and adopted for steel-mill use, and are performing some important control functions. One of the most general applications involves the use of photoelectric tubes for the determination of temperatures of steel during the rolling process, and for such indications as the temperatures of furnace walls or roofs.

There are other interesting electrical devices which, though small in size, are important in function. Among these should be mentioned the so-called selsyn devices for remote indication or for synchronized angular movements. The "thrustor," which utilizes a centifugal oil-pump to translate the rotary motion of a small motor into linear motion with considerable force, is finding numerous applications in the steel mill. Lifting magnets have been modified to dispose the poles and the flux more advantageously for special applications, such as handling coiled strip.

Electricity is being applied in the field of measuring and regulating devices. Sensitive electromagnet gauges are being utilized for determining with extreme accuracy the thickness of strip by continuous gauging while rolling. Strain gauges are being inserted in mill housings to measure minutely their stretch, and thereby to indicate rolling pressures. Tension rolls are being installed between the stands of tandem cold-rolling mills to indicate the tension on the piece between stands and to control the drive motors of the respective stands to maintain advantageous tension. Small winches driven by torque motors are being applied to the continuous measurement of the depth of burden in blast furnaces. These are but a few examples.

I hope that this incomplete and abbreviated résumé of high lights in the use of electricity in the steel industry will give to those of you who are not familiar with the steel mills, a glimpse of the wide range of problems which confront the steel mill electrical engineer. To those of you within the industry, I trust that this review may have suggested some new and useful slant at your every-day tasks. Of one thing we may be sure, namely: there is much that is new yet ahead.

Two-Reaction Theory of Synchronous Machines With Any Balanced Terminal Impedance

By C. CONCORDIA
MEMBER AIEE

Introduction

R. H. PARK[1,2,3] has presented a general analysis of salient-pole synchronous machines* in terms of direct, quadrature, and zero axis quantities. This analysis is very well adapted to the case of a machine with balanced series inductance and resistance in the armature circuit, but requires modification for any other terminal impedance. S. B. Crary[4] has recently extended the method of Park to include the effects of balanced armature circuit capacitance. In the present paper the same method of analysis is further extended to include the effects of *any* balanced impedance network connected to the armature terminals. Every concept (impedance operators, per-unit quantities, d and q axis quantities, etc.) of the previous synchronous machine theory is preserved unchanged in this extended theory, and every problem solvable by the previous theory is now susceptible to an exactly similar solution in the extended case. Such problems are, e.g.: 3-phase short circuit, sudden application or change of load, self-excitation, asynchronous operation, hunting, pulling into step, any prescribed change of field or armature voltage, torque under any of the conditions given above.

Application of the present theory has been made to the determination of the regions of self-excitation occurring in induction and synchronous machine circuits containing series capacitors. This is described in a related paper.[6]

Results

The results of the analysis may be stated as follows: First let

$$\begin{aligned} e_d - p\psi_d + ri_d + \psi_q(p\theta) &= D \\ e_q - p\psi_q + ri_q - \psi_d(p\theta) &= Q \\ e_0 - p\psi_0 + ri_0 &= N \end{aligned} \tag{1}$$

where $D = 0$, $Q = 0$, $N = 0$ are recognized as the 3 equations 8, 9, 10 of reference 2.

Next let the voltage of phase a due to the external network be

$$z_{11}i_a + z_{12}i_b + z_{13}i_c \tag{2}$$

with similar expressions for the voltages of phases b and c. For any balanced, 6-terminal network containing inductances, resistances, and capacitances connected in any arbitrary way there is, by ordinary circuit theory,

$$\begin{aligned} z_{ii} &= w(p)/y(p) \\ z_{ij} &= v(p)/y(p) \end{aligned} \tag{3}$$

for $i, j = 1, 2, 3$; $i \neq j$ where $w(p)$, $v(p)$, $y(p)$ are polynomials in the operator p. For example, let

$$\begin{aligned} w(p) &= d_0 + d_1p + d_2p^2 + \ldots \\ v(p) &= c_0 + c_1p + c_2p^2 + \ldots \\ y(p) &= b_0 + b_1p + b_2p^2 + \ldots \end{aligned} \tag{4}$$

Further, let

$$\begin{aligned} u(p) &= w(p) - v(p) &= a_0 + a_1p + a_2p^2 + \ldots \\ n(p) &= w(p) + 2v(p) &= e_0 + e_1p + e_2p^2 + \ldots \end{aligned}$$

Then it may be shown that the general equations of a machine, to the terminals of which the balanced impedance has been connected, are

$$\begin{aligned} & b_0D + a_0i_d + p(b_1D + a_1i_d) - (p\theta)(b_1Q + a_1i_q) + \\ & [p^2 - (p\theta)^2](b_2D + a_2i_d) - [2(p\theta)p + (p^2\theta)](b_2Q + a_2i_q) + \\ & [p^3 - 3(p\theta)^2p - 3(p\theta)(p^2\theta)](b_3D + a_3i_d) - \\ & [3(p\theta)p^2 + 3(p^2\theta)p - (p\theta)^3 + (p^3\theta)](b_3Q + a_3i_q) + \\ & [p^4 - 6(p\theta)^2p^2 - 12(p\theta)(p^2\theta)p + (p\theta)^4 - 3(p^2\theta)^2 - 4(p\theta)(p^3\theta)] \times \\ & (b_4D + a_4i_d) - \\ & [4(p\theta)p^3 + 6(p^2\theta)p^2 - 4(p\theta)^3p + 4(p^3\theta)p - 6(p\theta)^2(p^2\theta) + (p^4\theta)] \times \\ & (b_4Q + a_4i_q) + \ldots = 0 \end{aligned} \tag{5a}$$

$$\begin{aligned} & b_0Q + a_0i_q + p(b_1Q + a_1i_q) + (p\theta)(b_1D + a_1i_d) + \\ & [p^2 - (p\theta)^2](b_2Q + a_2i_q) + [2(p\theta)p + (p^2\theta)](b_2D + a_2i_d) + \\ & [p^3 - 3(p\theta)^2p - 3(p\theta)(p^2\theta)](b_3Q + a_3iq) + \\ & [3(p\theta)p^2 + 3(p^2\theta)p - (p\theta)^3 + (p^3\theta)](b_3D + a_3i_d) + \\ & [p^4 - 6(p\theta)^2p^2 - 12(p\theta)(p^2\theta)p + (p\theta)^4 - 3(p^2\theta)^2 - 4(p\theta)(p^3\theta)] \times \\ & (b_4Q + a_4i_q) + \\ & [4(p\theta)p^3 + 6(p^2\theta)p^2 - 4(p\theta)^3p + 4(p^3\theta)p - 6(p\theta)^2(p^2\theta) + (p^4\theta) \times \\ & (b_4D + a_4i_d) + \ldots = 0 \end{aligned} \tag{5b}$$

$$b_0N + e_0i_0 + p(b_1N + e_1i_0) + p^2(b_2N + e_2i_0) + \ldots = 0 \tag{5c}$$

or the zero-sequence equation may be written as

$$y(p)N + n(p)i_0 = 0 \tag{5d}$$

The derivation of these general equations is given in the appendix. Equations 5 may be used directly in place of equations 8, 9, 10 of reference 2 to determine the transient performance of a synchronous or induction machine with any connected balanced network.

Discussion

It may be shown that equations 5 yield all the previous results as special cases. For example, if there is no external impedance, all the constants a, c, d, e are zero, while

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* It is assumed that the reader is completely familiar with the first 4 of the references given at the end of the paper.

[1] For all numbered references, see list at end of paper.

b_0 may be any constant number. Then equations 5 reduce to

$$\begin{aligned} D &= 0\\ Q &= 0\\ N &= 0 \end{aligned} \qquad (6)$$

which are exactly Park's equations.[2] Similarly, if there is series capacitance in the armature circuit,

$$\frac{w(p)}{y(p)} = \frac{x_c}{p}, \quad \frac{v(p)}{y(p)} = 0$$

so $d_0 = a_0 = e_0 = x_c$, $b_1 = 1$, while all other constants vanish. Then equations 5 reduce to

$$\begin{aligned} x_ci_d + pD - (p\theta)Q &= 0\\ x_ci_q + pQ + (p\theta)D &= 0\\ x_ci_0 + pN &= 0 \end{aligned} \qquad (7)$$

which are exactly Crary's[4] equations.

The present analysis has been restricted to the case of balanced circuits. This is because the balanced case is the only one for which the use of *d*, *q*, 0 axis quantities offers any simplification in the actual solution of problems. In all cases of operation with unbalanced impedances in the armature circuit, e.g., unbalanced faults, the phase quantities should be used directly. The phase voltage equations are given in the appendix and the expressions for phase flux linkages in terms of phase currents are given in equations 10, 11, 12 of reference 1 or equations 2 of reference 2. From the point of view of the practical engineer one reason for the use of *d*, *q*, 0 axis quantities is that such use results in the formulation of his problem as a set of linear differential equations with constant coefficients. These equations may be simply solved by the ordinary operational methods with which he is familiar. However, when the armature impedances are not balanced the machine voltage equations have variable coefficients regardless of the co-ordinate system used. Thus, it is usually simpler in such cases to use phase quantities for voltage, flux, and current; on the other hand the machine impedances should *always* be expressed along the *d* and *q* axes.

For a large class of problems the armature and its connected circuits may be treated as operating in the steady state and only the rotor circuit transients need be considered. This form of treatment is the basis of the practical utility of the definitions of transient and subtransient reactances and is analytically equivalent to the neglect of the $p\psi$ terms of the voltage equations. It is obvious that when the armature circuits are in steady state operation the presence of any connected impedance does not complicate the fundamental equations, and equations 6, with $p\psi = 0$, will apply to any case of balanced armature impedance. From this point of view the exact equations derived here are seen as a refinement and need be used only for exceptional cases. However, in these exceptional cases the refinement is all-important, as entirely misleading and useless results may be obtained without it. Such, e.g., was the case in the calculation of the regions of self-excitation in references 4 and 6.

Examples

1. Suppose the external impedance is simply a balanced series self- and mutual inductance and resistance, the phase *a* voltage being

$$(r_s + pl_s)i_a - pm_s(i_b + i_c) \qquad (8)$$

then

$$\begin{aligned} w(p) &= r_s + l_sp\\ v(p) &= -m_sp\\ y(p) &= 1 \end{aligned}$$

Therefore,

$$\begin{array}{llll} a_0 = r_s, & a_1 = (l - m), & a_2 = 0, & \ldots\\ e_0 = r_s, & e_1 = (l + 2m), & e_2 = 0, & \ldots\\ b_0 = 1, & b_1 = 0, & \ldots & \end{array}$$

and by equations 5,

$$\begin{aligned} D + r_si_d + p(l - m)i_d - (p\theta)(l - m)i_q &= 0\\ Q + r_si_q + p(l - m)i_q + (p\theta)(l - m)i_d &= 0\\ N + r_si_0 + p(l + 2m)i_0 &= 0 \end{aligned} \qquad (9)$$

It is evident that these equations may be reduced at once to the elementary form

$$\begin{aligned} D' &= e_d - p\psi_d' + r'i_d + \psi_q'(p\theta) = 0\\ Q' &= e_q - p\psi_q' + r'i_q - \psi_d'(p\theta) = 0\\ N' &= e_0 - p\psi_0' + r'i_0 = 0 \end{aligned} \qquad (10)$$

where the primed quantities are of the same nature as the unprimed quantities, and

$$\begin{aligned} r' &= r + r_s\\ \psi_d' &= \psi_d - (l - m)i_d\\ \psi_q' &= \psi_q - (l - m)i_q\\ \psi_0' &= \psi_0 - (l + 2m)i_0 \end{aligned} \qquad (11)$$

2. Suppose the external circuit consists of an inductance coil (having also resistance) in parallel with a capacitor in each phase. Then

$$\frac{w(p)}{y(p)} = \frac{1}{\dfrac{1}{r_l + x_lp} + \dfrac{p}{x_c}} = \frac{x_cr_l + x_cx_lp}{x_c + r_lp + x_lp^2} \qquad (12)$$

and

$$\frac{v(p)}{y(p)} = 0$$

Therefore

$$\begin{array}{lllll} a_0 = x_cr_l, & a_1 = x_cx_l, & a_2 = 0, & \ldots & \\ e_0 = x_cr_l, & e_1 = x_cx_l, & e_2 = 0, & \ldots & \\ b_0 = x_c, & b_1 = r_l, & b_2 = x_l, & b_3 = 0, & \ldots \end{array}$$

and by equations 5

$$\begin{aligned} &x_cD + x_cr_li_d + p(r_lD + x_cx_li_d) - (p\theta)(r_lQ + x_cx_li_q) +\\ &\qquad [p^2 - (p\theta)^2]x_lD - [2(p\theta)p + (p^2\theta)]x_lQ = 0\\ &x_cQ + x_cr_li_q + p(r_lQ + x_cx_li_q) + (p\theta)(r_lD + x_cx_li_d) +\\ &\qquad [p^2 - (p\theta)^2]x_lQ + [2(p\theta)p + (p^2\theta)]x_lD = 0\\ &(x_c + r_lp + x_lp^2)N + (x_cr_l + x_cx_lp)i_0 = 0 \end{aligned} \qquad (13)$$

These equations may be used to determine the transient performance of, e.g., a synchronous machine supplying an open-circuited transformer and shunt capacitor. In many cases it will be possible to neglect the transformer resistance, thus effecting a considerable simplification.

3. Let the external *inductance* be zero in the previous case. Then

$$\begin{aligned} x_cD + x_cr_li_d + pr_lD - (p\theta)r_lQ &= 0 \\ x_cQ + x_cr_li_q + pr_lQ + (p\theta)r_lD &= 0 \\ (x_c + r_lp)N + x_cr_li_0 &= 0 \end{aligned} \tag{14}$$

These equations, when put in the form

$$\begin{aligned} \left(p + \frac{x_c}{r_l}\right) D + x_ci_d - (p\theta)Q &= 0 \\ \left(p + \frac{x_c}{r_l}\right) Q + x_ci_q + (p\theta)D &= 0 \\ \left(p + \frac{x_c}{r_l}\right) N + x_ci_0 &= 0 \end{aligned} \tag{15}$$

are seen to be of exactly the same form as Crary's[4] equations but with p replaced by $(p+x_c/r_l)$ wherever it operates on the elementary functions D, Q, N. Putting the resistor in shunt with the capacitor thus does not complicate the problem but only introduces the ratio x_c/r_l where r_l/x_c may be interpreted as the time constant of the resistor-capacitor circuit considered by itself.

Appendix I

1. We first derive Park's equations in their original form from the phase equations.

We have

$$e_a - p\psi_a + ri_a = 0 \tag{16}$$

with 2 similar equations for phases b and c. There are also the 9 transformation equations (equations 14, 15, 16 of reference 2).

$$_a = i_d \cos\theta - i_q \sin\theta + i_0, \text{ etc.} \tag{17}$$

Substitute 17 in 16 and obtain

$$e_d \cos\theta - e_q \sin\theta + e_0 - p(\psi_d \cos\theta - \psi_q \sin\theta + \psi_0) + r(i_d \cos\theta - i_q \sin\theta + i_0) = 0$$

Perform the indicated differentiations and segregate the coefficients of the trigonometric functions.

$$[e_d - p\psi_d + ri_d + \psi_q(p\theta)] \cos\theta - [e_q - p\psi_q + ri_q - \psi_d(p\theta)] \sin\theta + [e_0 - p\psi_0 + ri_0] = 0 \tag{18}$$

By combining equation 18 with the 2 similar equations for phases b and c it may be shown that the coefficients of 18 must be separately equal to zero. Then,

$$\begin{aligned} e_d - p\psi_d + ri_d + \psi_q(p\theta) &= 0 \\ e_q - p\psi_q + ri_q - \psi_d(p\theta) &= 0 \\ e_0 - p\psi_0 + ri_0 &= 0 \end{aligned} \tag{19}$$

which are the familiar stationary axis equations.

2. Now suppose there is connected to the terminals of the machine a balanced impedance network containing inductances, resistances, and capacitances connected in any manner. The network may be represented by the operational expression $w(p)/y(p)$ where $w(p)$, $y(p)$ are polynomials in p as shown in equations 4. Equations 16 are now replaced by

$$e_a - p\psi_a + ri_a + \frac{w(p)}{y(p)} i_a = 0 \tag{20}$$

If the impedance network has, in addition, mutual effects which cannot be eliminated by the use of an equivalent circuit there is also a mutual impedance $v(p)/y(p)$ where $v(p)$ is a polynomial as shown in equation 4. Since the circuit is balanced all the mutual coefficients are the same. The phase a voltage equation now becomes

$$e_a - p\psi_a + ri_a + \frac{w(p)}{y(p)} i_a + \frac{v(p)}{y(p)} (i_b + i_c) = 0 \tag{21}$$

By the substitution

$$i_b + i_c = 3i_0 - i_a \tag{22}$$

we obtain after clearing of fractions,

$$y(p)(e_a - p\psi_a + ri_a) + [w(p) - v(p)]i_a + 3v(p)i_0 = 0 \tag{23}$$

The result of the previous section for the special case of no external impedance can still be used now, i.e., expression 18 can be substituted directly for $(e_a - p\psi_a + ri_a)$ in 23. Then

$$y(p)\{[e_d - p\psi_d + ri_d + \psi_q(p\theta)] \cos\theta - [e_q - p\psi_q + ri_q - \psi_d(p\theta)] \times \sin\theta + [e_0 - p\psi_0 + ri_0]\} + [w(p) - v(p)](i_d \cos\theta - i_q \sin\theta + i_0) + 3v(p)i_0 = 0 \tag{24}$$

By equations 1

$$y(p)(D\cos\theta - Q\sin\theta + N) + [w(p) - v(p)] \times (i_d \cos\theta - i_q \sin\theta + i_0) + 3v(p)i_0 = 0 \tag{25}$$

and by (4)

$$(b_0 + b_1p + b_2p^2 + \ldots)(D\cos\theta - Q\sin\theta + N) + (a_0 + a_1p + a_2p^2 + \ldots)(i_d \cos\theta - i_q \sin\theta + i_0) + 3v(p)i_0 = 0 \tag{26}$$

As before, perform the differentiations required to segregate the trigonometric coefficients.

$$b_0(D\cos\theta - Q\sin\theta + N) + b_1[(pD)\cos\theta - D\sin\theta(p\theta) - (pQ)\sin\theta - Q\cos\theta(p\theta) + pN] + \ldots + a_0(i_d\cos\theta - i_q\sin\theta + i_0) + a_1[(pi_d)\cos\theta - i_d\sin\theta(p\theta) - (pi_q)\sin\theta - i_q\cos\theta(p\theta) + pi_0] + \ldots + 3v(p)i_0 = 0 \tag{27}$$

Again utilizing the result of the previous section, the coefficients of $\cos\theta$, $\sin\theta$, and the constant term are equated separately to zero. The result is the set of equations 5. These equations have been given explicitly for polynomials of up to the fourth degree in p, but further terms may be formed as indicated in equation 27. It is probable, however, that the mathematical complexity encountered will preclude the solution of the equations for polynomials of any higher degree and perhaps even for the fourth degree. It is therefore believed that the equations given are as complete as will be necessary for any practical case.

Appendix II

Nomenclature

The notation is in general the same as that of Park[1,2,3] and Crary.[4]

e = armature voltage
i = armature current
ψ = armature flux linkage
d, q, 0 = subscripts denoting direct-axis, quadrature-axis, and zero-sequence quantities, respectively
p = d/dt
r = armature phase resistance
θ = rotor angular displacement
a, b, c = subscripts denoting the 3 armature phases
x_c = capacitive reactance

Appendix III

In the last few years considerable interest has been shown in the application of tensor analysis to electrical engineering problems.[5] A derivation of the general equations of this paper in tensor symbolism is therefore appended below. It is believed that this form of derivation has the great virtue that, to one familiar with the notation, the principal and necessary steps of the derivation stand out clearly. That is, the main line of reasoning is not obscured by lengthy algebraic manipulations.

The voltage equations 21 in terms of phase quantities may be written as:

$$\boldsymbol{e} + \boldsymbol{R}\cdot\boldsymbol{i} - p\boldsymbol{\psi} + \boldsymbol{Z}\cdot\boldsymbol{i} = 0 \qquad (28)$$

where $\boldsymbol{Z}$ is the impedance tensor of the outside network having components specified by equation 3.

Now if voltage and current are expressed along the d, q, 0 axes by transforming $\boldsymbol{e}$, $\boldsymbol{i}$, $\boldsymbol{\psi}$ by Park's transformation $\boldsymbol{C}$, where $\boldsymbol{C}$ is the tensor whose components are the coefficients of equations 14, 15, or 16 of reference 2, equation 28 becomes:

$$\boldsymbol{C}\cdot\boldsymbol{e}' + \boldsymbol{R}\cdot\boldsymbol{C}\cdot\boldsymbol{i}' - p\boldsymbol{C}\cdot\boldsymbol{\psi}' + \boldsymbol{Z}\cdot\boldsymbol{C}\cdot\boldsymbol{i}' = 0 \qquad (29)$$

where the primed quantities are measured along the d, q, 0 axes.

By multiplying the first 3 terms of 29 by $\boldsymbol{C}\cdot\boldsymbol{C}^{-1}$, where $\boldsymbol{C}^{-1}$ is the inverse of $\boldsymbol{C}$, equation 29 becomes:

$$\boldsymbol{C}\cdot\boldsymbol{P} + \boldsymbol{Z}\cdot\boldsymbol{C}\cdot\boldsymbol{i}' = 0 \qquad (30)$$

where

$$\boldsymbol{P} = \boldsymbol{e}' + \boldsymbol{R}\cdot\boldsymbol{i}' - p\boldsymbol{\psi}' - \boldsymbol{J}\cdot\boldsymbol{\psi}'p\theta \qquad (30a)$$

$\boldsymbol{P}$ is seen to be the expression which in Park's analysis is equal to zero. The tensor $\boldsymbol{J}$ is

$$\boldsymbol{J} = \boldsymbol{C}^{-1}\cdot\frac{d\boldsymbol{C}}{d\theta} = \begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline 1 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline\end{array} \qquad (31)$$

The external impedance $\boldsymbol{Z}$ may be expressed as

$$\boldsymbol{Z} = \frac{1}{y(p)}\boldsymbol{A} \qquad (32)$$

where $\boldsymbol{A}$ is the tensor whose elements are $a_{ii} = w(p)$, $a_{ij} = v(p)$ $i \neq j$. Then 30 becomes

$$y(p)\boldsymbol{C}\cdot\boldsymbol{P} + \boldsymbol{A}\cdot\boldsymbol{C}\cdot\boldsymbol{i}' = 0 \qquad (33)$$

But this may be expressed as

$$(b_0 + b_1p + \ldots)\boldsymbol{C}\cdot\boldsymbol{P} + (\boldsymbol{A}_0 + \boldsymbol{A}_1p + \ldots)\cdot\boldsymbol{C}\cdot\boldsymbol{i}' = 0 \qquad (34)$$

where

$$\boldsymbol{A}_0 = \begin{array}{|c|c|c|}\hline d_0 & c_0 & c_0 \\ \hline c_0 & d_0 & c_0 \\ \hline c_0 & c_0 & d_0 \\ \hline\end{array} \qquad \boldsymbol{A}_1 = \begin{array}{|c|c|c|}\hline d_1 & c_1 & c_1 \\ \hline c_1 & d_1 & c_1 \\ \hline c_1 & c_1 & d_1 \\ \hline\end{array} \quad \text{etc.} \qquad (35)$$

If in equation 34 the indicated differentiations are performed, the result is

$$b_0\boldsymbol{C}\cdot\boldsymbol{P} + b_1\left[\frac{d\boldsymbol{C}}{d\theta}\cdot\boldsymbol{P}p\theta + \boldsymbol{C}\cdot p\boldsymbol{P}\right] + \ldots + \boldsymbol{A}_0\cdot\boldsymbol{C}\cdot\boldsymbol{i}' + \boldsymbol{A}\cdot\frac{d\boldsymbol{C}}{d\theta}\cdot\boldsymbol{i}'p\theta + \ldots = 0 \qquad (36)$$

It remains only to multiply the whole equation 36 by the inverse transformation tensor $\boldsymbol{C}^{-1}$ to obtain the set of equations 5. The elements of $\boldsymbol{C}^{-1}$ are the coefficients of the set of equations 3, 6, or 7 of reference 2. That this multiplication eliminates the $\boldsymbol{C}$'s is evident since:

$$\boldsymbol{C}^{-1}\cdot\boldsymbol{C} = \begin{array}{|c|c|c|}\hline 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ \hline\end{array} \qquad (37)$$

$$\boldsymbol{C}^{-1}\cdot\frac{d\boldsymbol{C}}{d\theta} = -\boldsymbol{C}^{-1}\cdot\frac{d^3\boldsymbol{C}}{d\theta^3} = \boldsymbol{C}^{-1}\cdot\frac{d^5\boldsymbol{C}}{d\theta^5} = \text{etc.,} = \begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline 1 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline\end{array} = \boldsymbol{J} \qquad (38)$$

$$-\boldsymbol{C}^{-1}\cdot\frac{d^2\boldsymbol{C}}{d\theta^2} = \boldsymbol{C}^{-1}\cdot\frac{d^4\boldsymbol{C}}{d\theta^4} = -\boldsymbol{C}^{-1}\cdot\frac{d^6\boldsymbol{C}}{d\theta^6} = \text{etc.,} = \begin{array}{|c|c|c|}\hline 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline\end{array} \qquad (39)$$

The final equation is thus:

$$b_0\boldsymbol{P} + b_1[\boldsymbol{J}\cdot\boldsymbol{P}p\theta + p\boldsymbol{P}] + \ldots + \boldsymbol{A}_0'\cdot\boldsymbol{i}' + \boldsymbol{A}_1''\cdot\boldsymbol{i}'p\theta + \boldsymbol{A}_1'\cdot p\boldsymbol{i}' + \ldots = 0 \qquad (40)$$

where

$$\boldsymbol{A}_0' = \boldsymbol{C}^{-1}\cdot\boldsymbol{A}_0\cdot\boldsymbol{C} = \begin{array}{|c|c|c|}\hline a_0 & 0 & 0 \\ \hline 0 & a_0 & 0 \\ \hline 0 & 0 & e_0 \\ \hline\end{array}$$

$$\boldsymbol{A}_1'' = \boldsymbol{C}^{-1}\cdot\boldsymbol{A}_1\cdot\frac{d\boldsymbol{C}}{d\theta} = \begin{array}{|c|c|c|}\hline 0 & -a_1 & 0 \\ \hline a_1 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline\end{array}$$

$$\boldsymbol{A}_1' = \boldsymbol{C}^{-1}\cdot\boldsymbol{A}_1\cdot\boldsymbol{C} = \begin{array}{|c|c|c|}\hline a_1 & 0 & 0 \\ \hline 0 & a_1 & 0 \\ \hline 0 & 0 & e_1 \\ \hline\end{array}$$

It will be noted that here the same transformation $\boldsymbol{C}$ has been applied to both $\boldsymbol{e}$ and $\boldsymbol{i}$, rather than applying the transformation $\boldsymbol{C}_t^{-1}$ to $\boldsymbol{e}$. The latter transformation has the property of maintaining the power $\boldsymbol{e}\cdot\boldsymbol{i}$ as an invariant, while in the former the only requirement has been that the trigonometric functions be eliminated from the differential equation $\boldsymbol{e} = \boldsymbol{Z}\cdot\boldsymbol{i}$, without regard to the formula for power. A transformation tensor can, of course, easily be found which will resemble as nearly as possible Park's transformation yet which has the further property of invariance of power. This transformation tensor is:

$$\boldsymbol{C} = \sqrt{\frac{2}{3}}\begin{array}{|c|c|c|}\hline \cos\theta_1 & -\sin\theta_1 & \sqrt{1/2} \\ \hline \cos\theta_2 & -\sin\theta_2 & \sqrt{1/2} \\ \hline \cos\theta_3 & -\sin\theta_3 & \sqrt{1/2} \\ \hline\end{array} \qquad (41)$$

where $\theta_2 = \theta_1 - 120°$, $\theta_3 = \theta_1 + 120°$. It will be found that now $\boldsymbol{C}_t^{-1} = \boldsymbol{C}$, so that it is immaterial which is used in the derivation of the equations.

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Resistance Characteristics of Tellurium and Silver-Tellurium Alloys

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Synopsis

The problem of correcting temperature errors in electrical instruments with copper windings is explained and the high electrical efficiency obtainable by correcting these errors with series resistors having negative temperature coefficients of resistance is noted. A method is described for making such resistors of tellurium and of silver-tellurium alloy. Tests at various loads are described as a result of which the suitability of these resistors for various applications is determined.

General

THE WINDINGS of electrical instruments and meters must usually be made of copper or aluminum wire to obtain high conductivity. Where, as in a millivoltmeter, the resistance of the winding has an effect on the accuracy of the instrument, the high temperature coefficient of resistance of the winding causes the instrument to have a correspondingly high temperature error.

A simple method commonly used for correcting this error is to connect a resistor of some material of low temperature coefficient, such as manganin, in series with the winding. The chief disadvantage of this method is that it increases the full-scale millivolts of the instrument in nearly an inverse ratio to the decrease in the temperature error, a total increase of 19 times[1] in the full-scale millivolts being required for complete compensation.

Another method which is more efficient and also more complicated makes use of a network of manganin and copper, iron, or nickel resistors. Brooks[1] has shown that by this method it is possible to compensate accurately the temperature error of a millivoltmeter by increasing the full-scale millivolts to nearly 5 times the millivolts of the uncompensated instrument.

Compensation by means of a series resistor having a negative temperature coefficient of resistance is an efficient method which has been used in instruments for many years. To the writer's knowledge, the only materials of negative temperature coefficient which have previously been known to have the required linear temperature-resistance curves are carbon and certain carbides. These materials as used in commercial instruments have negative temperature coefficients of resistance nearly equal numerically to the positive temperature coefficient of resistance of copper. Complete compensation of the temperature error is obtained at the expense of a 100 per cent increase in the full-scale millivolts of the uncompensated instrument. (Temperature errors due to the magnet and springs are disregarded because they are much smaller than the error due to the resistance change and may be compensated by the same method.) While instruments compensated with these materials are more sensitive than are instruments compensated by the other methods, a further gain in sensitivity may be obtained by the use of materials of higher negative temperature coefficients of resistance. The materials which are to be described have temperature coefficients which are numerically about 3 times as great as the temperature coefficient of copper. This makes it possible to obtain complete compensation at the expense of only a 33 per cent increase in the full-scale millivolts of the instrument.

In a search which the writer made for information on materials of negative temperature coefficient of resistance, he noted the values given in the International Critical Tables[2] for silver-tellurium alloys slowly cooled from their melting temperatures. These data were quoted from a thesis written by Riviere at the University of Paris in 1912.

All available data on tellurium indicated that it was inconstant in resistance and that small traces of impurities affected its resistivity so greatly as to preclude the use of commercial tellurium as a resistor material.[3]

Method of Fabricating and Testing Resistors

As the result of tests it was found that the silver-tellurium alloys could be readily formed into rods by melting the constituents in a porcelain crucible, stirring the melt thoroughly, and drawing the molten alloy into a long glass tube of which one end was plunged under the surface of the melt and the other was attached to a rubber bulb used to create a partial vacuum in the tube. The rods could sometimes be pushed out of the tubes without breaking, but it was often necessary to dissolve the tubes in hydrofluoric acid in order to obtain the rods.

The method of attaching the leads was to electroplate the ends of the resistors with nickel, cobalt, platinum, or rhodium and solder them on with ordinary lead-tin solder. No failures of any of the resistors were observed to be traceable to plated joints in which any of these four metals were used. Most of the tests were made on resistors with platinum- or rhodium-plated joints.

The temperature-resistance tests were made by connecting the resistor to a Wheatstone bridge and immersing it in a bath of water for temperatures above 0 degrees

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[1] For all numbered references, see list at end of paper.

centigrade and of alcohol cooled with solid carbon dioxide for temperatures below zero degrees centigrade. Check tests with several different voltages applied to the bridge were frequently taken in order to detect any variation in resistance with current density. No changes in resistance were detected in this way except those attributable to self heating.

Improvement in Alloy to Obtain Linear Temperature-Resistance Curve

Tests on resistors made from an alloy of 15 per cent Ag with 85 per cent Te melted under illuminating gas to prevent oxidation gave temperature-resistance curves similar to that shown in figure 1. The method by which the resistors were heat treated will be explained later. It will be noted that, like those of Riviere's alloys which were all melted in a nonoxidizing atmosphere, the curve shows a sudden departure from a linear form near −15 degrees centigrade and below this temperature the slope changes rapidly. When resistors so made were suddenly cooled to any temperature between −15 and −60 degrees centigrade it was noted that their resistances did not at once assume definite values but that they slowly increased for an hour or more. The resistance corresponding to the temperature on any linear portion of the curve was attained quickly, and no subsequent variation was observed to occur while the temperature was held constant.

A melt which had not been protected from oxidation was found to have a linear temperature-resistance curve from −70 to +70 degrees centigrade and no resistance variation occurred while the temperature was held at any value in this range. As a result of tests on several melts of each type, the nonlinear and erratic resistance variation was found to be characteristic of the alloy melted in illuminating gas, and the linear temperature-resistance curve with a definite resistance for each temperature was found in every melt that had been melted in air.

All melts subsequently made were stirred in the presence of air until the formation of a scum of undissolved oxide indicated that no more would dissolve.

Development of Heat-Treating Process

In view of recent developments in the age hardening of alloys it appeared that the results that Riviere obtained by slowly cooling the Ag-Te alloys might be due to precipitation and that they could be more effectively accomplished by annealing at a constant temperature than by slow cooling. The results of subsequent tests tend to confirm this assumption.

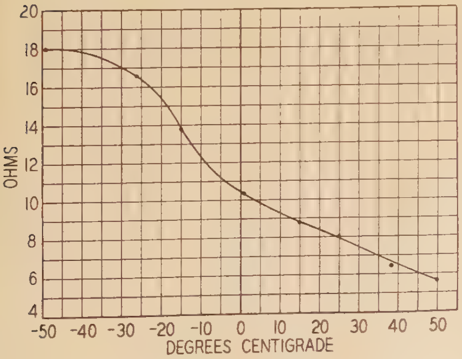

Figure 1. Temperature - resistance curve of a 15 per cent silver-tellurium alloy resistor melted in an atmosphere of illuminating gas and annealed 15 hours at 115 degrees centigrade

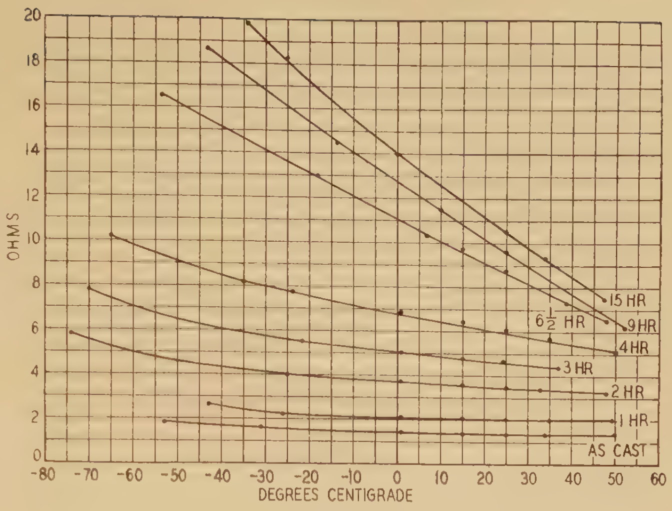

Figure 2. Effect of annealing time at 115 degrees centigrade on the temperature-resistance curve of a resistor of 15 per cent silver with 85 per cent tellurium. Effective length of resistor, $^9/_{32}$ inch; diameter of resistor, 0.102 inch

Figure 2 shows the effect of annealing at a constant temperature of 115 degrees centigrade on the temperature-resistance curve of a 15 per cent Ag, 85 per cent Te alloy. In order to get the data in this form, it was necessary to perform the annealing in a series of steps, taking a temperature-resistance curve between each of the steps. No difference has been observed between resistors annealed in in this way and resistors which were given the same total hours annealing time in a single treatment.

Groups of curves similar to those shown in figure 2 were taken for annealing temperatures from 105 degrees centigrade to 145 degrees centigrade and the results are summarized in figures 3 and 4. Some data were taken at 95 degrees centigrade, but at this temperature the rate of change was so slow that it would apparently have required a period of weeks to effect the entire change.

The increase in resistivity during the heat treatment, with a few exceptions which may be due to cracks, almost exactly parallels the increase in the negative temperature coefficient of resistance of both in 15 per cent Ag-Te alloy and of the purest grade of tellurium which we could readily obtain (Eimer and Amend *CP*). This tends to show that the same physical change is responsible for both effects.

Effect of Direct Current on Polycrystalline Resistors of 15 Per Cent Silver-Tellurium Alloy

In the course of tests made under various conditions on resistors prepared and annealed as previously described, it was found that when a direct current of 25 milliamperes was continuously passed through a resistor of approximately 0.10 inch in diameter its resistance decreased for a period of from 2 to 4 weeks when it had decreased about

10 per cent after which time its resistance began to increase. This increase in resistance was evidently due to cracks which occurred spontaneously and gradually extended until they caused a fracture of the resistor. It was also found that the same effect could be produced by immersing the resistor in either oil or water for cooling and passing a direct current of 0.9 ampere through it for one to 2 hours.

In studying this effect it was noted that these resistors had a structure as shown in figure 5. The first visible change in the resistor previous to failure was usually a widening of the longitudinal crack, followed by the development of cracks which branched from the longitudinal crack, usually at right angles to it. These cracks appeared to develop at crystal boundaries, so it was thought that if the resistor were so made that the current did not pass through the boundaries between crystals, this effect might be eliminated.

Effect of Direct Current on Single-Crystal Resistors of 15 Per Cent Silver-Tellurium Alloy

In order to determine whether the type of failure just described was related to the crystal structure, tests were conducted as follows: Sealed glass tubes containing the 15 per cent alloy were passed through a 480 degree-centigrade furnace at a rate of $^3/_8$ inch per minute. The alloy was molten at this temperature, and the time that it remained in the furnace after melting was not important. Where the tube emerged from the lower end of the furnace it passed into an oil bath. Rods made in this manner were found to have a single-crystal structure. Resistors made from these rods showed practically the same temperature-resistance characteristics and responded in the same manner to heat treatment as did those made by the simple casting process.

A number of these resistors were placed on a life test with a direct current of 25 milliamperes flowing continuously through them. Their resistances continued to decrease for a period of several weeks, the total decrease ranging from 8 per cent to 20 per cent and the length of time required to attain minimum resistance ranging from 3 weeks to 7 months, most of the change in every case taking place in the first month. After attaining the minimum resistance some of the resistors tended to remain constant in resistance for several months and others increased slowly in resistance. After a period of 257 days all of a group of 4 resistors which were tested under these conditions showed a bulge at a distance of $^1/_8$ inch from the cathode end of the resistor. Cracks appeared in these bulged sections and the resistors broke spontaneously. Figure 6 shows photographs of a resistor in which this bulge has developed, together with a similar resistor which has not been subjected to direct current long enough for this type of failure to develop.

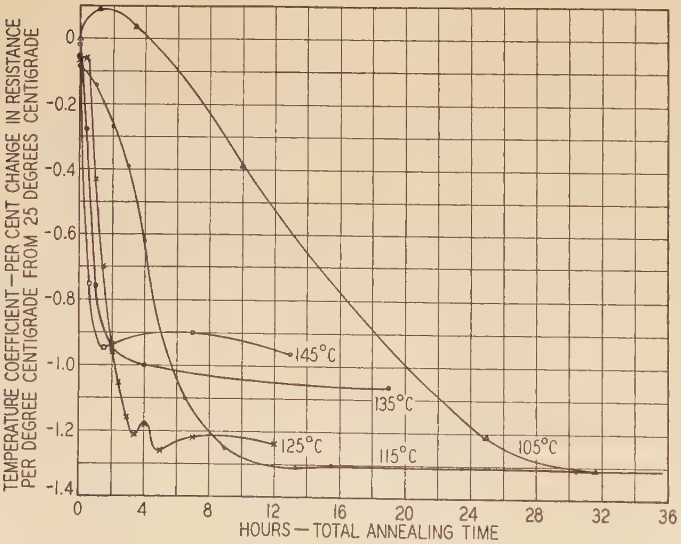

Figure 3. Effects of time and temperature of annealing on the temperature-resistance coefficient of an alloy of 15 per cent silver with 85 per cent tellurium

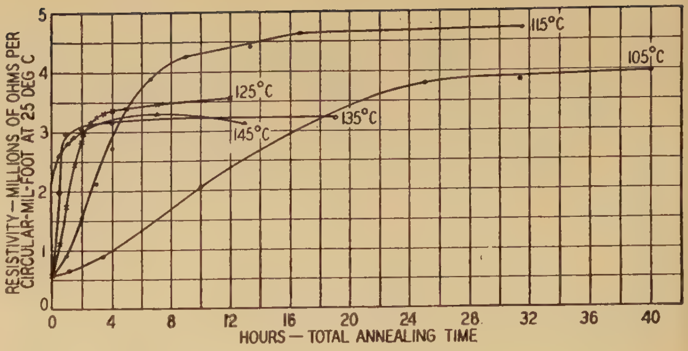

Figure 4. Effects of time and temperature of annealing on the resistivity at 25 degrees centigrade of an alloy of 15 per cent silver with 85 per cent tellurium

Effect of Alternating Current on Single-Crystal Resistors of 15 Per Cent Silver-Tellurium Alloy

Since the failures of the resistors were evidently due to an effect resembling electrolysis, tests were made similar to those just described except that 60-cycle alternating current was used instead of direct current. In the test at 0.9 ampere with the resistor immersed in a cooling liquid, the resistance increased slowly, the total increase in resistance being 12 per cent for a period of 14 days during which time the current was passed continuously through the resistor except for the brief periods during which it was removed from the circuit to have its resistance measured. At the end of this period it was put through a temperature cycle from −62 degrees centigrade to +74 degrees centigrade after which its resistance was only 7 per cent above the original value.

Since the chemical as well as the heating effect of the current is involved in the effect of direct current on the resistors, comparisons should be made on the basis of average as well as on effective values of current. However, in this case the difference in the effects of alternating and of direct current is so great that the comparison is little affected by a change in the alternating current equal

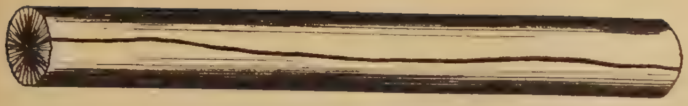

Figure 5. Arrangement of crystals and longitudinal crack in a cast resistor of 15 per cent silver-tellurium alloy

to the difference between the effective and the average values. Comparatively stated: a direct current of 0.9 ampere applied to a 15 per cent Ag-Te resistor which has been annealed 15 hours or more at 115 degrees centigrade caused it to decrease 40 per cent in resistance in 3 hours and to fracture spontaneously in $4^1/_2$ hours. An equivalent (root-mean-square) value of alternating current applied to a similar resistor under similar conditions for over 300 hours caused a semipermanent increase of 12 per cent in its resistance and no mechanical damage. It is reasonable to suppose that this increase in resistance may have been due to heat rather than to any other effect of the current.

Effect of Direct Current on Resistors With Various Percentages of Silver

Annealed resistors of single-crystal tellurium of both the *CP* and the technical grades withstood direct current for a much longer period than did the alloy resistors. The tests at 0.9 ampere direct current were not continued longer than 10 hours, and at the end of this time the resistances had decreased only 2 per cent and there was no evidence of physical change in the resistors.

Tests at 0.9 ampere direct current on resistors with 5 per cent and 10 per cent of silver showed that the 5 per cent alloy resistors withstood the current longer than did the resistors with 10 per cent of silver. Failure of resistors containing 10 per cent of 15 per cent silver occurred by fracture, while with resistors containing 5 per cent silver a separation of the electroplated joints occurred at the cathode ends of the resistors. This separation was accompanied by the formation of a black coating on the separated surfaces.

For reasons to be explained later the 10 per cent alloy was the only one which appeared to have substantially as good temperature-resistance characteristics as those of the 15 per cent alloy. Hence it was the only one to be tested with a small current over a long period of time. Under these conditions it failed in a shorter time than did the 15 per cent alloy.

Effect of Composition on the Temperature-Resistance Characteristics of Silver-Tellurium Alloys

Preliminary tests on alloys with different percentages of silver and tellurium had indicated that compositions low in silver content gave temperature-resistance curves which were concave downward below −10 degrees centigrade while compositions which were high in silver content gave curves which were concave upward over the same temperature range. Since the composition which gave the most nearly linear curve over the range from −70 degrees centigrade to +70 degrees centigrade was that containing 15 per cent silver, most of the previously described work was done with an alloy of this composition.

The temperature-resistance characteristics of compositions ranging from zero (*CP* tellurium) to 50 per cent were determined by making a series of resistors of the required compositions in the manner which had been found to give the best results with the 15 per cent alloy. This procedure comprised melting in air, casting in glass tubes, progressive solidification, and annealing for 15 hours at 115 degrees centigrade.

The temperature-resistance curves for this range of compositions are shown in figures 7 and 8. In figure 8 it will be noted that portions of the curves are indicated by dotted lines. Over the temperature ranges so indicated the resistance is not constant at any given temperature. When the temperature of the resistor is lowered to a value on the dotted curve the resistance assumes the value so indicated, but when the temperature is held constant in this region, the resistance continues to increase at a diminishing rate for a period of several hours or more. In one case a resistance increase of 70 per cent in 45 minutes was observed. In all observed instances, this increase in resistance was not permanent; that is, the original resistance at any point on the solid portion of the curve could be again obtained by bringing the temperature to the corresponding value.

This behavior resembles that of the 15 per cent alloy melted in a reducing atmosphere, although the alloys with the higher percentages of silver were melted in an oxidizing atmosphere, using the same procedure as that used in producing the 15 per cent alloy which had a linear temperature-resistance curve.

It will be noted that the resistivity of the 15 per cent alloy resistor in figure 7 is lower than that of the resistor of the same composition in figure 4. This difference may be accounted for by the fact that the former is a single crystal, while the latter is polycrystalline in structure.

When the single-crystal resistors were found to be superior to the polycrystalline resistors, sufficient tests were made to determine the fact that they responded to heat treatment in a manner similar to that of the polycrystalline

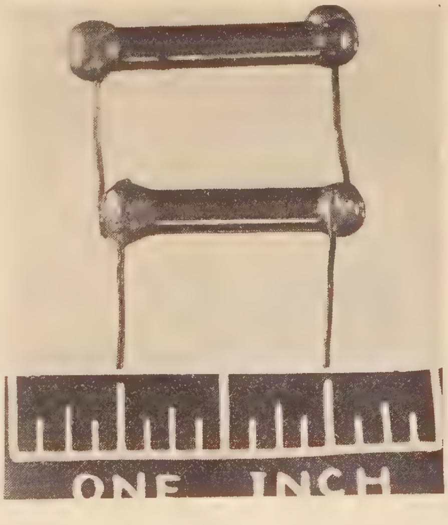

Figure 6. At top, a silver-tellurium resistor in which bulge has formed due to passage of direct current over a long period of time; at bottom, a similar resistor which has not been subjected to current

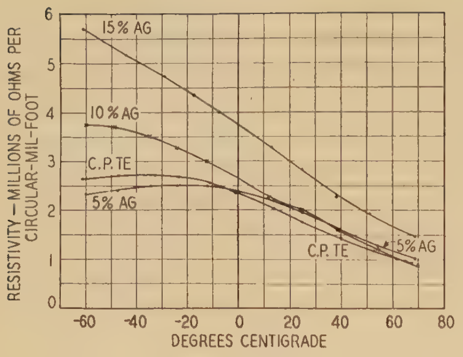

Figure 7. Temperature - resistivity curves of Ag-Te alloys with zero to 15 per cent Ag annealed 16 hours at 115 degrees centigrade

resistors, although the resistivities of the single-crystal resistors were the lower. The series of tests summarized in figures 1 to 4 inclusive were made with polycrystalline resistors and were not repeated with single-crystal resistors. Figures 7 and 8 were made with single-crystal resistors or at least with resistors which had been solidified progressively in a manner which had been found to produce single crystals.

Referring to figure 7, the temperature-resistivity curve of annealed chemically pure tellurium is linear between −10 degrees centigrade and +70 degrees centigrade. Tests on commercial tellurium from 4 different sources showed that unannealed single-crystal resistors of these materials had temperature coefficients within the range +0.1 to −0.1 per cent per degree at 25 degrees centigrade. Unannealed single-crystal resistors of chemically pure tellurium had temperature coefficients of approximately −0.7 per cent per degree at 25 degrees centigrade. After the annealing treatment, the temperature-resistivity curves of the commercial and of the chemically pure tellurium resistors were practically the same. Both had temperature coefficients of −1.2 per cent per degree at 25 degrees centigrade.

Thermal Electromotive Force

The thermal electromotive force as measured on an unannealed sample of 15 per cent Ag-Te alloy was found to be 0.46 millivolts per degree with the hot junction at 50 degrees centigrade and the cold junction at 25 degrees centigrade. This is close to the value usually given for tellurium.[4]

The annealing treatment which had such a great effect on the resistivity and temperature coefficient of resistance of the alloy lowered the thermal electromotive force as measured over the above temperature range to 0.25 millivolt per degree.

Aging Tests

The accelerated life tests which have been described were made sufficiently severe to destroy the resistors. While these tests did not represent service conditions, they were valuable guides in selecting the compositions and treatments of the resistors on which aging tests were made.

The following test results indicate the constancy that may be expected under the specified conditions of use:

1. In a group of 9 single-crystal tellurium resistors of 0.1 inch diameter, tested at 0.025 ampere direct current for 81 weeks, the maximum change was +1.2 per cent in one resistor, and the average change for the group was +0.15 per cent.
2. In a group of 3 single-crystal Ag-Te resistors of 0.1 inch diameter, tested at 0.10 ampere alternating current for 78 weeks, the maximum change was +2.1 per cent in one resistor, and the average change for the group was +1.4 per cent.
3. In a group of 4 single-crystal Ag-Te resistors of 0.1 inch diameter, tested at 0.001 ampere direct current for 80 weeks, the maximum change was +2.2 per cent in one resistor, and the average change for the group was +1.4 per cent.

In all of these groups most of the change took place during the first part of the period, indicating a tendency to stabilize at a higher value of resistance. It should be noted that in the previously described tests in which the Ag-Te resistors were broken by electrolytic action, a considerable decrease in resistance always preceded failure. The tests at one milliampere direct current indicate that this effect is not occurring at this low current density, since the resistance is increasing.

Aging tests on polycrystalline Ag-Te resistors of the type shown in figure 5 showed resistance changes several times as great as those in the single-crystal resistors.

Applications

The heat-treated tellurium resistors of single-crystal structure are more constant than are the similarly treated Ag-Te alloy resistors, their temperature coefficient is practically the same from −10 degrees centigrade to +70 degrees centigrade, and they are not subject to electrolytic effects. Therefore, they are to be preferred to the Ag-Te alloy resistors for all applications in which compensation is not required at temperatures below −10 degrees centigrade. Since the resistance of these resistors should not be more than 25 per cent of the resistances of the instrument circuits in which they are used, the maximum change of 1.2 per cent which was noted in one resistor during the aging test would have caused a maximum instrument error of 0.3 per cent. From these data it appears that this material may be used in instruments with rated accuracies of one per cent to 2 per cent, provided that other sources of error are kept sufficiently low.

The 15 per cent Ag-Te alloy is suitable for long-range

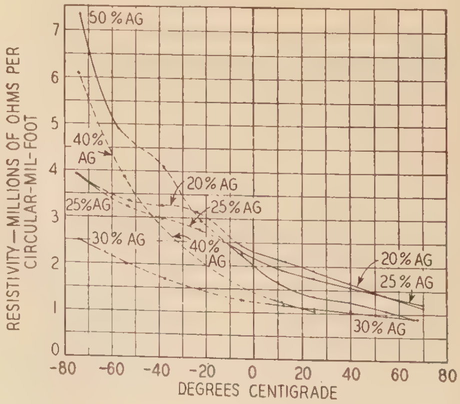

Figure 8. Temperature - resistivity curves of Ag-Te alloys with 20 per cent to 50 per cent Ag annealed 16 hours at 115 degrees centigrade

temperature compensation, but when used with direct current the current density should not exceed one milliampere in a 0.1 inch diameter resistor. A suitable application of this kind is for compensating the resistance change in a thermocouple temperature indicator with cold-junction compensation for measuring temperatures in aircraft engines or for portable use in measuring temperatures of molds, bearings, and molten metals.

An application in which the 15 per cent Ag-Te alloy has been tried with good results is in the compensation of "class 2" temperature error in watt-hour meters. This error is principally due to the change in phase angle of the flux of the potential coil due to the change in resistance of the potential winding with changing temperature. It has been entirely corrected by substituting for the usual solid copper lag plate a wound copper lag coil with a resistor of Ag-Te alloy connected across its terminals. The maximum aging change of 2.1 per cent for this alloy as tested under similar conditions with alternating current would cause an error not greater than 0.1 per cent in the meter accuracy. Another method, as described by Kinnard and Trekell,[5] which has recently been developed for correcting this error has the advantage of a better mechanical design.

In compensating millivoltmeters with these materials, it is necessary on account of the high thermal electromotive force of Te and of Ag-Te to shield them from rapid temperature fluctuations and to mount them so as to equalize the temperatures at the ends of the resistor.

Figure 9 illustrates 2 resistors which have been made for use in instruments.

Theory

Because the results just described were obtained with tellurium which was not investigated spectroscopically to determine its purity and which was known to contain dissolved oxide, they are not directly comparable with those of Cartwright and Haberfeld-Schwarz[3] who used tellurium of high purity melted in vacuum. They considered the increase in resistivity which occurred during annealing to be due to the formation of cracks. This theory does not explain the increase in temperature coefficient which accompanied the increase in resistivity in our tests. Neither pressure up to the crushing strength of the material, nor a current which would have fused the resistor had it not been cooled in liquid, decreased the resistance of our annealed tellurium resistors.

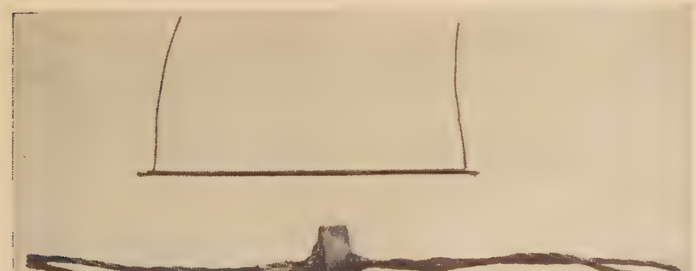

Figure 9. At top, a silver-tellurium resistor of 200 ohms resistance (length 3 inches, diameter 0.059 inch); at bottom, a tellurium resistor of 0.06 ohm resistance

Table I. Summary of Electrical Characteristics of Resistor Materials

Single Crystals Annealed 15 Hours at 115 Degrees Centigrade

Composition Per Cent Ag	Composition Per Cent Te	Resistivity at 25 Degrees Centigrade in Megohms Per Circular-Mil-Foot	Temperature Coefficient of Resistance at 25 Degrees Centigrade	Effect of Direct Current	Effect of 60-Cycle Alternating Current
0	100	1.7	—1.2	Thermal	Thermal
5	95	1.9*	—1.1*	Thermal and electrolytic	Thermal
10	90	2.0	—1.3	Thermal and electrolytic	Thermal
15	85	2.8	—1.3	Thermal and electrolytic	Thermal

* These data based on tests on only one sample.

The behavior of all the resistors tested by the writer may be explained as follows: In rapid cooling from the melting temperature, silver telluride and other impurities are held in solid solution in the tellurium, which solid solution has the property of metallic conduction with a very small temperature coefficient of resistance. Assuming that the impurities are precipitated during the heat treatment, the conductivity decreases due to the reduction in the number of free electrons. This theory is supported not only by the fact that the temperature-resistivity curves of the impure and the pure tellurium are made to coincide by heat treatment, but also by the fact that the thermal electromotive force of the Ag-Te alloy is lowered by the heat treatment. Cartwright and Haberfeld-Schwarz[3] have shown that the thermal electromotive force of pure tellurium is much lower than the value usually given and that is increased by the additions of small amounts of silver and of other metals.

The breakdown of the Ag-Te resistors produced by direct current may be due to electrolysis of silver telluride accompanied by migration of the silver ions toward the cathode. Evidence has been shown to indicate that this effect does not take place when the voltage across the resistor is very low.

The observations here recorded were obtained in determining the utility of tellurium and of its alloys with silver as special resistor materials. It is hoped that the data thus obtained may be of some value in the further determination of the nature of the conductivity of semiconductors.

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Regulation of Grid-Controlled Rectifier

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Synopsis

This paper discusses several factors which have not been included in the previously published regulation formulas and which should be considered in the practical application of rectifiers. Approximate solutions are given to take system reactance, and the load inductance into account. The effect of grid pickup characteristics on regulation, and means of modifying the inherent regulation characteristics by regulators or compensators are discussed.

Introduction

SEVERAL PAPERS[3,4] have been written on the general characteristics of grid-control rectifiers, and formulas have been derived for the regulation. The conventional formula, equation 1, is based on the assumptions that a sinusoidal voltage exists at the point where the rectifier is connected to the system, and that the inductance in the load circuit is sufficiently large to prevent the current from going to zero when the voltage drops below the counter electromotive force of the load circuit. These assumptions are sufficiently accurate for some cases, but in many practical cases other factors, such as the effect of system reactance and the effect of a limited amount of inductance in the load circuit must be considered.

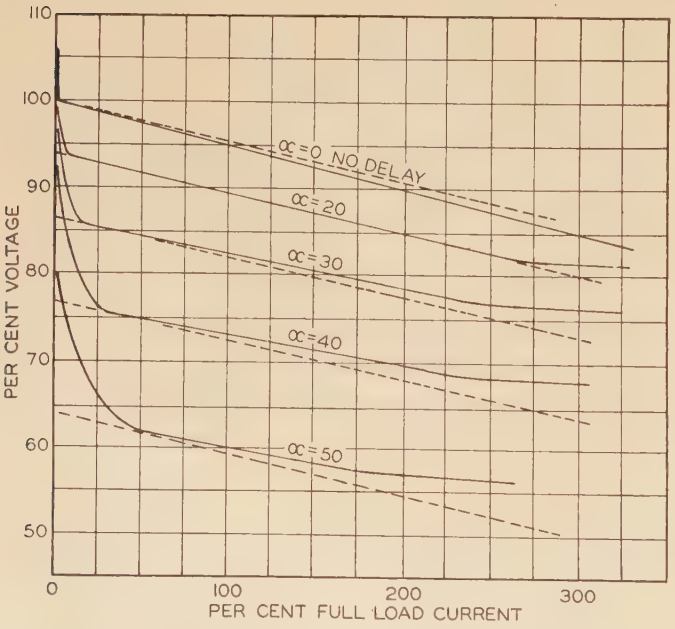

Figure 1. Test and calculated regulation curves of experimental grid-controlled rectifier

Double 3-phase connection. Dotted lines show calculated values for fixed delay

The conventional formulas and regulation curves involve the angle of delay (α). In the practical application of these formulas it may not be correct to assume that the actual angle of delay is constant for a certain setting of the grid-control apparatus. The angle of delay may vary with the load due to a variation in the grid pickup voltage, and in some cases it may vary due to a shift of phase of the grid voltage due to system reactive drop.

It is the purpose of this paper to consider these other factors and to present approximate methods of including them in the determination of regulation. A more exact solution is outlined in the appendix. The use of regulators and compensators for the purpose of modifying the inherent regulation of grid-controlled rectifiers and ignitrons is also discussed, and results obtained with certain types are given.

Conventional Regulation Formula

What might be termed the conventional regulation formula for grid-controlled rectifiers (equation 1) has been derived several times;[2,3] and the derivation will not be repeated.

$$E_d = \left[\frac{\sqrt{2}EP \sin\frac{\pi}{P}}{\pi}\right]\cos\alpha - \frac{IX_TP}{g2\pi} - \frac{W_T}{I} - A \qquad (1)$$

This formula is exact within the limits of the assumptions made in deriving it. These assumptions are: (1) a sinusoidal voltage at the terminals of the rectifier transformer, (2) a load inductance sufficiently high to prevent the current from going to zero, (3) the effect of all the transformer and lead resistance to be equivalent to a series resistance in the d-c circuit giving the same loss, and (4) only 2 anodes in one group firing at one time. This formula gives straight-line regulation curves as shown by the dotted lines of figure 1, providing the curve of arc drop against current is straight. It should be noted that the curves are parallel; that is, a given angle of grid delay (α) drops the output voltage the same number of volts whether at no load or at full load.

If a rectifier is supplied through a line of appreciable reactance directly from a large system whose voltage is

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[3] For all numbered references, see list at end of paper.

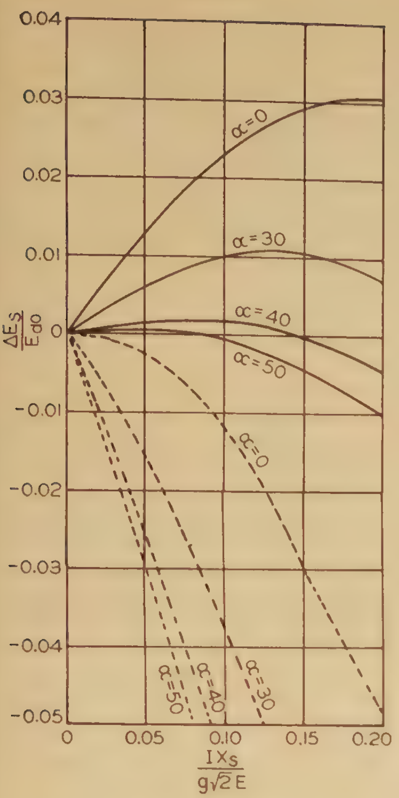


Figure 2. Apparent increase in regulation due to system reactance

A normal transformer with about 3 per cent reactive drop is assumed. Full lines are for double 3-phase and dotted lines are for 12-phase quadruple zigzag

not affected by the rectifier load, the line reactance can obviously be included in the total commutating reactance. In this case E in equation 1, is the system voltage. The additional commutating reactance due to the line is calculated by multiplying the ohms per phase (line to neutral) by the square of the ratio of primary to secondary line to neutral voltages.

For the more common case where the system voltage is affected appreciably by the rectifier load, equation 1 can still be used as an approximation if the primary voltage is known. However, a further analysis as presented below is necessary to obtain an accurate method of dealing with this case.

Effect of System Reactance

In the general case of a rectifier connected to an a-c system there is no one source of constant sinusoidal voltage. It greatly simplifies the problem if an equivalent sinusoidal source voltage can be found which may be considered as unaffected by rectifier currents. The principle of superposition, which has proved such a useful tool in other system problems, may be applied to determine a voltage which can be considered as acting through the combined impedance of the system and the rectifier transformer.

Since the back current of a power rectifier is negligible, the effect is equivalent to opening each anode circuit whenever the current tends to reverse. A method of applying superposition to obtain as complete a solution as desired is outlined in the appendix. For practical purposes, the capacity and resistance of the a-c circuit and any effect of harmonics produced by other rectifiers, will be neglected.

As shown in the appendix, the voltages acting in the secondary to produce the anode currents are the voltages which would exist with no rectifier currents flowing. This voltage acts through the reactance of the transformer and the reactance of the system measured from the point of connection. This voltage may be treated as an equivalent sinusoidal source voltage and the system reactance may be added to the commutating reactance of the transformer as described for the line reactance in the simpler case mentioned previously.

In general the voltage E at the primary of the rectifier transformer is known or can be calculated. The system reactance X_s can be measured or estimated. As shown in appendix B, the system reactance to be used is the same as that used for calculating single-phase short-circuit current; that is, the average of the negative-sequence and positive-sequence subtransient reactances: $X_s = (X_2 + X_d'')/2$.

The fundamental frequency voltage E, at the primary terminals may be taken equal to the root-mean-square value for practical purposes. If the fundamental-frequency system reactive drop be added vectorially to the terminal voltage the equivalent sine-wave source voltage E'' is obtained. Using E'' instead of E and $(X_T + X_s)$ in place of X_T in equation 1, gives the general equation;

$$E_d = \left[\frac{\sqrt{2}\,E''P\sin\frac{\pi}{P}}{\pi}\right]\cos\alpha - \frac{I(X_T + X_s)}{g2\pi} - \frac{W_T}{I} - A \qquad (2)$$

where

$$E'' = \sqrt{(E + I_1X_s\sin\theta_1)^2 + I_1^2X_s^2\cos^2\theta_1} \qquad (3)$$

With good approximation,

$$I_1 = \frac{I\sqrt{2}}{\pi g}\sin\frac{\pi}{P} \qquad (4)$$

The fundamental-frequency power factor is approximately:

$$\cos\theta_1 = \cos\alpha - \frac{\text{transformer reactive drop}}{\text{voltage at no load no delay}} \qquad (5)$$

This may be demonstrated as follows. All the power is supplied by the fundamental since the source voltage is

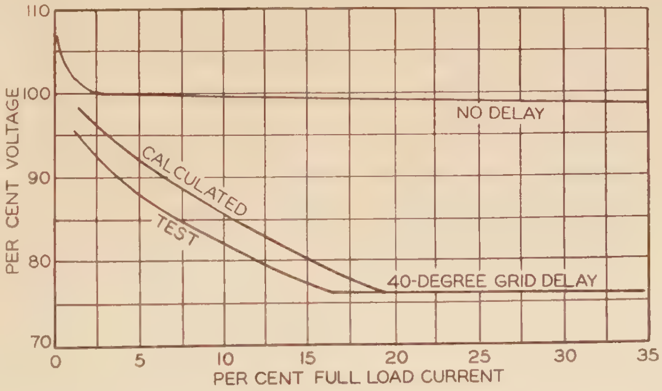


Figure 3. Light-load voltage regulation curves

Experimental grid-controlled rectifier, double 3-phase connection assumed sinusoidal. This power factor can then be written as power divided by fundamental-frequency volt-amperes:

$$\cos\theta_1 = \frac{E_d I}{E I_1 p g}$$

and neglecting arc drop and resistance losses, this reduces to equation 5.

The fundamental reactive drop adds to E the amount nearly equal to $I_1 X_s \sin\theta_1$. If the average of the power factor angles at the source and at the terminals be used the accuracy of this approximation is improved. The effect of the system reactance is to add to the source voltage and to the reactive drop. The net correction expressed as an addition to the regulation (to be subtracted from the right-hand side of equation 1) is:

$$\Delta E_s = \frac{I X_s P}{2\pi g}\left[1 - \frac{4g}{\pi}\sin^2\frac{\pi}{p} \times \cos\alpha \sin\cos^{-1}\left\{\cos\alpha - \frac{I\left(X_T + \dfrac{X_s}{2}\right)}{2\sqrt{2}gE\sin\dfrac{\pi}{p}}\right\}\right] \qquad (6)$$

Curves of $\Delta E_s/E_{d0}$ for different values of $IX_s/g\sqrt{2}\,E$ are given in figure 2, for a rectifier with about 3 per cent regulation in the transformer. It will be seen that the correction can be either positive or not depending on the type of connection and the angle of delay. It should be noted that the angle of delay (α) is measured relative to the source voltage, and that the angle of overlap (u) as measured on an oscillogram will be determined by the total reactance $(X_T + X_s)$ thus:

$$\cos(u + \alpha) = \cos\alpha - \frac{I(X_T + X_s)}{g\sqrt{2}\,E''\sin\dfrac{\pi}{P}} \qquad (7)$$

To illustrate the importance of the correction for system reactance, consider a rectifier where the primary voltage is held constant as it might be in a regulation test. If the system reactance is 10 per cent based on the rectifier kilovolt-amperes and the transformer reactive drop is 3 per cent, the ratio of X_s to X_T is 1.74 for a double 3-phase connection and the measured regulation may be 2.3 per cent less with zero delay than the value which would be obtained if a sine-wave voltage were impressed. That is, the regulation might measure 2.7 instead of 5 per cent. As another example take a 12-phase quadruple zigzag connection with the same per cent line reactance and transformer reactive drop. In this case the ratio of X_s to X_T is 0.87 and the regulation will be increased only 0.3 per cent at no delay; but at 20 per cent reduction in voltage by grid delay the measured regulation will be decreased 4 per cent, that is; it may measure one per cent instead of 5 per cent.

Light Load Voltage Rise

The rise in voltage at very light loads on a double 3-phase rectifier is a well-known phenomenon. This has been explained by saying that below some transition value of load, at which the load current just equals the interphase transformer excitation current, the rectifier is operating 6 phase; and that the commutating current flows between anodes of alternate groups through the windings of the interphase transformer. This is an accurate explanation for the case of straight resistance load or for the theoretical case of infinite inductance in the load circuit.

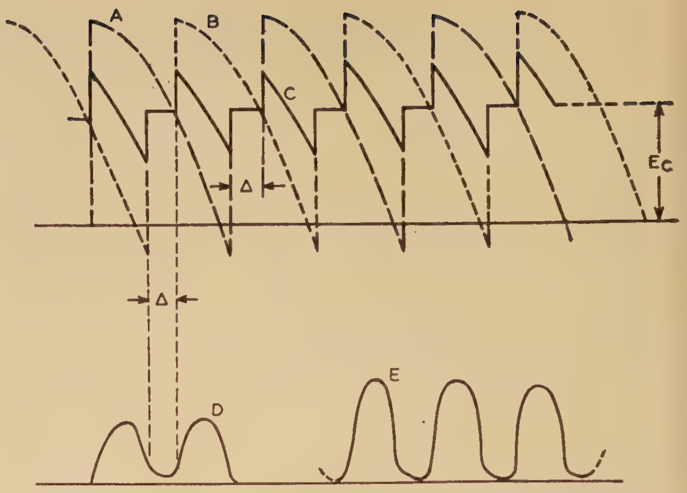


Figure 4. Current and voltage waves when operating at light load with counter electromotive force

Curve A—Voltage of one group of double 3-phase winding
Curve B—Voltage of other group
Curve C—Net voltage output
Curve D—Current in first group
Curve E—Total current output

However, for the case of load with a counter electromotive force and with normal inductance a different phenomenon appears which radically alters the light-load regulation characteristics, especially when operating with appreciable grid delay. Whenever the output voltage of the rectifier dips below the counter electromotive force the total load current tends to go to zero but is restricted in this action by the inductance of the circuit. Above a certain critical load the current is prevented from reaching zero. This phenomenon occurs for all type of rectifiers both with and without interphase transformers.

When interphase transformers are used the action is more complicated. Interphase transformers are designed to have a high inductance (L_i) between the 2 transformer neutrals so as to prevent one group from dropping the current ahead of the other except at light loads. However, it is the leakage inductance ($\mathcal{L}_i$) between the 2 halves of the winding which opposes the rise and fall of total current so long as both groups are conducting. This leakage inductance, measured with the windings in parallel, the magnetomotive forces being opposed, is very much smaller than L_i. This is true because the windings are normally made with close coupling.

As long as both groups are firing, it is the total inductance ($\mathcal{L}$) of the load circuit, including L_i and the paralleled inductance of the phase circuits of the anodes which are conducting at the same time, which damps out the pulsations of load current.

As discussed in the appendix, a complete solution would involve consideration of the effective resistance and reactance of fundamental frequency as well as the impedance to suddenly applied voltages. The resistance in the load circuit causes a voltage drop both on the rising and dropping parts of current cycle and thus has relatively little effect on the total swing of the current unless the total line drop is 10 per cent or more at the load considered.

For most practical purposes the following equation determines the amplitude of total current pulsation:

$$i_\Delta = \frac{1}{\mathcal{L}}\int (e_d - E_c)dt \tag{8}$$

At a critical value of load this pulsation of current (i_Δ) reaches a value equal to the average current, and the total current tends to go out between each pulse.

Consider the voltage waves for a double 3-phase circuit at light load and some delay, as shown in figure 4, for the interval of $^1/_6$ of a cycle. The total current will rise and fall during this period, but since there is a slight difference in the currents of the 2 groups due to difference in voltage acting through the high inductance, L_i, only the current in one group goes out, and the current in the group having the last anode to pickup is maintained but at some very low value. One group at a time will be out for a period Δ and the current wave of each group will be as shown by curve D of figure 4. The total current shown by curve E will not go completely out but will be very low for the period Δ. The output voltage at the neutral point of the interphase rises to the counter electromotive force value (E_c), assuming the inductance of half the interphase to be large compared with that of the load circuit. Equations could be written for the critical load value in terms of inductance and the angle of delay, but for practical purposes a graphical solution is satisfactory. Plotting the output voltage for an assumed value of α and Δ as shown in figure 4, the current can be solved by taking the area of one half cycle of the voltage ripple in volt seconds divided by the inductance $\mathcal{L}$ as indicated by equation 8. Also from this curve the average voltage may be determined graphically.

The results of this method are given in figure 5 plotted as the rise in voltage (ΔE_L) versus ($I\mathcal{L}\omega/\sqrt{2}E$). Curves obtained in a laboratory test with counter electromotive force are shown in figure 3. This rise in voltage at light loads with appreciable angles of delay is much more pronounced than the usual light-load voltage rise at no delay. However, in practice this is not generally a serious matter since danger due to overvoltage exists in the great majority of cases only when the normal voltage at no delay is exceeded.

If exactly straight-line regulation curves are desired at very light loads and appreciable angles of delay it may be necessary to use more than normal load inductance. An iron-core reactor may be used in series with the d-c bus to provide a high inductance at low values of current.

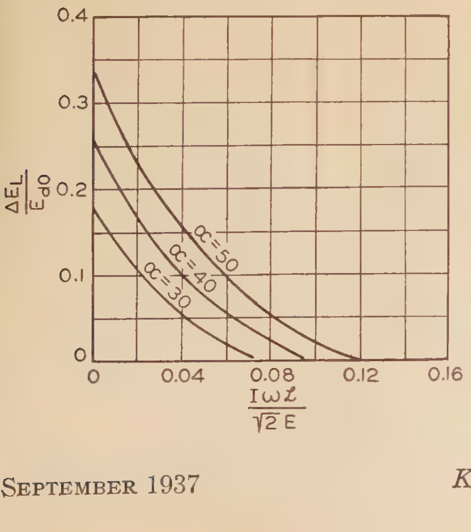


Figure 5. Rise in voltage at light load as a function of d-c circuit inductance and load current

Effect of Grid Characteristics

The regulation formula and conventional regulation curves are based on the actual angle of delay. However, the angle of delay for a given phase shifter or bias setting may vary appreciably with load. This may be due to variation of the pickup voltage or an actual shift of the grid voltage with respect to the source voltage.

The pickup voltage will vary with the degree of ionization and the vapor pressure in the tank. At light loads a slight positive voltage relative to the cathode may be required to make the grids pickup, and at higher loads an appreciable negative voltage may be required to prevent them from picking up.

A simplified explanation of the action of the grids is as follows: A positive ion space charge sheath is formed around the grids when they are negative with respect to the ionized gas preventing the ionization from reaching the anode. The thickness of this layer[6] varies roughly as the $^3/_4$ power of voltage and inversely as the $^1/_2$ power of the ionization. When the thickness of the space charge sheath just fails to close the holes in the control-grid structure, positive voltage on the anode accelerates the electrons sufficiently to increase the ionization and further reduce the space charge thickness and cause complete pickup.

It is difficult to test for the pickup voltage of the grid under actual operation but a method of testing the grid characteristics statically was developed. An arrangement was made whereby the load could suddenly be removed from one anode and definite continuous voltages applied to that anode and control grid. Curves of grid voltage at which the anode picked up for different values of total load were plotted as shown in figure 6. These characteristics were found to vary some with anode voltage and the temperature of the cooling water. Obviously conditions were not identical with those under actual operating conditions. However, it was found that these static curves could be used to predict approximately the deviation of the test regulation curves from a straight line under overload conditions as shown in figure 1.

In order to explore a range of grid characteristics an experimental rectifier was built with grids having large holes so that the variation in pickup voltage caused a noticeable effect on the regulation curves even at rated load when using a sine-wave grid voltage. Without appreciably increasing the arc drop, it was possible to build a grid structure with sufficiently limited variation in pickup voltage so that it had no appreciable effect on regulation out to at least 200 per cent load. The test curves of figure 1 show a flattening out of the regulation curves at about 250 per cent load at large angles of delay. However, grids sufficiently tight to give straight line regulation curves at very high overloads result in excessive arc drop.

When paralleled with other apparatus having a straight or drooping characteristic on high overloads, this effect will add to the tendency for the rectifier to take more than its share of the overload. However, so long as the departure from straight line regulation occurs at well over 150 per cent load, the rectifier will return to normal after the load is decreased, and no damage is done since the rectifier is suited to short time overloads.

By the use of steep-wave-front impulses on the grids it is possible to obtain straight-line regulation even with comparatively loose grid structures, since in this case a variation in the pickup voltage has little influence on the time of pickup.

The voltage control characteristics of an ignitron rectifier are similar to those of a multianode rectifier except that the pickup of the auxiliary tubes is not affected by the load current. Tests on a 6-phase ignitron rectifier, both as grid-controlled rectifier and as an inverter, show straight line regulation curves at all angles of delay and advance out to very high overloads.

Where the system reactance is comparable with the transformer reactance an increase in delay with load, due to a shift of the fundamental voltage at the point where the grid transformers are attached to the system, might be expected. For the sine wave grid transformer with only resistance in the grid circuits, tests show the effect on regulation to be negligible, even though the fundamental voltage shifts several degrees. This may be explained by the fact that the voltage applied to each grid through a resistance has almost identically the wave shape of the primary voltage. That is, it will have a wave proportional to the source voltage with nicks cut into it due to commutating currents, rather than a sine wave shifted in phase. Since the nicks in the voltage wave occur only during commutation they do not affect the voltage just before pickup provided only 2 anodes are commutating at one time, as is normally true.

Regulators and Compensators

It is possible to regulate the output voltage of a grid-controlled rectifier, or ignitron, with the same types of current or voltage regulators that are used for rotating machines. The regulator may act to change the angle of delay, either by shifting the phase of the grid voltage or by varying the bias voltage applied to the neutral of a sine-wave grid transformer, which causes pickup to take place at a different point on the wave. The regulator may be made to give any desired regulation curve by using current compensation, and by cross current compensation units may be made to divide load under varying conditions. In some installations regulators have been applied so as to come into play only at high load for the purpose of limiting the overload. In electrochemical service current regulators have been used to control the output at any voltage.

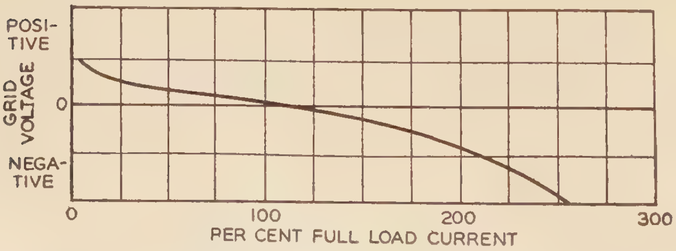


Figure 6. Curve showing variation of grid pickup voltage with load current. Experimental rectifier

In considering voltage regulation by grid control, either automatic or otherwise, it is essential to keep in mind its chief limitations. The transformer must provide the maximum voltage desired and any reduction in voltage is accomplished, first, at the expense of an almost proportional reduction in power factor and, second, an increase in d-c harmonics. For these reasons it may in some cases be more desirable to regulate the output voltage over the normal range by an induction regulator in the primary or by tap changing under load. A combination of tap changing and grid control can be made to give continuous control and good power factor.

Inherent compensation or compounding can be used on a grid controlled rectifier or ignitron with results similar to compounding in a d-c generator. The action of such a compensator can be made practically instantaneous, thus eliminating the tendency to hunt or overshoot.

There are several methods of obtaining this compounding effect. One method is to use small series transformers, in the primary of the supply transformer, with a rectifier connected to the neutral of a sine-wave grid transformer. When adjusted for some grid delay at no load, the load current will cause an increase in positive bias which advances the phase and tends to raise the output voltage. This may be adjusted to compensate for the regulation of the transformer and line reactance or it may be made to more than compensate for the natural regulation, thus giving an overcompound regulation characteristic.

Another method of obtaining some compounding effect, and at the same time reducing the variation in continuous voltage due to variations in the a-c supply, is to connect the neutral of the grid transformer to the negative bus through a source of fixed positive bias. This gives a net bias on the grids which is proportional to the difference between the output voltage and the constant positive bias. An increase in output voltage will cause an increase in net negative bias and so increases the angle of delay and reduces the voltage. The sensitivity of this scheme will be limited by the lowest grid impulse voltage which may be used and still obtain accurate pickup of the anodes so as to insure balanced currents.

A combination of the 2 schemes of inherent regulation mentioned above may be used to give almost any regulation characteristic desired. Results similar to those discussed here were described in a recent paper by Goodhue and Power.[1] The method used by them differed in that they employed no grid transformer, but relied on the tube characteristics and the ripple of the output voltage to time the pickup of the anodes.

The regulation curves for an ignitron rectifier with current and voltage compensation are shown in figure 7. In this case as shown by the curves, the regulation is reduced from 17 per cent to 3.5 per cent and the variations in output voltage with the high line are reduced except at higher loads. The desirability of such characteristics for certain applications is obvious.

Conclusions

The following conclusions may be drawn from the work presented in this paper:

1. Accurate determination of the regulation of a rectifier connected to system, whose voltage is affected by the rectifier current, requires a modification of the conventional regulation formula as given in equation 2. The effect of the system reactance may also be taken into account by adding a correction term given by equation 6 and by the curves of figure 2.

2. With a normal amount of load inductance the conventional formula (equation 1) is not accurate at light loads where there is a counter electromotive force. This effect is much more pronounced with appreciable grid delay than at no delay. A graphical method of evaluating this effect is given and the results plotted in figure 5 as a correction term which is a function of delay and load circuit inductance. A comparison with test is given in figure 3. This is generally not important but may be corrected by adding inductance.

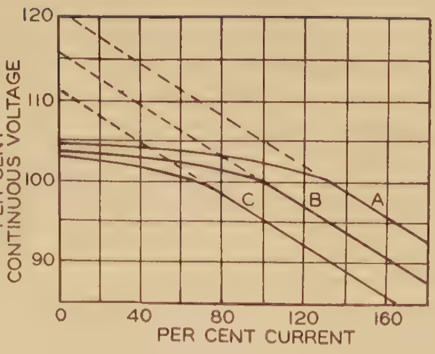
3. There is a tendency for the regulation curves of a rectifier, operating with a fixed phase-shifter or grid-bias setting, to curve up at overloads. This effect is due to the fact that with high ionization in the tank the grids pick up at a more negative voltage. The results of this effect on the regulation curves of the rectifier are shown in figure 1. This effect may be minimized by applying steep-wave-front impulses to the grids, or making a grid structure whose pickup voltage is only slightly affected by the amount of ionization in the tank.

4. Current and voltage regulators of the same general type as used for controlling the fields of rotating machines may be used to control the output voltage of grid-controlled rectifiers and ignitrons. It is also possible to use current and voltage compensation inherently to modify the regulation characteristics giving results similar to compounding on a d-c machine.

List of Symbols

A = arc drop
e_d = instantaneous output voltage
E = root-mean-square voltage anode to neutral, at no load
E'' = fundamental-frequency voltage back of system reactance
E_c = counter electromotive force
E_d = average output voltage
E_{d0} = theoretical no-load output voltage
f = fundamental frequency
g = number of groups
I = average total current output
I_1 = fundamental of secondary current
L_i = total interphase inductance, from one transformer group to the other
$\mathcal{L}$ = total output circuit inductance
$\mathcal{L}_i$ = interphase leakage inductance, measured with windings in parallel, magnetomotive forces opposed
P = number of phases per group
P' = total number of phases for the rectifier
u = angle of overlap
W_T = copper loss of main and interphase transformers

Figure 7. Rectifier regulation curves, showing the effect of compounding to correct for a high reactive drop and variations in high line voltage



Curves *A*, *B*, and *C* are for 104, 100, and 96 per cent high line voltage

X_e = effective value of generator reactance to be used in rectifier calculations
X_d'' = direct-axis subtransient reactance
X_q'' = quadrature-axis subtransient reactance
X_s = system commutating reactance converted to secondary ohms per phase
X_T = transformer commutating reactance in ohms per phase in secondary
X_2 = negative-phase-sequence reactance
α = angle of retard or delay
ΔE_L = increase in voltage due to lack of inductance in load circuit
ΔE_s = increase in voltage regulation due to system reactance
θ_1 = power factor angle of fundamental-frequency power

Appendix A. Outline of a General Method of Analysis

The discussion of the paper has been confined to practical solutions which involved numerous assumptions and approximations. It is interesting therefore, to consider what might be done in the way of a more general solution taking into account many more factors. Although the mathematical formulas are not given, such a general method of analysis is outlined here in order to give a physical picture of what is involved and to aid in judging the simplifying assumptions made in the practical formulas.

The principle of superposition has been previously used[6] in the calculation of commutating currents. This principle can be extended to the solution of the currents and voltages in the complete rectifier system including both a-c and d-c circuits by solving a series of transient states. The a-c system impedance including capacity and resistance as well as reactance, may be accurately dealt with. Also the counter electromotive force, the inductance of the d-c system, and the shunt capacity of any filter circuits may be included in the analysis.

The only limitations of this method of analysis, other than the general limitation that the circuit constants must remain fixed are: (1) that the currents drawn by the rectifier being analyzed must have negligible effect on any other sources of harmonics such as those from another rectifier and, (2) that the arc drop is constant for a given value of average current. Where several similar rectifiers are connected to the same source and are similarly loaded they may be analyzed as if they were a single rectifier.

For this exact analysis the voltage on each anode at no load, and the actual angle of delay must be known. The picking up and the extinction of the arcs in the rectifier may be thought of as switching operations. In the analysis for any given condition the arc drop may be considered as a voltage source in series with each anode.

The ripple on the output voltage wave has a period equal to $1/P_1$ times the period of the fundamental frequency. This period may be further broken down into several time intervals, one for each different combination of anodes firing. The principle of superposition can be used in solving the transient within each interval. In general a transient resulting from a change in the circuit may be solved by assuming an imaginary voltage equal and opposite to the applied no-load voltage so that the circuit may be left closed with no current flowing. A second imaginary voltage is then applied to cancel the first and to give the transient current.

When 2 anodes are firing they are in effect connected together. Using the above method the average of their no-load voltages opposed by the counter electromotive force for the load may be thought of as acting through the paralleled impedances of the transformer windings and their associated primary and system impedances, and through the load impedance.

An accurate analysis would require that equivalent circuits be constructed for the system as viewed from the rectifier terminals. It may also be necessary to set up an equivalent circuit for the load if there are several branches. If there are several sources of counter electromotive force an equivalent value may be obtained.

If the no-load voltages consist of several appreciable harmonics, as well as a suddenly applied fundamental, the equivalent circuit must be solved for each. If the harmonics are negligible the solution can be greatly simplified.

In order to determine the length of time an anode fires it is necessary to determine when the resultant current goes to zero. When the current reaches zero there is no reverse current (except in case of an arcback), hence the circuit is in effect switched open. The total current in an anode circuit is the resultant of the load component calculated as described above and a commutation current which is circulating due to the difference in voltages of any anodes that are firing at the same time. This commutating current can be solved by the same general methods outlined above, in which case the load circuit is not involved. The interphase transformer may be involved if the commutating current between 2 groups is considered, as in calculating the light-load voltage rise.

From this outline it is apparent that we can resolve the calculation of all voltages and currents involved into the solving of a series of transient states. Because it is necessary to determine the time intervals from the solutions for the total current in an anode, the mathematical determination of the boundary conditions may be difficult.

Appendix B. Generator Reactance to Be Used in Rectifier Calculations

If a rectifier is fed from a single generator the generator adds to the commutating reactance increasing the angle of overlap and affecting the output voltage. The generator reactance drop to fundamental-frequency current may be added to the fundamental of the terminal voltage to get the internal sine-wave voltage. Using this internal voltage and the total commutating reactance the regulation can be calculated by the conventional formulas as indicated in the first part of the paper.

A question arises as to what generator reactance should be used. Since commutation is equivalent to a momentary short circuit the same reactance is effective as for the first fraction of a cycle of a single-phase short circuit. This reactance may vary between the direct-axis subtransient reactance (X_d'') and the quadrature-axis subtransient reactance (X_q'') depending on the load and the instant of short circuit. For zero delay and no load on the generator, X_d'' would be the value at the first instant of short circuit. For a delay angle α, and load angle γ between internal voltage and the terminal voltage, the effective reactance would be:

$$X_e = \frac{X_d'' + X_q''}{2} - \frac{X_q'' - X_d''}{2} \cos(2\alpha + 2\gamma) \qquad (1B)$$

If we use the method of symmetrical components to calculate the fundamental component of single-phase short circuit for a fundamental voltage applied, the effective reactance

$$X_e = \frac{X_d'' + X_2}{2} = \frac{3X_d'' + X_q''}{4} \qquad (2B)$$

This corresponds in the first equation to $\alpha + \gamma = 30$ degrees which is probably an average for conditions in which α does not exceed 30 degrees.

For the great majority of cases X_d'' and X_q'' will be so nearly equal that the approximate equation $2B$, consistent with the method of symmetrical components, is sufficiently accurate and should be used.

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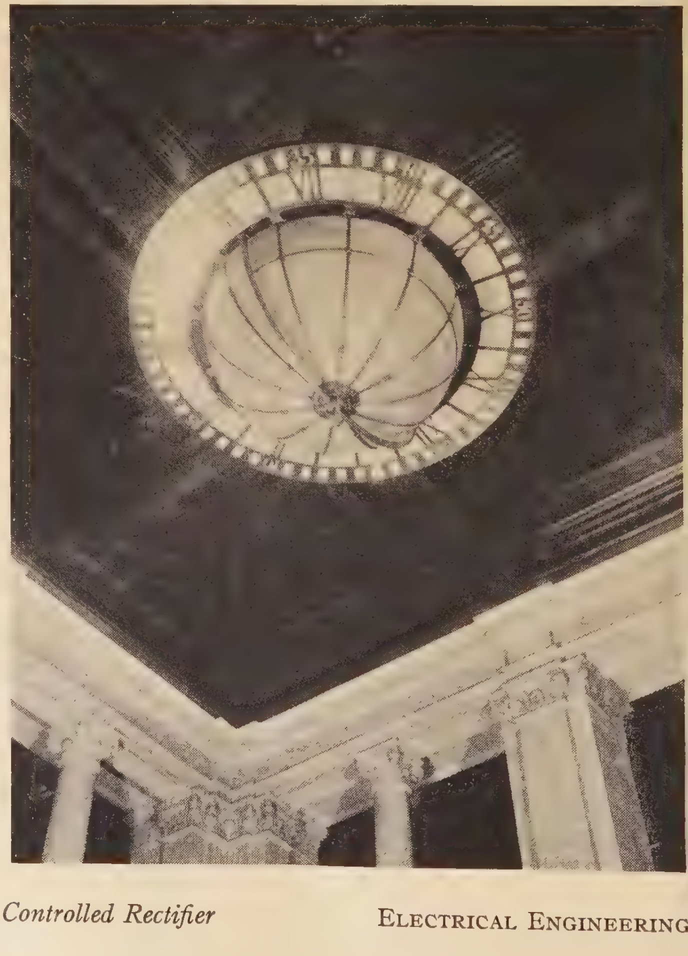
Unusual Electric Clock

Beauty and individuality are combined with utility in this unique terrestrial-globe electric clock hanging in the foyer of the Christian Science Monitor Building in Boston, Mass. Lights in the dial around the globe indicate the hours by large Roman numerals and minutes by the smaller Arabic numerals around the edge. A twin celestial globe at the other end of the foyer gives the day of the month. Both are driven by Telechron motors.



Tension Measurement and Control in Cold-Strip Mills

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Introduction

THE COLD rolling of steel strip on tandem mills requires the measurement and control of tension in the strip, and to meet this requirement the tensiometer herein described has been developed. The tensiometer produces a continuous indication of the tension in the strip during the rolling operation, and functions in connection with auxiliary control equipment to automatically maintain the tension at any desired value. An additional function of the tensiometer is the instantaneous indication of any difference in tension between the 2 edges of the strip.

Value of the Tensiometer

A brief outline of the process of cold-strip rolling and of the problem involved is as follows:

A tandem mill consists of 2 or more mill stands placed in line so that the strip passes from one mill directly into the other. It is highly desirable to maintain tension in the strip between these stands. Greater reductions can be obtained, a greater number of elongations can be made without intermediate annealing to soften the strip, and a flatter and smoother strip can be rolled if the proper tension in the strip is maintained. Furthermore, much can be done to compensate for defects in roll shape, strip lubrication, mill-housing stretch, etc., by properly manipulating the tension.

However, in order to control tension it must be measured, and the control cannot be more accurate than the measurement. So far only 2 methods have been discovered for measuring this tension. One method is by electrically measuring the input to the motor producing the tension, and the other is by means of the tensiometer to be described. A brief comparison of these methods of control follows.

INPUT CONTROL

The input to a motor can be measured by measuring the armature voltage and current, and if the voltage is constant, it is only necessary to measure the current. The current in the armature of the motor driving the mill stand is therefore a function of the tension in the strip, provided all the input to the motor goes into tension.

This is not the case in a tandem mill, however. In this case the rolling load may represent about 90 per cent, and the tension 10 per cent, of the motor input. It is hardly conceivable that the rolling load does not vary at least plus and minus 10 per cent. This is equivalent to a variation in tension of 200 per cent if the total input to the motor is maintained at 100 per cent by a current regulator. The difficulties involved in attempting to control the strip tension by means of motor input control are apparent.

MECHANICAL CONTROL

Because of the difficulties involved in controlling the strip tension by controlling motor input, various mechanical schemes of measuring tension have been tried, but the major difficulty with such devices has centered around the inertia of the moving parts. These devices usually involve a roller over which the strip passes in such a way that the strip tension produces a force on the roller. If the roller is resiliently mounted and connected to some device for measuring its displacement when the tension is applied, a measure of the tension can be obtained. If this roller has to move an appreciable distance troubles are experienced in high-speed rolling due to inertia forces of the roller itself, so that the strip tension could change considerably before the roller could be accelerated and moved to its new position. The tensiometer makes use of this general scheme but uses a sensitive electric gauge for measuring the roller deflections, so that very small deflections are required. The natural frequency of the roller can then be made high enough to eliminate troubles resulting from inertia of the moving parts.

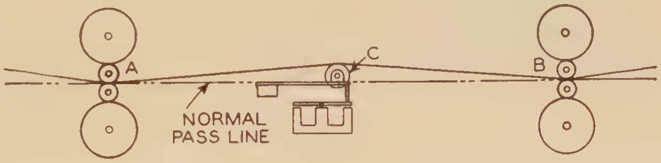

Figure 1. Diagram showing location of pass roller C between 2 stands A and B

LEVEL INDICATION

In addition to the tension problem, there has been a great need for some method of indicating whether the strip was the same thickness throughout its width. One edge can be measured with a flying micrometer but not the entire width. If one edge is the proper thickness and the tension is the same in both edges of the strip, it is reasonable to assume that the thickness of the other side is the same as that measured. If the other side is thicker, the tension will be greater on that side, and vice versa.

A paper recommended for publication by the AIEE committees on instruments and measurements and applications to iron and steel production. Manuscript submitted March 27, 1937; released for publication July 20, 1937.

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This differential tension or "level" indication can also be obtained with the tensiometer.

The Tensiometer

The need for a device for the measurement of total strip tension and of differences in tension between the 2 edges of the strip has been pointed out. The tensiometer produces a continuous indication of the total tension, provides a sensitive indication of any out-of-level condition, and operates in conjunction with standard control equipment to automatically maintain the total tension at any desired value by controlling the input to the mill motors. The tensiometer measures the total tension by measuring the force necessary to deflect the strip a given distance from the normal pass line. The strip as it passes from one mill to the next is caused to pass over a roller located so as to deflect the strip a small distance from the normal pass line. A force is thus produced on the roller at right angles to the normal pass line, and this force is proportional to the total strip tension. The arrangement is shown schematically in figure 1 in which the strip is shown passing from *A* to *B* over the pass roller *C* of the tensiometer.

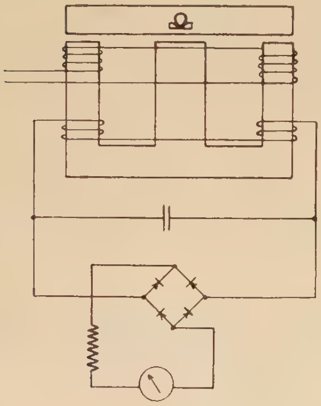
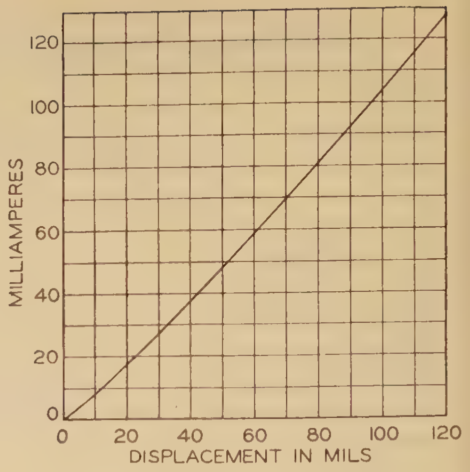


Figure 2. Electric gauge for measurement of pass-roller deflection

Each end of the pass roller *C* is mounted by means of antifriction self-aligning bearings on a stiff cantilever beam, and the deflection of each beam when load is applied to the pass roller is measured by means of an electric gauge. The electric gauge is a device capable of modulating or controlling an electric current in proportion to a small linear displacement. Each electric gauge has an output directly proportional to the deflection of the beam to which it is attached, and therefore, to the vertical load supported by the beam. The sum of the outputs of the 2 electric gauges is proportional to the total vertical load applied to the pass roller, and this is in turn proportional to the total strip tension if the deflections of the cantilever beams are small enough to be negligible in comparison to the elevation of the pass roller *C* above the normal pass line. The outputs of the 2 gauges are added by an instrument calibrated directly in thousands of pounds total strip tension. A second instrument is used to indicate any difference in output between the 2 gauges so that an indication can be obtained of any out-of-level condition that may exist in the mill.

Figure 3. Electric gauge output current versus armature displacement



The electric gauges used for the measurement of the deflection of the pass-roller supports consist of a stack of E-shaped laminations mounted adjacent to a laminated armature as shown in figure 2, so that a small air gap exists between the 2. The armature is supported at its center on knife-edge bearings, so that a movement of the armature on its bearings will increase the air gap at one end and decrease it at the other. On each of the outer legs is a primary and a secondary winding. The primary windings are connected in series across the a-c power source, and in such a direction that the resultant primary flux passes through the center leg. The secondaries are also connected in series, but in such a direction that the resultant flux due to a secondary current will flow around the outside of the laminated structure rather than through the center leg. With the primary and secondary windings connected in this manner, the induced voltage across the secondary terminals will be zero when the armature is in its mid-position and the air gaps at its ends are equal. For this condition, the voltages induced in the 2 secondary coils are equal and opposite. A displacement of the armature from this position will decrease one air gap and increase the other causing an unequal distribution of primary flux between the 2 secondary coils, and a voltage will appear across the secondary terminals that is a function of the armature displacement.

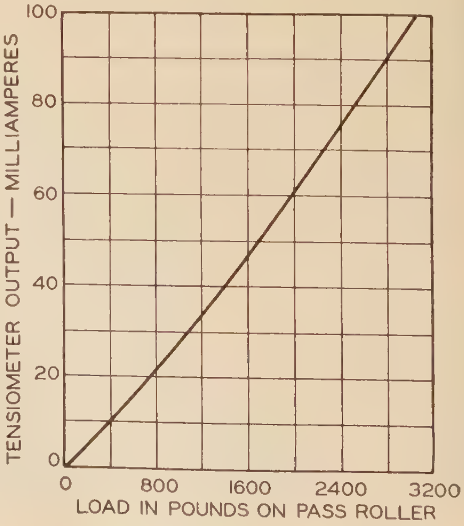


Figure 4. Tensiometer output current versus pressure on pass roller

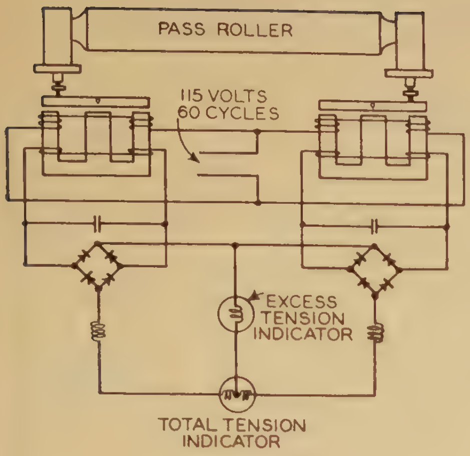


Figure 5. Connection diagram of tensiometer indicating instruments

The electric gauge unit can, therefore, be used to measure the deflection of the springs supporting the pass roller. The secondary output is rectified by a copper-oxide rectifier as shown in figure 2, and the rectified output is used to operate the indicating instrument and the control equipment. A capacitor across the secondary terminals greatly increases the sensitivity of the gauge because of the voltage rise produced by the passage of the leading current of the capacitor through the leakage reactance of the secondary.

The relationship between armature deflection from the neutral position and the rectified current in the secondary circuit is shown in figure 3. In figure 4 is shown the rectified current as a function of pressure on the pass roller as obtained by actual measurement on a typical tensiometer equipment.

If it is desired to indicate unequal tension between the 2 edges of the strip as well as the total tension, each end of the pass roller must be mounted on a separate spring and the deflection of each spring measured separately so that 2 electric-gauge units are required. The outputs of the 2 gauges can be added to obtain the total tension, and any difference between the outputs will be an indication of an out-of-level condition. These indications are obtained as shown in figure 5, in which *A* is the instrument for indicating total strip tension. This instrument carries 2 separate windings on its moving element. Each winding is connected in one of the detector unit circuits so that the instrument torque is proportional to the sum of the 2 electric-gauge outputs or to the total strip tension. Any difference in current between the 2 circuits will flow through the instrument *B* and will produce a sensitive indication of an out-of-level condition.

In some cases it may be desired to roll strip of a wide range of width and thickness in the same mill, and in such cases the total tension may vary over such wide limits that it is necessary to provide a means for changing the scale of the tensiometer. This could, of course, be done by changing the springs on which the pass roller is mounted when it is desired to change scales, but it can be done more easily by changing the mechanical ratio of the

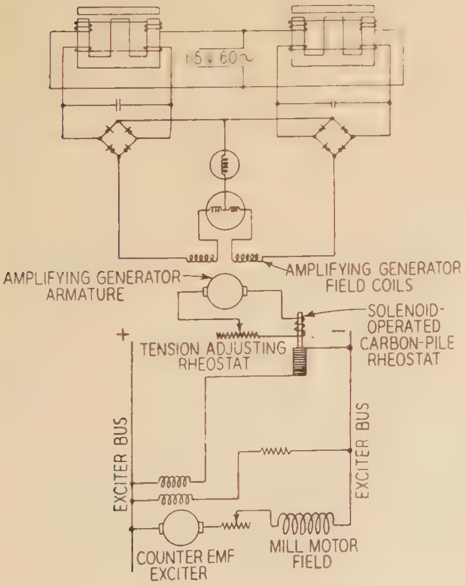


Figure 6. Tension control system

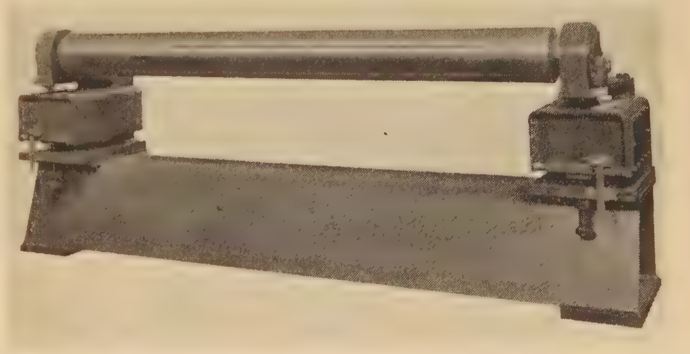


Figure 7. Tensiometer stand with 93-inch pass roller

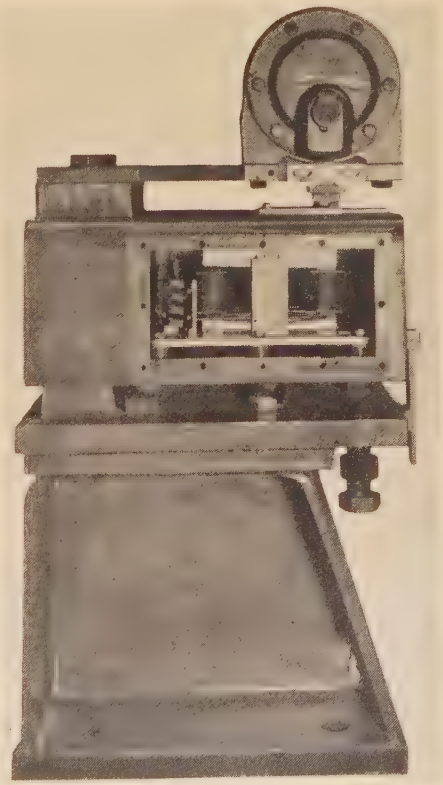


Figure 8. Tensiometer stand with cover removed, showing electric-gauge unit

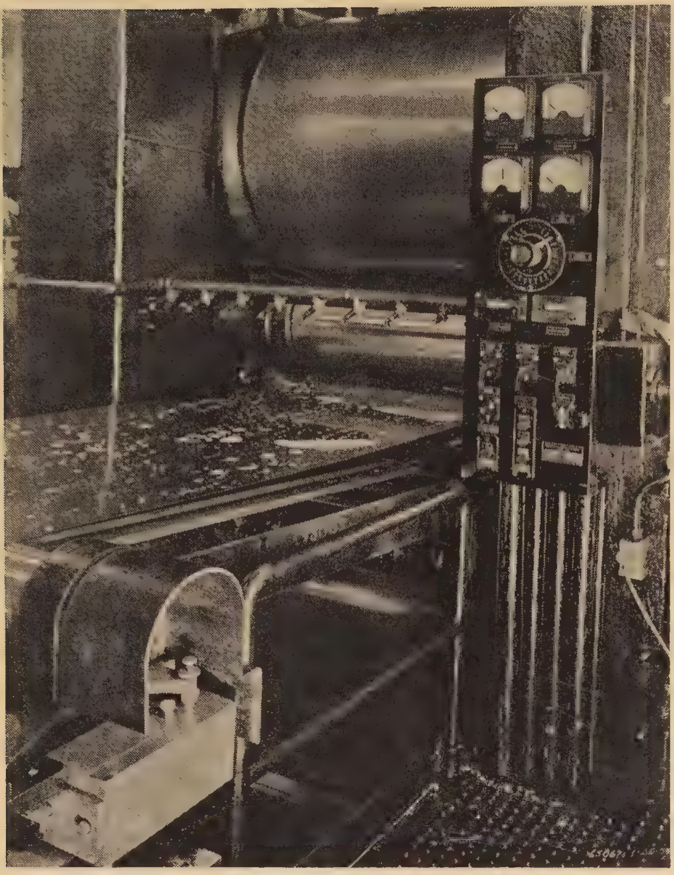


Figure 9. Tensiometer installation

linkage between the pass roller bearings and the electric-gauge armatures.

The upper element of the pass roller is located 2 or 3 inches above the normal pass line and the distance between stands is usually of the order of 10 or 20 feet. The diameters of the working rolls on the mills on each side of the tensiometer roller are not fixed but decrease with time due to wearing and resurfacing, so that the normal pass line is variable by as much as several inches. Provision must be made, therefore, for conveniently adjusting the elevation of the pass roller above the normal pass line, so that this value can be maintained at standard. To aid in making this adjustment, vernier height gauges are attached to each of the pass roller bearings. These gauges are calibrated directly in working-roll diameter, and must at all times be set to coincide with the average measured diameter of the 2 lower working rolls.

Tension-Control System

A satisfactory control system for use in association with the tensiometer equipment for automatically maintaining the total strip tension at any desired value is shown schematically in figure 6. Referring to this figure the electric gauge outputs are amplified by means of a small 150-watt amplifying generator so that sufficient output is available for operation of a constant-current regulator. The regulator is used to control the field current in one winding of the counter-electromotive-force generator in the field circuit of the mill motor of the stand preceding the tensiometer.

Typical Installations

In figure 7 is shown a tensiometer stand equipped with a pass roller 93 inches long. The electric-gauge units are enclosed in the steel boxes at each end of the roller, and the cantilever springs on which the pass roller is mounted are clearly shown. The cover is removed in figure 8, showing one of the electric-gauge units. Figure 9 shows a tensiometer and its associated control cabinet installed in a mill. One end of the tensiometer roller can be seen under the strip at the extreme left side of the photograph.

Conclusions

A device termed a tensiometer has been described for the measurement of the tension in the strip during rolling in a cold-steel mill, and for the indication of any difference in tension between the 2 edges of the strip. The tensiometer can function in connection with auxiliary control equipment for the automatic maintenance of the total tension to any desired value. The value of tension measurement and "level" indication has been outlined, and the use of the tensiometer will aid to a more scientific study and operation of the cold-strip mill.

"Science More or Less"

EVERY technical change brings about maladjustments, more or less serious depending on the number of people involved. Much more is involved than mere unemployment or the lowering of price to consumers. New processes may destroy the capital investments of sleeping corporations; they render men and equipment obsolete; they uproot whole industries and ways of life. To date, invention and scientific discovery have kept to the black side of the ledger; they have been credits to civilization. We should like to keep them there.

No one has yet solved the problems of human obsolescence, mass motivation, or technological unemployment. Unless our sociological inventiveness can show great improvement in the next 25 years we will not have made much progress no matter how pleasant and comfortable the air-conditioned lives of a few of us may be. I do not mean to imply that to get out of the woods we must have rigidly planned economy. The problem is much too complicated for that. The best plan in the world would be useless without the proper personnel, and with the proper personnel the plan, with its attending unpleasantness, is unnecessary. Education is far more important than coercion.

From an article by C. C. Furnas of the Yale University school of engineering, New Haven, Conn., published in volume 1, number 1, of *Amateur Scientist.*

Mechanical High-Speed Resistance-Welder Control

By F. H. ROBY
ASSOCIATE AIEE

Operating speeds of 350 or more welds per minute are made practical with a new type of control equipment for pneumatically operated portable welding tools.

A DEMAND for higher production rates in the automobile and related industries has resulted in the adoption of pneumatically operated portable type resistance welding machines.[1] Fully automatic repeat control devices are needed to obtain maximum operating speeds without sacrificing the quality of the welds. To be practical this control must be simple and inexpensive.

This paper is concerned with a control device developed for use in connection with a high-speed portable air-operated machine now commonly used in the automobile industry. The principles outlined apply as well to other types of equipment.

A "stitch" welder consists of a pneumatically operated welding tool, a solenoid-operated valve and a separate welding transformer connected to the tool by means of water-cooled cables, 6 feet or more in length. When the operator squeezes a momentary contact push button in the pistol-grip handle of the tool, the automatic control device closes the electrodes, applies power, disconnects it, opens the electrodes and keeps them open for a definite period of time before repeating the cycle. Repeat operation continues for as long as the push button is closed. The term "stitch" welder has been applied because of similarity to an electric sewing machine.

Those connected with the application of resistance welding to the fabrication of sheet metal are familiar with the principle upon which the art depends—that the heat generated in the weld is proportional to the product of the welding current squared, the resistance between pieces of work and the time during which the current flows.[2] Few realize the difficulties encountered in satisfactorily controlling these 3 variables.

The current is determined by selecting the turn ratio of the welding transformer by means of a tap switch on the primary side. The resistance of the weld decreases or increases as the pressure applied to the work increases or decreases, respectively, and is controlled by regulating that pressure. Timing has previously involved the use of expensive and complicated control equipment. A recent trend is toward simplified but more comprehensive control.

Control Equipment Now Used

Electronic control devices (power interrupted by electronic tubes instead of magnetic contactors) are made to accurately control the period of time during which the welding current is allowed to flow.[3] These devices select the point on the current wave at which the circuit is made and broken. Such equipment is necessary for some special applications but the first cost is high and complicated circuits make highly trained maintenance men necessary.

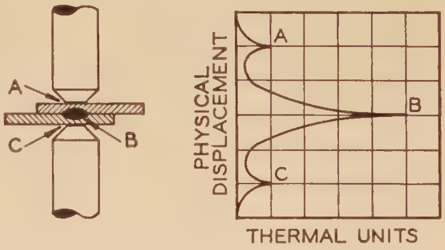
Other less-expensive systems of control consist of a magnetic contactor and some form of timing device. The timer may be mechanical, electromagnetic, or electronic.[4,5,6] Most designs determine only the period of time during which the welding current flows. A few control the interval of time between welds as well as the welding period.

Four Independent Timing Periods Required

Recent experience has made clear that the factor of time in the weld has 4 component parts. For purpose of identification these have been named "delay," "weld," "hold," and "off."

The "delay" period is the interval required between the operation of the solenoid valve and the application of power. Two conditions may result from insufficient time delay. If the electrodes are energized before they touch the work a "flash" will result, leaving the surface of the work mutilated and the electrodes damaged. If the electrodes are closed but are under insufficient pressure when power is applied, the weld will appear to be satisfactory but it will not hold. Reference to the thermal gradient chart (figure 1) will make clear that heat is generated at 3 points in the secondary circuit. If the pressure on the work is low when the transformer is energized, the heat generated at points *A* and *C* will be large compared to that generated at *B*. The surfaces of the work will be burned before the temperature at point *B* is raised to the fusion point.

Figure 1. Thermal gradient chart showing distribution of heat in weld



Some manufacturers of welding equipment have recognized the need for obtaining a sufficient amount of pressure

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[1] For all numbered references, see list at end of paper.

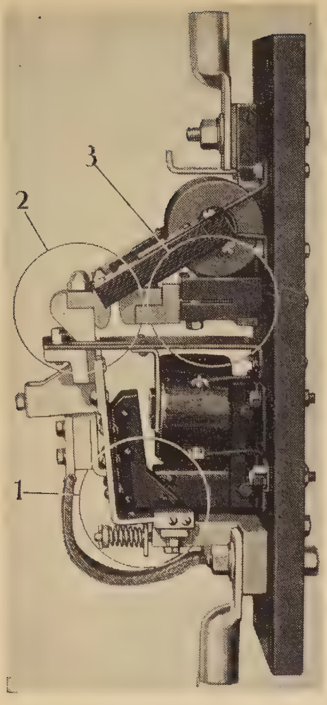


Figure 2. Profile view of high-speed magretic contactor

on the electrodes within the shortest possible time and to this end have designed tools with favorable operating characteristics. Short strokes, large ports, and operating valves mounted as close to the tool as possible are essential. At least one automobile body manufacturer speeded up production by designing a solenoid valve in the handle of the tool itself, thereby eliminating the delay involved in building up pressure in the line between the valve and the tool.

A pressure switch is ordinarily connected in the air line in such a manner that the contacts of the switch close when the pressure on the operating piston has reached the desired value. The closing of the switch initiates the remainder of the welding cycle. Although satisfactory for most applications, the pressure switch has been eliminated from stitch-welder control. It represents an additional delay in the time required for the pressure to build up and exhaust in the line between the tool and the switch. Also, experience has determined that pressure switches will close prematurely because of turbulent air conditions, "slugs" of water in the line or because the electrodes are called upon to force together 2 or more pieces of work which have been formed improperly and do not fit perfectly. The solution of the problem lies in including in the control means for providing a definite time delay between the operation of the valve controlling the electrodes and the application of power. This period must be adjustable to compensate for the difference in the design of tools and the idiosyncrasies of the installation.

Much has been said about the "weld" time in other published information.[3,4,5,6] It is sufficient to point out that in the interest of high-speed production and improved welds, this period has been continually shortened until 5-cycle welds (60-cycle frequency) are considered average in industries using sheet metal ranging in thickness from 12 to 22 gauge. Shortened timing periods give less time for the heat generated in the weld to be conducted to the surface of the metal, thereby defacing or deforming the work.

In general, "fixed" timing is used. There is little need for "current responsive" timing which, in one manner or another, compensates for variations in the welding current.[5] The value of this current is essentially predetermined by the secondary voltage and the short-circuited impedance of the secondary circuit. The resistance of the weld itself has little effect on the impedance of the circuit formed by a pair of electrodes at the end of 2 cables at least 6 feet in length. Then, too, high-grade body stock is not usually subject to scale, rust, or other imperfections which materially influence the resistance of the weld. Changes in resistance of the weld affect results because of the presence of the resistance factor in the heat equation. Even accurate compensation for minor current changes would not alter this condition.

The shortest timing period practical when magnetic contactors are used to interrupt the primary of the welder transformer is between 2 and 3 cycles. There are 3 limitations. The so-called high-speed welder contactor requires slightly less than a cycle to close after the operating coil has been energized, and a little more than a cycle to open after it has been de-energized. If a contactor, when once closed, remains so for over a cycle, it cannot be used to energize a transformer for a period of time less than this amount. In fact, a longer period of time should be allowed to insure that the pole pieces of the magnet seal each operation. Otherwise, extremely erratic performance will result.

Secondly, magnetizing current transients[3,7] which result when the transformer is energized at the zero point of the voltage wave, cause wide variations in results if shortened timing periods are used. One welder application studied had peak primary currents of 217 and 1,030 amperes, respectively, on 2 consecutive spot welds. Without a doubt, this case is extreme but it serves to illustrate a limiting factor in shortened weld periods. The transient currents contribute to a lowered secondary voltage and a "cold" spot is the result. Fortunately, such transients die out along an exponential curve with a very steep characteristic, and they are effective for only one or 2 cycles.

Finally, a magnetic contactor will not respond accurately to a given electrical impulse unless that impulse begins each operation at the same point on the reference voltage wave. Otherwise, the transient in the operating coil may cause the contactor to fail to respond.

One solution to the transient problem is the use of a control device having electronic tubes as interrupters instead of magnetic contactors. The point at which the circuit

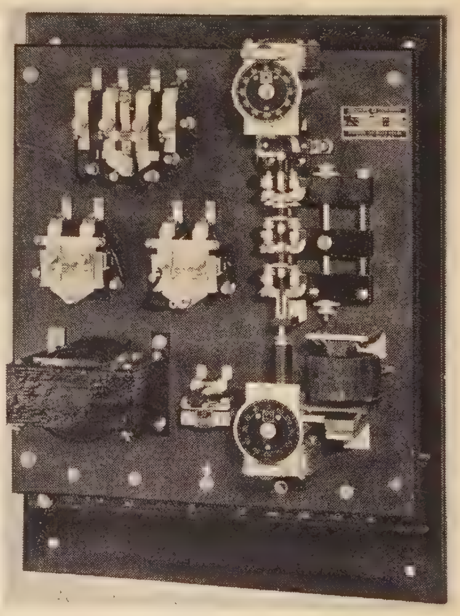


Figure 3. Mechanical high-speed resistance-welder control unit

is made and broken can then be controlled. The use of such complicated and expensive control equipment is not necessary when ordinary ferrous metals are being welded because the plastic range of the metal is wide enough to absorb reasonable variations in control. The timing period can be lengthened to 3 or more cycles to make the variations small when compared to the total.

The design of a magnetic contactor which will consistently perform at the rate of 350 or more operations per minute and will have the operating characteristics needed for short timing is a problem in itself. Long mechanical life, immediate interruption of the arc, rapid operation, and accessibility are imperative. The high-speed heavy-duty welder contactor illustrated (figure 2) has knife edge bearings (1); short gap (2); strong blowout coil and large copper contacts with gap adjustment (3).

The "hold" time is the interval least discussed in the timing cycle and yet is one of the most importance. Until recently, it was general practice to de-energize the operating coils of the welding contactor and the solenoid valve at the same instant. It was assumed that the welding contactor would open before the electrodes parted. Such practice was effective for as long as tools were sluggish and exhaust lines restricted. Modern tools, with valve in the handle, open as fast or faster than the welding contactor, making a predetermined "hold" time necessary.

When the electrodes are allowed to remain on the work a period of time after the power is off, the appearance of the weld is improved. The water-cooled electrodes conduct away the heat as it comes to the surface, thereby preventing abnormal discoloration. A weld allowed to cool under pressure also has finer grain structure and greater strength.

Two additional advantages of "hold" time have made themselves apparent in the automobile industry. Welds made on pieces of work not properly fitted together are prevented from breaking open before the weld is cooled below the plastic stage and heat is prevented from traveling ahead of the spot being welded. The electrodes of the tool are apt to stick to the surfaces of work made plastic by heat from a weld immediately adjacent.

The "off" period needs little discussion. When repeat operation is desired, the operator continues to hold down the momentary contact push button in the handle of the tool and the timing device regulates the period during which the electrodes remain open before the cycle is repeated. The minimum "off" time is determined by the interval required to open the gun and the delay involved in moving the tool to a point at which the next weld is to be made.

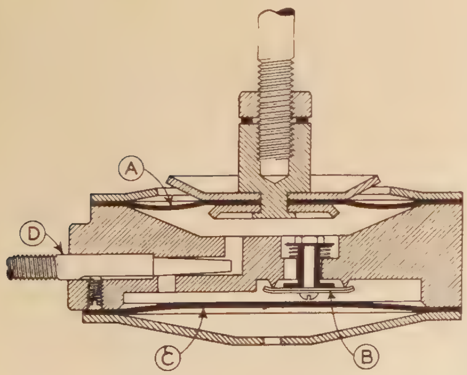


Figure 4. Cross-sectional view of time-delay unit

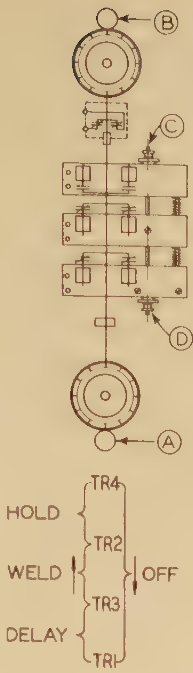


Figure 5. Graphical representation of control adjustments

Description of Control Unit

The need for separately adjustable timing periods is clear. To obtain them without seriously complicating the control device has presented a problem. One solution is illustrated in figure 3. The nucleus of the device is formed by 2 diaphragm-type pneumatic time-delay assemblies joined by means of a rod upon which a series of contacts is mounted. With the rod in its normal position, the top diaphragm *A* of the lower assembly (figure 4) is cupped downward, forcing the air in the upper chamber down through the spring-held release valve *B*. Lower diaphragm *C* responds to the movement of the top diaphragm confining the air to the lower chamber and making it unnecessary to expose the mechanism to the danger of faulty operation because of dust or other foreign matter in the air surrounding the installation. When force is applied to the contact rod, tending to move it upward, air must be drawn into the upper chamber from the lower one. Because valve *B* has seated itself the only possible passage is regulated by needle valve *D*. A similar mechanism coupled to the upper end of the rod regulates its downward travel.

The interval off time required for the rod to complete its upward stroke must be long enough to include the "delay," "weld," and "hold" periods (figure 5). The "off" period is determined by the interval required for the rod's return to its initial position.

Once the rod speed has been established, adjustment *C* can be regulated to provide spacing between contacts *TR*2 and *TR*3 and obtain the "weld" time needed. Adjustment *D* proportions the "delay" and "hold" periods to meet the requirements of the application. The "weld" time is not changed by this adjustment inasmuch as contacts *TR*2 and *TR*3 are moved the same amount.

Schematic Diagram of Circuit

Figure 6 is a schematic diagram of the electrical connections used. Closing the push button energizes control relay *CR*1 which in turn applies power to the check valve and the operating coil of control relay *CR*2. This relay energizes the solenoid operated 4-way valve which applies air to the tool and also connects the operating coil of timing relay *TR*1 to the line. The armature of the timing relay magnet closes instantly, compressing a spring which applies force to the contact rod. Timing is, therefore, independent of voltage.

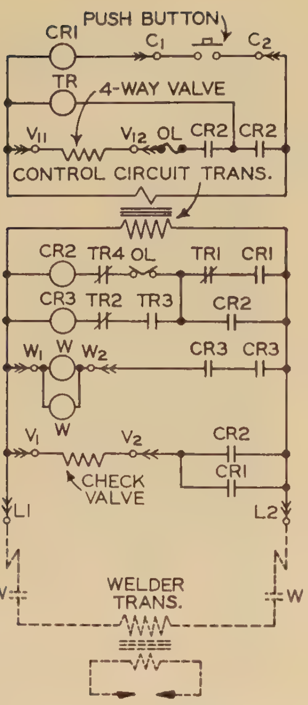


Figure 6. Schematic diagram of electrical connections

During the upward travel of the rod, contact *TR*1 is opened first, disconnecting the pole of relay *CR*1 which functions from the push button. Operation continues because relay *CR*2 has formed its own maintaining circuit. Later in the rod travel contact *TR*3 closes, energizing relay *CR*3 which in closing energizes the magnetic contactor that applies power to the machine. Still later contact *TR*2 opens, dropping out relay *CR*3 and de-energizing the welding contactor. The pressure on the electrodes is not removed until the rod reaches the end of its upward travel, opening contact *TR*4 and de-energizing control relay *CR*2. The cycle is not repeated again until the rod is returned to its initial position, reclosing contact *TR*1.

Features not ordinarily required can be included in the control device. The installation of a check valve to close the air-supply line during the period in which the tool is not in use is not difficult. Double action air cylinders are ordinarily used to speed up operation and considerable leakage occurs in the head end of such cylinders.

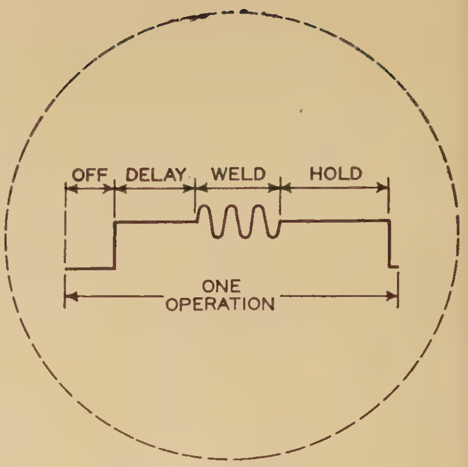
A low-voltage transformer large enough to supply the solenoid operating valve can be mounted on the panel. It is considered good practice to energize the valve solenoid from a low-voltage supply source if it is located in the handle of the tool where a ground would endanger the operator. To prevent the burning out of the valve solenoid if it should lodge in the open position, a thermal protective device can be included in the circuit.

Maintenance is simplified by the use of a separable connector on the panel to make it unnecessary to disconnect any wiring if the panel is removed. Panels can be interchanged with minimum delay and repairs or adjustments on the defective unit can be made later in the electrician's shop.

Test Procedure

The information contained in this paper has been verified in a series of field tests conducted by one of the large automobile-body manufacturers. A cathode-ray oscillograph was used to analyze the various components of the timing period previously described. The circuit of the oscillograph was arranged to show simultaneously on the viewing screen 4 elements of the weld cycle. To accomplish this the a-c voltage applied to the solenoid-operated air valve was fed through a rectifier to the vertical plates of the cathode-ray tube. A second a-c signal having as its source the voltage drop across the work itself was applied directly to the same set of plates. Horizontal plates of the cathode-ray tube were energized from a sweep circuit which forms a part of the oscillograph. A pattern similar to that illustrated in figure 7 resulted. Synchronizing contacts on the control device being studied caused the image to appear at the same place on the screen each operation.

Figure 7. Representation of oscillograph screen showing method of measuring weld cycle



Practical problems in welding were considered and a theoretical solution to each problem was advanced. Adjustment of the timing device to meet the specifications outlined in the theoretical solution was accomplished with the aid of the oscillograph. Samples of materials were welded and results checked. In each instance it was determined that satisfactory welds could be made if the 4 component parts of the welding cycle were properly proportioned. At least 25 installations of the type described have been in operation for several months on an automobile-body assembly line. Reports indicate that the conclusions reached during laboratory tests were justified.

Conclusions

Under laboratory controlled conditions good welds have been made at the rate of 600 per minute. Commercial results will depend upon the design of the equipment being used and the application involved. It is certain, however, that good welds will be made at the maximum rate only if each portion of the welding cycle is accurately and independently controlled.

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Double-Line-to-Neutral Short Circuit of an Alternator

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Starting with R. H. Park's formulas for the flux linkages of an ideal synchronous machine, the case of a fault from 2 line terminals of a generator to neutral is treated in a manner similar to that used by Doherty and Nickle for other unsymmetrical faults. Expressions are given for the armature currents in the faulted phases and for the main field current, at any time after the short circuit. An expression for the voltage across the open phase a short time after the fault is also developed.

Introduction

THE PROBLEM of short circuits at the terminals of synchronous machines is one that has been studied extensively for a number of years. R. H. Park[1] has given a rigorous solution of the 3-phase symmetrical fault, and Doherty and Nickle[2] have developed expressions for the currents flowing during single-phase short circuits both between lines and from line to neutral which give excellent checks with experimental results. The present paper derives similar expressions for currents flowing in a machine when 2 lines are faulted to neutral. Park's formulas for the armature and field flux linkages of an ideal synchronous machine are used as a starting point. They are simplified by the additional assumption that all resistances are negligible. Decrement factors are applied to the resulting expressions for current and voltage in a manner analogous to that used by Doherty and Nickle. The final formulas are found to give results which are in good agreement with experiment.

The basic assumptions made in the treatment of this problem are:

1. The assumptions made by R. H. Park in his definition of an ideal synchronous machine.[5]
2. That the resistances of all field and armature circuits are negligible except where they appear in decrement factors.
3. That the speed of the rotor is constant at all times.

The second assumption is equivalent to saying that the flux linkages in both the direct and quadrature axes remain constant, that is, the effects of the terms $p\psi_d$ and $p\psi_q$ of reference 1 are neglected. If this assumption were strictly adhered to, the initial values of the currents as given by equations 11 would persist indefinitely. We know, however, that if assumption 2 were not made the fluxes and consequently the currents would decrease according to certain decrement factors obtained from the roots of a determinantal equation arising in the solution of equations 11, 12, and 13 of reference 1. Since in the case of a double-line-to-neutral fault this equation cannot be solved algebraically these decrement factors are obtained approximately and applied to the appropriate terms of equations 11. This treatment should not produce much error unless either the decrement factor of some circuit or the armature circuit resistance is large.

The third assumption should not introduce much error except in the case of a machine with a small moment of inertia which might slow down appreciably at the instant of short circuit.

The machine is considered to have 2 field circuits in the direct axis, referred to as the main and the additional field. In the quadrature axis there is no main field, so only the additional (amortisseur) field is present. Direct current excitation is applied only in the main field.

The short circuit is assumed to occur directly at the terminals of the machine at a time when it is carrying no load. If the short circuit occurred through external reactance this reactance could be added to the direct- and quadrature-axis reactances and the sums used in equations 35. If there is much external resistance the equations of this paper cannot be used because of assumption 2.

System of Units

All quantities said to be in armature terms are in a per-unit system based on the following normal values:

e_n = rated phase voltage of the machine in volts
i_n = rated phase current in amperes
$X_n = r_n = \dfrac{e_n}{i_n}$ = normal reactance or resistance in ohms
$\omega_n = 2\pi f_n$ = normal angular velocity in radians corresponding to a normal frequency of f_n cycles
$L_n = \dfrac{X_n}{\omega_n}$ = normal inductance in henrys
$\psi_n = L_n I_n$ = normal flux linkages in volt seconds

The term "per unit" means that the quantity in question is expressed as a fraction of the appropriate normal value. Note that at normal frequency per-unit reactance equals per-unit inductance. The symbol x as used in the paper can therefore be thought of as either an inductance or a reactance, whichever is most convenient.

For reasons that will be brought out later it is sometimes convenient to express quantities in field terms. Quantities said to be in main field terms have the following normal values:

$E_{fn} = \dfrac{e_n R_{fd}}{x_{afd}}$ = voltage in volts which if connected to main field terminals will cause normal field current to flow

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1. For all numbered references, see list at end of paper.

where

R_{fd} = per-unit resistance of the main field in armature terms
x_{afd} = per-unit mutual inductance of the armature with the main field in armature terms
$I_{fn} = \dfrac{i_n}{x_{afd}}$ = normal main-field current, that is main-field current which will produce rated terminal voltage on open circuit

$R_{fn} = \dfrac{e_n}{i_n} R_{fd}$ = normal main-field resistance

Quantities said to be on additional field terms are based on similarly defined normal values except that the subscript *f* is replaced by one.

To change a per-unit quantity from one system to another simply multiply the per-unit quantity in the old system by the ratio of its normal value in the old system to its normal value in the new system. Thus for example:

$$I = I_{fd} \times \frac{i_n}{I_{fn}} = x_{afd} I_{fd}$$

where:

I = main-field current in main-field terms, direct axis
I_{fd} = main-field current in armature terms, direct axis

Development of General Equations

Let the following symbols represent per-unit quantities in armature terms:

ψ_a, ψ_b, ψ_c = armature phase flux linkages in phases *a*, *b*, and *c*
i_a, i_b, i_c = armature phase currents in phases *a*, *b*, and *c*
x_d'', x_d', x_d = subtransient, transient, and synchronous reactances in the direct axis
x_q'', x_q', x_q = subtransient, transient, and synchronous reactances in the quadrature axis
x_0 = zero-sequence reactance
θ = angular position of the rotor in degrees measured from the axis of phase *a*
e = open-circuit terminal voltage for one phase
r = resistance of one armature phase

Let the following symbols represent per-unit quantities in main field terms:

I = current in main field, direct axis
E = excitation voltage in main field, direct axis. It is also numerically equal to the sustained steady state d-c current in main field, direct axis

Let the following symbols represent per-unit quantities in additional field terms:

I_{d1} = current in additional field, direct axis
I_{q1} = current in additional field, quadrature axis

It is also convenient to talk about the total excitation due to all fields in an axis. In terms of the above defined quantities there is:

$I_d = I + I_{d1}$ = excitation in direct axis
$I_q = I_{q1}$ = excitation in quadrature axis

Park gives the following formulas for armature flux linkages:

$$\psi_a = I_d \cos\theta - I_q \sin\theta - \frac{x_0}{3}(i_a + i_b + i_c) - \frac{x_d + x_q}{3}\left(i_a - \frac{i_b + i_c}{2}\right) - \frac{x_d - x_q}{3}[i_a \cos 2\theta + i_b \cos(2\theta - 120) + i_c \cos(2\theta + 120)] \quad \textbf{(1a)}$$

$$\psi_b = I_d \cos(\theta - 120) - I_q \sin(\theta - 120) - \frac{x_0}{3}(i_a + i_b + i_c) - \frac{x_d + x_q}{3}\left(i_b - \frac{i_c + i_a}{2}\right) - \frac{x_d - x_q}{3} \times [i_a \cos(2\theta - 120) + i_b \cos(2\theta + 120) + i_c \cos 2\theta] \quad \textbf{(1b)}$$

$$\psi_c = I_d \cos(\theta + 120) - I_q \sin(\theta + 120) - \frac{x_0}{3}(i_a + i_b + i_c) - \frac{x_d + x_q}{3}\left(i_c - \frac{i_a + i_b}{2}\right) - \frac{x_d - x_q}{3}[i_a \cos(2\theta + 120) + i_b \cos 2\theta + i_c \cos(2\theta - 120)] \quad \textbf{(1c)}$$

When the assumption is made that $r = 0$, equations 4*a* and 5 of reference 1 become:

$$I_d = E + (x_d - x_d'')i_d \quad (2)$$

$$I_q = (x_q - x_q'')i_q \quad (3)$$

where

$$i_d = \frac{2}{3}[i_a \cos\theta + i_b \cos(\theta - 120) + i_c \cos(\theta + 120)] \quad (4)$$

$$i_q = -\frac{2}{3}[i_a \sin\theta + i_b \sin(\theta - 120) + i_c \sin(\theta + 120)] \quad (5)$$

also in the per-unit systems used

$$E = e \quad (6)$$

After substituting equations 2 to 6 into equations 1 there results:

$$\psi_a = e\cos\theta - \frac{x_0}{3}(i_a + i_b + i_c) - \frac{x_d'' + x_q''}{3}\left(i_a - \frac{i_b + i_c}{2}\right) - \frac{x_d'' - x_q''}{3}[i_a \cos 2\theta + i_b \cos(2\theta - 120) + i_c \cos(2\theta + 120)] \quad \textbf{(7a)}$$

$$\psi_b = e\cos(\theta - 120) - \frac{x_0}{3}(i_a + i_b + i_c) - \frac{x_d'' + x_q''}{3}\left(i_b - \frac{i_c + i_a}{2}\right) - \frac{x_d'' - x_q''}{3}[i_a \cos(2\theta - 120) + i_b \cos(2\theta + 120) + i_c \cos 2\theta)] \quad \textbf{(7b)}$$

$$\psi_c = e\cos(\theta + 120) - \frac{x_0}{3}(i_a + i_b + i_c) - \frac{x_d'' + x_q''}{3}\left(i_c - \frac{i_a + i_b}{2}\right) - \frac{x_d'' - x_q''}{3}[i_a \cos(2\theta + 120) + i_b \cos 2\theta + i_c \cos(2\theta - 120)] \quad \textbf{(7c)}$$

If θ_0 is the angular position of the rotor at the last instant before short circuit, then at that instant

$$\left.\begin{aligned} \psi_a &= e\cos\theta_0 \\ \psi_b &= e\cos(\theta_0 - 120) \\ \psi_c &= e\cos(\theta_0 + 120) \end{aligned}\right\} \quad (8)$$

since because of our assumption that the armature resistance is negligible the flux linkages before the short circuit must equal the flux linkages after the short circuit. When equations 7*a*, 7*b*, 7*c*, and 8 are combined and solved for

i_a and i_b under the special condition for a double-line-to-neutral fault, $i_c = 0$, there results:

$$i_a = \frac{1}{2}\sqrt{3}e\left\{\frac{(x_d'' + x_q'')[\cos(\theta - 30) - \cos(\theta_0 - 30)]}{D} + \frac{(x_d'' - x_q'')[\cos(2\theta - \theta_0 - 30) - \cos(\theta - 30)]}{D} + \frac{2x_0[\cos(\theta + 30) - \cos(\theta_0 + 30)]}{D}\right\} \quad (9a)$$

$$i_b = \frac{1}{2}\sqrt{3}\,e\left\{\frac{(x_d'' - x_q'')[\cos(\theta - 90) - \cos(\theta_0 - 90)]}{D} + \frac{(x_d'' - x_q'')[\cos(2\theta - \theta_0 - 90) - \cos(\theta - 90)]}{D} - \frac{2x_0[\cos(\theta + 30) - \cos(\theta_0 + 30)]}{D}\right\} \quad (9b)$$

where

$$D = x_d''\left(x_0 + \frac{1}{2}x_q''\right) + x_q''\left(x_0 + \frac{1}{2}x_d''\right) + \left[x_d''\left(x_0 + \frac{1}{2}x_q''\right) - x_q''\left(x_0 + \frac{1}{2}x_d''\right)\right]\cos(2\theta + 60) \quad (10)$$

It can be seen that each of the phase currents is made up of 3 components. Each of these components may be expanded into a harmonic series by assuming a Fourier series and finding the appropriate coefficients. When this work is carried out there results:

$$i_a = \frac{1}{2}\sqrt{3}e\,[M(x_d'' + x_q'')(A_{ep} + A_{op}) + M(x_d'' - x_q'') \times (A_{en} + A_{on}) + Mx_0(A_{e0} + A_{00})] \quad (11a)$$

There is a similar expression for i_b:

$$i_b = \frac{1}{2}\sqrt{3}e\,[M(x_d'' + x_q'')(B_{ep} + B_{op}) + M(x_d'' - x_q'') \times (B_{en} + B_{on}) - Mx_0(A_{e0} + A_{00})] \quad (11b)$$

where

$$M = \frac{1}{2\sqrt{x_d''\left(x_0 + \frac{1}{2}x_q''\right)x_q''\left(x_0 + \frac{1}{2}x_d''\right)}}$$

$$A_{ep} = -2\cos(\theta_0 - 30)\left(\sum_{n=2}^{\infty} b^{\frac{n}{2}}\cos n(\theta + 30) + \frac{1}{2}\right)$$

The subscripts indicate that this is the series of *even* harmonics which is multiplied by x_d'' *plus* x_q''.

$$A_{op} = +\frac{1}{2}(1 + b)\sum_{n=1}^{\infty} b^{\frac{n-1}{2}}\cos n(\theta + 30) + \frac{1}{2}\sqrt{3}(1 - b)\sum_{n=1}^{\infty} b^{\frac{n-1}{2}}\sin n(\theta + 30)$$

$$A_{en} = +\cos\theta_0(1 - b^2)\sum_{n=2}^{\infty} b^{\frac{n-2}{2}}\sin n(\theta + 30) - b\sin\theta_0 - \sin\theta_0(1 + b^2)\sum_{n=2}^{\infty} b^{\frac{n-2}{2}}\cos n(\theta + 30)$$

$$A_{on} = -A_{op}$$

$$A_{e0} = -4\cos(\theta_0 + 30)\left[\sum_{n=2}^{\infty} b^{\frac{n}{2}}\cos n(\theta + 30) + \frac{1}{2}\right]$$

$$A_{00} = 2(1 + b)\sum_{n=1}^{\infty} b^{\frac{n-1}{2}}\cos n(\theta + 30)$$

$$B_{ep} = -2\cos(\theta_0 - 90)\left[\sum_{n=2}^{\infty} b^{\frac{n}{2}}\cos n(\theta + 30) + \frac{1}{2}\right]$$

$$B_{op} = -\frac{1}{2}(1 + b)\sum_{n=1}^{\infty} b^{\frac{n-1}{2}}\cos n(\theta + 30) + \frac{1}{2}\sqrt{3}(1 - b)\sum_{n=1}^{\infty} b^{\frac{n-1}{2}}\sin n(\theta + 30)$$

$$B_{en} = +\cos(\theta_0 + 60)(1 - b^2)\sum_{n=2}^{\infty} b^{\frac{n-2}{2}}\sin n(\theta + 30) - b\sin(\theta_0 + 60) - b\sin(\theta_0 + 60) - \sin(\theta_0 + 60)(1 + b^2) \times \sum_{n=2}^{\infty} b^{\frac{n-2}{2}}\cos n(\theta + 30)$$

$$B_{on} = -B_{op}$$

$$b = \frac{\sqrt{x_q''\left(x_0 + \frac{1}{2}x_d''\right)} - \sqrt{x_d''\left(x_0 + \frac{1}{2}x_q''\right)}}{\sqrt{x_q''\left(x_0 + \frac{1}{2}x_d''\right)} + \sqrt{x_d''\left(x_0 + \frac{1}{2}x_q''\right)}}$$

Only odd values of n are to be used in the summations from n = one to $n = \infty$. Only even values of n are to be used in the summations from $n = 2$ to $n = \infty$. Doherty and Nickle have shown in the case of single-phase short circuits that all d-c and even harmonic terms in the armature current die to zero according to a time constant depending on constants of the armature. Odd harmonic terms die to a steady state value according to time constants depending on constants of the field circuits. In the double-line-to-neutral short there are 3 armature time constants, one to be used with the even harmonic terms of each of the components of the armature current. Before proceeding with the derivation of these time constants it will first be necessary to find an expression for the field current.

Field Current

Let the following symbols represent per-unit quantities in armature terms:

ψ_{fd} = flux linkages with the main field
ψ_{1d} = flux linkages with the additional field
I_{fd} = main-field current
I_{1d} = additional field current
i_d = current flowing in the direct-axis armature circuit
X_{ffd} = self-inductance of main field
X_{f1d} = mutual inductance of main field with additional field
X_{11d} = self-inductance of additional field, direct axis
x_{afd} = mutual reactance of armature with main field
x_{a1d} = mutual reactance of armature with additional field
E_{fd} = voltage across main-field terminals
R_{fd} = resistance of main field
R_{11d} = resistance of additional field

Waring and Crary[3] have given expressions for the field flux linkages in armature terms. In the main field, direct axis:

$$\psi_{fd} = X_{ffd}I_{fd} + X_{f1d}I_{1d} - x_{afd}i_d \qquad (12)$$

and in the additional field, direct axis:

$$\psi_{1d} = X_{f1d}I_{fd} + X_{11d}I_{1d} - x_{a1d}i_d \qquad (13)$$

before the fault

$$\psi_{fd} = E_{fd}/R_{fd}X_{ffd} \qquad (14)$$

$$\psi_{1d} = E_{fd}/R_{fd}\, X_{1fd} \qquad (15)$$

Assuming constant flux linkages these four equations may be solved simultaneously and there results:

$$I_{fd} = \frac{E_{fd}}{R_{fd}} - \frac{X_{a1d}X_{f1d} - x_{afd}X_{11d}}{X_{11d}X_{ffd} - X_{f1d}^2}\, i_d \qquad (16)$$

$$I_{1d} = -\frac{x_{afd}X_{f1d} - x_{a1d}X_{ffd}}{X_{11d}X_{ffd} - X_{f1d}^2}\, i_d \qquad (17)$$

Equations 16 and 17 can be rewritten in individual field terms and there results:

$$I = E - \frac{X_{a1d}X_{f1d} - x_{afd}X_{11d}}{X_{11d}X_{ffd} - X_{f1d}^2}(x_{afd})i_d = E + A_{fd}x_d i_d \qquad (18)$$

$$I_{d1} = -\frac{X_{afd}X_{f1d} - x_{a1d}X_{ffd}}{X_{11d}X_{ffd} - X_{f1d}^2}(x_{a1d})i_d = A_{1d}x_d i_d \qquad (19)$$

If 16 is multiplied by x_{afd} and 17 by x_{ald} and the resulting equations are added it is found that:[3]

$$x_{afd}I_{fd} + x_{a1d}I_{1d} = \frac{E_{fd}}{R_{fd}}x_{afd} + (x_d - x_d'')i_d \qquad (20)$$

If the field quantities in this equation are now expressed in terms of their individual fields 20 becomes:

$$I + I_{d1} = E + (x_d - x_d'')i_d \qquad (21)$$

where

$$x_d'' = x_d - \frac{X_{11d}x_{afd}^2 - 2X_{f1d}x_{a1d}x_{afd} + X_{ffd}x_{a1d}^2}{X_{11d}X_{ffd} - X_{f1d}^2}$$

Note that equation 21 is the same as equation 2.

The factors A_{fd} and A_{1d} serve to divide the rotor current into the portions which flow in the main and amortisseur fields. For the transient d-c components of the rotor currents these factors can be put in a simpler form. In most practical machines the transient d-c component of I_{1d} dies out very rapidly so that it is zero for most of the time that the transient part of I_{fd} is flowing. Therefore we may consider that for d-c transients X_{f1d} in equations 18 and 19 is equal zero. Under these conditions

$$x_dA_{fd} = x_d - x_d' \qquad (22)$$

$$x_dA_{1d} = x_d' - x_d'' \qquad (23)$$

Substituting in equation 18 for i_d and E their values from equations 4, 9, and 6 there results:

$$I = e + ex_dA_{fd}\left\{\frac{x_d'' + x_q''}{2}\left[\frac{1 - \cos(\theta - \theta_0)}{D}\right] - \frac{x_d'' - x_q''}{2}\left[\frac{1 - \cos(\theta - \theta_0)}{D}\right] + \frac{x_0[1 - \cos(\theta - \theta_0) - \cos(\theta + \theta_0 + 60) + \cos(2\theta + 60)]}{D}\right\} \qquad (24)$$

The current in the additional field is the same except that the constant term is missing and A_{fd} is replaced by A_{1d}.

Equation 24 may be expanded into a set of harmonic series in a manner similar to that used for equation 9. It then becomes:

$$I = e + ex_dA_{fd}[M(x_d'' + x_q'')(C_{ep} + C_{op}) + M(x_d'' - x_q'') \times (C_{en} + C_{on}) + Mx_0(C_{eo} + C_{oo})] + eM(x_d - x_d') \times (x_q'' + x_0 + bx_0) \qquad (25)$$

Note that the d-c component is multiplied by its special value of A_{fd}.

In equation 25:

$$C_{ep} = \sum_{n=2}^{\infty} b^{\frac{n}{2}} \cos n\,(\theta + 30)$$

$$C_{op} = -\frac{1}{2}\cos(\theta_0 + 30)\,(1 + b)\sum_{n=1}^{\infty} b^{\frac{n-1}{2}} \cos n(\theta + 30) - \frac{1}{2}\sin(\theta_0 + 30)\,(1 - b)\sum_{n=1}^{\infty} b^{\frac{n-1}{2}} \sin n\,(\theta + 30)$$

$$C_{en} = -C_{ep}$$

$$C_{on} = -C_{op}$$

$$C_{e0} = 2\sum_{n=2}^{\infty} b^{\frac{n}{2}} \cos n\,(\theta + 30) + (1 + b^2)\sum_{n=2}^{\infty} b^{\frac{n-2}{2}} \cos n\,(\theta + 30)$$

$$C_{00} = -2(1 + b)\cos(\theta_0 + 30)\sum_{n=1}^{\infty} b^{\frac{n-1}{2}} \cos n\,(\theta + 30)$$

Similarly the current in an additional field in the direct axis is

$$I_{d1} = ex_dA_{1d}[M(x_d'' + x_q'')(C_{ep} + C_{op}) + M(x_d'' - x_q'') \times (C_{en} + C_{on}) + Mx_0(C_{e0} + C_{00})] + eM(x_d' - x_d'') \times (x_q'' + x_0 + bx_0) \qquad (26)$$

Decrement factors must be applied to the various terms in these expressions for field currents in the same way as to the armature currents except that the armature decrement factors apply to the odd harmonics and the field decrement factors to the even harmonics.

Decrement Factors

In order to apply field decrement factors to the odd harmonic terms of the armature current, it is necessary to divide the armature current into portions, each of which is dependent only on the current flowing in a particular field circuit. The decrement factor for each portion is then the reciprocal of the time constant of its field circuit. Since this division will be the same at all times it will be sufficient to determine it at the initial instant.

At the initial instant the total direct current field excitation in the direct axis is:

$$e + eM\,(x_d - x_d'')(x_q'' + x_0 + bx_0)$$

from equations 25 and 26.

The field excitation due to direct currents flowing in the additional field in the direct axis is:

$$eM\,(x_d' - x_d'')(x_q'' + x_0 + b_{x0})$$

The fraction of the total armature current dependent on this additional field current, that is the subtransient part, is then the ratio:

$$K''i_{a0} = \frac{M(x_d' - x_d'')(x_q'' + x_0 + bx_0)}{1 + M(x_d - x_d'')(x_q'' + x_0 + bx_0)}\, i_{a0} \tag{27}$$

where i_{a0} is the odd harmonic terms of i_a.

Similarly the fraction of the total armature current dependent on the transient current in the main field, that is the transient part, is:

$$K'i_{a0} = \frac{M(x_d - x_d')(x_q'' + x_0 + bx_0)}{1 + M(x_d - x_d'')(x_q'' + x_0 + bx_0)}\, i_{a0} \tag{28}$$

This leaves for the sustained part:

$$Ki_{a0} = \frac{1}{1 + M(x_d - x_d'')(x_q + x_0 + bx_0)}\, i_{a0} \tag{29}$$

The same factors of course serve to divide i_{b0}, the odd harmonic terms of i_b.

To calculate field time constants for a double-line-to-neutral short it is necessary to rely upon some physical reasoning rather than a strict mathematical derivation. When a short circuit of any kind occurs in a machine having 2 field circuits in the direct axis, the inductance of each field circuit is altered by the fact that there are 2 other closed circuits linking at least part of the flux linked by the circuit in question. However, the amortisseur circuit in most machines has such low reactance that its d-c component of current dies so rapidly that it is nearly zero for most of the time that a transient current is flowing in the main field. Therefore the amortisseur circuit may be considered open in calculating the time constant of the main field. Under these circumstances the per-unit self-inductance of the direct axis field circuit is X_{ffd}. The current flowing in the direct axis before the fault is equal to $E = e$. Shortly after the fault the average value of main field current is, from equation 25:

$$I = e + eM(x_d - x_d')(x_q'' + x_0 + bx_0)$$

Since the flux linkages remain constant the main field inductance after the fault must be:

$$\frac{1}{1 + M(x_d - x_d')(x_q'' + x_0 + bx_0)}\, X_{ffd}$$

The main field (transient) decrement factor will then be the ratio of the main field resistance to the main field inductance.

$$\alpha_d' = \frac{r_{fd}[1 + M(x_d - x_d')(x_q'' + x_0 + bx_0)]}{X_{ffd}} \tag{30}$$

The self-inductance of the additional field considering the main field as a closed circuit is:

$$X_{11d} - \frac{X_{1fd}^2}{X_{ffd}}$$

The average value of the current in the additional field immediately after the fault is, from equation 26:

$$I_{d1} = eM(x_d' - x_d'')(x_q'' + x_0 + bx_0)$$

Therefore the inductance of the additional field after the fault is:

$$\frac{1}{1 + M(x_d' - x_d'')(x_q'' + x_0 + bx_0)}\left(X_{11d} - \frac{X_{1fd}^2}{X_{ffd}}\right)$$

The additional field (subtransient) decrement factor is:

$$\alpha_d'' = \frac{X_{ffd} r_{1d}[1 + M(x_d' - x_d'')(x_q'' + x_0 + bx_0)]}{X_{11d}X_{ffd} - X_{1fd}^2} \tag{31}$$

The even harmonic terms of the armature current are grouped into 3 separate series. Each series may be considered as a current flowing in a closed armature circuit. The decrement factor to be used with each series is the ratio of the resistance of this circuit to its inductance. From an inspection of equation 11 the inductance met by the first even harmonic component of armature current is

$$\frac{1}{(x_d'' + x_q'')M}$$

The decrement factor for this component is therefore:

$$\alpha_{ap} = r(x_d'' + x_q'')M \tag{32}$$

Similarly for the other 2 components:

$$\alpha_{an} = r(x_q'' - x_d'')M \tag{33}$$

$$\alpha_{a0} = rx_0M \tag{34}$$

The final expression for the armature current in phase a at any instant is:

$$\begin{aligned} i_a = \frac{1}{2}\sqrt{3}\,eM[&(x_d'' + x_q'')\epsilon^{-\alpha_{ap}t}A_{ep} + (x_d'' - x_q'')\epsilon^{-\alpha_{an}t}A_{en} + \\ &x_0\epsilon^{-\alpha_{a0}t}A_{e0} + K''\epsilon^{-\alpha_d''t}(2x_q''A_{0p} + x_0A_{00}) + \\ &K'\epsilon^{-\alpha_d't}(2x_q''A_{0p} + x_0A_{00}) + K(2x_q''A_{0p} + x_0A_{00})] \end{aligned} \tag{35a}$$

The current in phase b is:

$$\begin{aligned} i_b = \frac{1}{2}\sqrt{3}\,eM[&(x_d'' + x_q'')\epsilon^{-\alpha_{ap}t}B_{ep} + (x_d'' - x_q'')\epsilon^{-\alpha_{an}t}B_{en} - \\ &x_0\epsilon^{-\alpha_{a0}t}A_{e0} + K''\epsilon^{-\alpha_d''t}(2x_q''B_{0p} - x_0A_{00}) - \\ &K'\epsilon^{-\alpha_d't}(2x_q''B_{0p} - x_0A_{00}) + K(2x_q''B_{0p} - x_0A_{00})] \end{aligned} \tag{35b}$$

The current in the main field is:

$$\begin{aligned} I = e + eMx_dA_{fd}[&(x_d'' + x_q'')\epsilon^{-\alpha_{ap}t}C_{0p} + (x_d'' - x_q'')\epsilon^{-\alpha_{an}}C_{0n} + \\ &x_0\epsilon^{-\alpha_{a0}t}C_{00} + (x_q'' + x_0 + bx_0)\epsilon^{-\alpha_d't} + K''\epsilon^{-\alpha_d''t}(2x_q''C_{ep} + x_0C_{e0}) + \\ &K'\epsilon^{-\alpha_d't}(2x_q''C_{ep} + x_0C_{e0}) + K(2x_q''C_{ep} + x_0C_{e0})] \end{aligned} \tag{35c}$$

The current in the additional field is the same except that the sustained d-c term is missing, and the transient d-c term has the decrement factor α_d'', and the factor A_{fd} is replaced by A_d.

If x_0 becomes infinitely large equations 35 become identical with the equations developed by Doherty and Nickle for a line-to-line fault, as should be expected.

The Voltage Across the Open Phase

To get an expression for the voltage across the open phase the flux linkages can be found by setting $i_c = 0$ in equation 7*c*. i_a and i_b can be eliminated from the resulting expression by substituting their values from equation 9. The result can then be expanded into a series and decrement factors applied as in the case of the currents and there is:

$$\psi_c = eM\,[3x_q''x_0K''\epsilon^{-\alpha_d''t}F_1 + 3x_q''x_0K'\epsilon^{-\alpha_d't}F_1 + 3x_qx_0KF_1 + \frac{x_d - x_0}{2}(x_d''+x_q'')\epsilon^{-\alpha_{ap}t}F_2 - \frac{(x_d''-x_q'')^2}{4}\epsilon^{-\alpha_{an}t}F_2 - \frac{x_d''+x_q''-2x_0}{4}(x_d''-x_q'')\epsilon^{-\alpha_{an}t}F_4 - x_0(x_d''-x_q'')\cos(\theta_0+30)\epsilon^{-\alpha_{ao}t}F_3] \quad (36)$$

To get the voltage equation 36 may now be differentiated with respect to time. It is much more convenient to expand the functions into Fourier series and then differentiate them term by term. This procedure is valid because the functions F satisfy the rigid conditions under which the term by term derivative of a Fourier series is also the derivative of the function.[4]

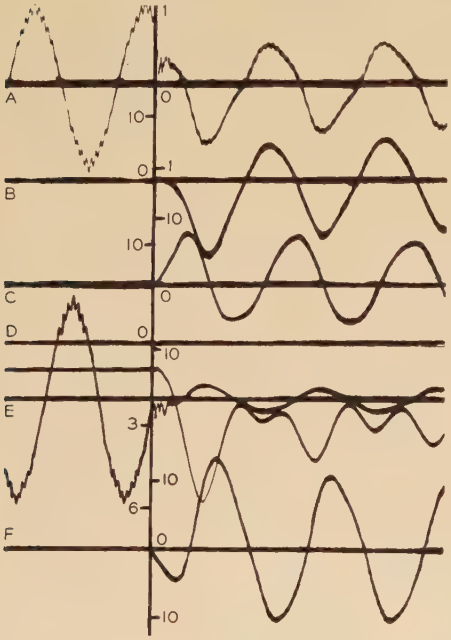

Figure 1a. $\theta=0$

A—Voltage across the open phase

C—Current in line *b*

E—Voltage of phase *a*

B—Current in line *a*

D—Main-field current

F—Neutral current

The final expression for the voltage across the open phase a short time after a double-line-to-neutral fault is:

$$v_c = -eM[3x_q''x_0K''\epsilon^{-\alpha_d''t}(pF_1 - \alpha_d''F_1) + 3x_q''x_0K'\epsilon^{-\alpha_d't}(pF_1 - \alpha_d'F_1) + 3x_q''x_0KpF_1 + \frac{x_d''-x_0}{2}(x_d''+x_q'')\epsilon^{-\alpha_{ap}t}(pF_2 - \alpha_{ap}F_2) - \frac{(x_d''-x_q'')^2}{4}\epsilon^{-\alpha_{an}t}(pF_2 - \alpha_{an}F_2) - \frac{x_d''+x_q''-2x_0}{4}(x_d''-x_q'')\epsilon^{-\alpha_{an}t}(pF_4 - \alpha_{an}F_4) - x_0(x_d''-x_q'')\cos(\theta_0+30)\epsilon^{-\alpha_{a0}t}(pF_3 - \alpha_{a0}F_3)] \quad (37)$$

where

$$F_1 = -(1-b)\sum_{n=1}^{\infty} b^{\frac{n-1}{2}}\sin n(\theta+30)$$

$$pF_1 = -(1-b)\sum_{n=1}^{\infty} b^{\frac{n-1}{2}}\cos n(\theta+30)$$

$$F_2 = -2\cos(\theta_0-60)\left[\sum_{n=2}^{\infty} b^{\frac{n}{2}}\cos n(\theta+30) + \frac{1}{2}\right]$$

$$pF_2 = 2\cos(\theta_0-60)\sum_{n=2}^{\infty} nb^{\frac{n}{2}}\sin n(\theta+30)$$

$$F_4 = b\cos(\theta_0-60) + \sin(\theta_0-60)(1-b^2)\sum_{n=2}^{\infty} b^{\frac{n-2}{2}} \times \sin n(\theta+30) + \cos(\theta_0-60)(1+b^2)\sum_{n=2}^{\infty} b^{\frac{n-2}{2}}\cos n(\theta+30)$$

$$pF_4 = \sin(\theta_0-60)(1-\mathrm{b}^2)\sum_{n=2}^{\infty} nb^{\frac{n-2}{2}}\cos n(\theta+30) - \cos(\theta_0-60)(1+b^2)\sum_{n=2}^{\infty} nb^{\frac{n-2}{2}}\sin n(\theta+30)$$

$$F_3 = (1-b^2)\sum_{n=2}^{\infty} b^{\frac{n-2}{2}}\sin n(\theta+30)$$

$$pF_3 = (1-b^2)\sum_{n=2}^{\infty} nb^{\frac{n-2}{2}}\cos n(\theta+30)$$

Steady-State Currents and Voltages

From equations 35 the sustained line currents are:

$$i_a = \frac{1}{2}\sqrt{3}\,eMK\,(2x_q''A_{0p} + x_0A_{00}) \quad (38)$$

$$i_b = \frac{1}{2}\sqrt{3}eMK\,(2x_q''B_{0p} - x_0A_{00}) \quad (39)$$

The neutral current is their sum:

$$i_N = \sqrt{3}\,eMKx_q''(A_{0p} + B_{0p}) \quad (40)$$

Substituting for A_{0p} and B_{0p} their values:

$$i_N = 3eMKx_q''(1-b)\sum_{n=1}^{\infty} b^{\frac{n-1}{2}}\sin n(\theta+30) \quad (41)$$

From equation 37 the sustained voltage across the open phase is:

$$v_c = -3\,eMKx_q''x_0(1-b)\sum_{n=1}^{\infty} nb^{\frac{n-1}{2}}\cos n(\theta+30) \quad (42)$$

If only the fundamental terms are considered, which can be done with very little approximation if b is small, then the root-mean-square values of the time functions in equations 41 and 42 are equal. Therefore the ratio of the root-mean-square values is

$$\frac{V_c}{I_N} = x_0$$

which we would expect since this is one of the methods of measuring x_o.

Comparisons of Calculated and Test Results

In order to check equations 35 and 37, oscillograms were taken of a number of short circuits on 2 different machines. Figure 1*a* is a typical oscillogram taken on the following machine: General Electric Company, 45 kva, type *ATB*, form P.B., frame 7,621, 118 amperes, 220 volts, 60 cycles, 1,200 rpm, serial number 302,689. The constants of this machine in per-unit on a 220-volt 118-ampere base are:

$x_d = 0.9$
$x_d' = 0.241$
$x_d'' = 0.220$
$T_{do}' = 339$
$x_q = x_q' = x_q'' = 0.470$
$x_0 = 0.07$
$r = 0.033$
$T_{do}'' = 7.6$

The amortisseur winding of the machine had been removed.

Figure 1*b* shows curves calculated from equations 35 and 37. The armature currents check very closely, especially in the first cycle where there is no difference of more than 8 per cent. The calculated curve of the field current shows too large a percentage of harmonics. This can be partly explained by the fact that in the actual calculation A_{fd} was computed from equation 22 because all the constants in equation 18 were not available. All of the calculated curves show too large a percentage of even harmonics in armature quantities and odd harmonics in the field current, especially after the first cycle. This indicates that the values of the armature decrement factors as calculated from equations 32, 33, and 34 are somewhat too small. The short circuits were all made at normal voltage, so that saturation of the iron undoubtedly affected the test curves. The constants used in the calculations were measured under unsaturated conditions.

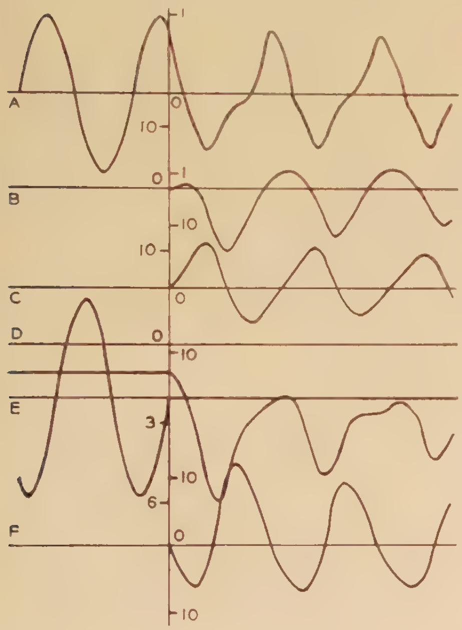

Figure 1b. $\theta = 0$

A—From equation 37

C—From equation 35*b*

E—$e \sin \theta$

B—From equation 35*a*

D—From equation 35*c*

F—From equations 35*a* and 35*b*

In order to find the largest possible value of current that can occur plots of both i_a and i_b were made for a number of different valus of θ_o. These plots showed that the largest peak of i_a occurred very near the point where $\theta_o = 0°$ and $\theta_o = 180°$. For i_b the largest peak occurred near $\theta_o = 90°$ and $\theta = 270°$.

When these values are substituted in equations 9 there is:

$$i_a = \frac{6e(x_q'' + x_0)}{3x_d''x_0 + x_q''x_0 + 2x_d''x_q''} \tag{43}$$

$$i_b = \frac{2\sqrt{3}\,e(2x_q'' + x_0)}{x_d''x_0 + 3x_q''x_0 + 2x_d''x_q''} \tag{44}$$

These values do not give quite the theoretical maximum values of equations 9, but since actually we should consider the decrements as well, these expressions, are close enough and have the advantage of being relatively simple. In table I values of current calculated from equations 43 and 44 for the machine described and another similar machine are compared with the largest peaks for other types of faults calculated from analogous expressions. For line-to-neutral[2] fault:

$$i_a = \frac{6e}{2x_d'' + x_0} \tag{45}$$

For line-to-line fault:

$$i_a = \frac{\sqrt{3}\,e}{x_d''} \tag{46}$$

For 3-phase fault:

$$i_a = \frac{2e}{x_d''} \tag{47}$$

Table I

Type of Fault	Current	Machine I	Machine II
$l-n$	i_a	11.8	14.2
$l-l$	i_a	7.9	10.0
$l-l-l$	i_a	9.1	11.5
$l-l-n$	i_a	11.4	14.0
$l-l-n$	i_b	10.7	13.0

The table shows that for 2 typical machines the theoretical peak value of the line current in a double line-to-neutral fault is of about the same order of magnitude as that in a line-to-neutral fault.

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Flywheels for A-C Generating Units

By F. K. BRAINARD
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SO MUCH has been written on the subject of flywheels and parallel operation of generating units, that the object of this paper is not to attempt to develop any entirely new method of flywheel calculation, but rather to collect and apply the available information and put it into convenient shape for practical use. The consideration of torsional vibrations in connection with the determination of flywheel sizes is frequently of the greatest importance, especially in the case of large multicylindered oil engines, and should not be overlooked. However it is beyond the scope of this paper and so will not be considered here.

In addition to the above, there are 3 quite distinct conditions which the flywheel of a proposed unit must satisfy.

The first is the very obvious condition that it must be large enough to prevent objectionable variations in frequency (and voltage) when the unit is the only generating unit on the system. Failure to provide for this will result in objectionable flickering of lighting handled by the generator. These variations may be (*a*) the result of violent changes in load combined with a somewhat sluggish prime mover or governor, or (*b*) the result of pulsating torque of the prime mover.

A convenient criterion for the (*a*) condition is the ratio of the kinetic energy of the rotating parts to the rated output of the unit. This will be the time required by the unit when operating at full load to develop energy equal to the kinetic energy of the rotating parts at full speed. There is nothing new about this as the equivalent of this has been used for many years. If this time is in seconds, then:

$$WR^2 = \frac{433\ kw\ T_1}{\left(\frac{\text{rpm}}{100}\right)^2} \tag{1}$$

For hydroelectric units this effect is important, T_1 in this case varying between about 1.4 and 2.0 seconds, for a normal machine, depending upon the head, length of penstock, speed of operation of governor as well as gates, etc. For other applications this effect is, as far as the writer is aware, never the limiting feature, although it would seem that it might be in the case of certain multicylinder steam engines.

The (*b*) condition is important in the case of reciprocating engine units, on account of the varying turning effort developed by the engine for different crank angles. A convenient criterion for this condition is the well-known "irregularity factor." The use of this is also very old. It is the variation in speed expressed as a fraction of normal speed, during a power cycle, the load being assumed constant, e.g., an irregularity factor of $^1/_{200}$ means a variation in speed of 0.5 per cent during one revolution of a 2-cycle engine or 2 revolutions of a 4-cycle engine. It can be easily shown that the following is a very close approximation for flywheel effect in terms of irregularity factor:

$$WR^2 = \frac{130\ kw\ Z}{\left(\frac{\text{rpm}}{100}\right)^3} \times \frac{A_2}{A_1} \tag{2}$$

Permissible values of irregularity factor depend upon the character of the load served and the frequency of the impulses causing the variations in speed, i.e., the number of power impulses per minute. For a miscellaneous factory load in which good lighting is not important an irregularity factor of $^1/_{100}$ will usually be satisfactory. However since office or residence lighting is almost always an important factor, and since low-wattage metallic filament lamps are used so generally, it is practically always necessary to make the irregularity factor low enough to avoid serious flickering of such lamps. The voltage variation necessary to produce an objectionable flicker varies with the frequency of the variation, being a maximum at about 500 cycles per minute. The available data are somewhat conflicting but it would seem that the following are reasonable values of irregularity factor:

Frequency of impulses in cycles per minute	$\frac{1}{Z}$
100	1/125
300	1/175
500	1/225
700	1/175
1,000	1/125

The value of A_2/A_1 depends upon the shape of the tangential effort diagram, being the ratio of certain areas (see figure 1), and so varies widely for different engines. The following table gives approximate values for single acting oil engines:

Number of Cylinders	$\frac{A_2}{A_1}$ 2 Cycle	4 Cycle
1	1.3	
2	0.40	1.4
3	0.19	0.45
4	0.11	0.20
5	0.075	0.14
6	0.055	0.10

The second condition is that the flywheel must be suitable for satisfactory parallel operation of the unit with a very large system. This limitation can best be expressed in terms of the quantity C, which is the same as the "com-

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pressor factor" as standardized by the National Electrical Manufacturer's Association. The expression for WR^2 in terms of C is:

$$WR^2 = \frac{1.34\, P_r f\, C}{\left(\frac{\text{rpm}}{100}\right)^4} \tag{3}$$

The limiting values of C can be determined from the tangential effort diagram of the engine as follows: Calculate the CY (Y = power pulsation of generator) curve by first assuming a reasonable unbalance between cylinders. Since the operation of internal combustion engines is somewhat irregular the amount of unbalance is largely a matter of judgment and hence the CY curves cannot be calculated with the same accuracy as, for instance, compressor curves are calculated. For this reason and also because the flywheels are relatively large (which results in the effect of damping being relatively small) the damping can usually be neglected entirely. This results in a considerable simplification of the equations as follows:[2] For a 2-cycle engine

$$P = 100 + \sum y_n \frac{9.25}{9.25 - n^2 C} \sin(n\omega t + \phi_n)$$

and for a 4-cycle engine

$$P = 100 + \sum y_n \frac{37}{37 - n^2 C} \sin(n\omega t + \phi_n)$$

Then after having analyzed the tangential effort curve and determined the coefficients in the Fourier series representing the same, the procedure is to select a value of C and calculate the curve of P from $t = 0$ to $t = 2\pi$, i.e., over one power cycle which will be one revolution for a 2-cycle engine or 2 revolutions for a 4-cycle engine. Then Y will be the maximum difference in the ordinates of this curve and the result one point on the CY curve. Repeat for different values of C until sufficient points are obtained to set limits of C corresponding to the power pulsation desired.

The following are approximate values of C based upon 66 per cent power pulsation and 10 per cent unbalance, for oil engines:

	C	
Number of Cylinders	2-Cycle Single-Acting	4-Cycle Single-Acting
1	100 or over	500 or over
2	20 or over	150 or over
3	16 or over	75 or over
4	3.5 to 4.5 or 14 or over	60 or over
5	3.3 to 5.0 or 13.5 or over	57 or over
6	3.0 to 6.0 or 12.5 or over	55 or over

The above values must not however be considered as final since as mentioned above they cannot be calculated with as much precision as would be desirable. Also the permissible values depend upon the maintenance of the engine. Hence the limits will have to be determined ultimately by experience, and so the above table must be considered as empirical and subject to some variation.

The third condition is that the flywheel must be suitable for parallel operation of the unit with certain particular units whose characteristics are known. A simple method of checking this after the other conditions are satisfied is to compute the natural frequency F of each of the units with an infinite system using the AIEE formula:

$$F = \frac{35{,}200}{\text{rpm}} \sqrt{\frac{P_r f}{WR^2}}$$

Then if these natural frequencies are all well below the lowest forced frequency, say 20 per cent below, the operation will be satisfactory. This is because the combined natural frequencies of any combination of units is always between the natural frequencies of the individual units on an infinite system. However if the lowest forced frequency of one of the units is below the natural frequency of another unit, the combined natural frequencies of all combinations should be calculated and care taken to see that none of the forced and natural frequencies are dangerously close to each other.

The expression for combined natural frequency[1] of 2 units is

$$F_{1-2} = \frac{35{,}200}{\text{rpm}_1} \sqrt{\frac{P_{r1} f}{WR_1^2} \times \frac{1 + \left(\frac{\text{rpm}_1}{\text{rpm}_2}\right)^2 \times \frac{WR_1^2}{WR_2^2}}{1 + \frac{P_{r1}}{P_{r2}}}}$$

and in the case of 3 units, the expression is

$$F_{1-2 \text{ and } 3} = 9.55 \sqrt{\frac{N_2}{2N_1} \pm \sqrt{\left(\frac{N_2}{2N_1}\right)^2 - \frac{N_3}{N_1}}}$$

where

$$c_1 = 4.22 \times 10^5 \frac{P_{r1} f}{(\text{rpm}_1)^2}$$

$$d_1 = 5.74 \times 10^{12} f^2 \left[\frac{P_{r1} P_{r2}}{(\text{rpm}_1)^2 (\text{rpm}_2)^2 WR_2^2} + \frac{P_{r1} P_{r3}}{(\text{rpm}_1)^2 (\text{rpm}_3)^2\; WR_3^2}\right]$$

$$e_1 = 7.85 \times 10^{19} f^3 \left[\frac{P_{r1} P_{r2} P_{r3}}{(\text{rpm}_1)^2 (\text{rpm}_2)^2 (\text{rpm}_3)^2\; WR_2^2\; WR_3^2}\right]$$

$$N_1 = c_1 + c_2 \left(\frac{\text{rpm}_2}{\text{rpm}_1}\right)^2 + c_3 \left(\frac{\text{rpm}_3}{\text{rpm}_1}\right)^2$$

$$N_2 = d_1 + d_2 \left(\frac{\text{rpm}_2}{\text{rpm}_1}\right)^2 + d_3 \left(\frac{\text{rpm}_3}{\text{rpm}_1}\right)^2$$

$$N_3 = e_1 + e_2 \left(\frac{\text{rpm}_2}{\text{rpm}_1}\right)^2 + e_3 \left(\frac{\text{rpm}_3}{\text{rpm}_1}\right)^2$$

Subscripts 1, 2, and 3 refer to units 1, 2, and 3 respectively.

For c_2, d_2, and e_2 interchange subscripts 1 and 2 in expressions for c_1, d_1, and e_1, respectively.

For c_3, d_3, and e_3 interchange subscripts 1 and 3 in expressions for c_1, d_1, and e_1, respectively.

It is sometimes assumed that parallel operation is always a more severe condition than operating alone and that if the flywheel is satisfactory from the former standpoint it will also be satisfactory from the latter. A few trials using equations 2 and 3 will show the fallacy of this,

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[2] For all numbered references, see list at end of paper.

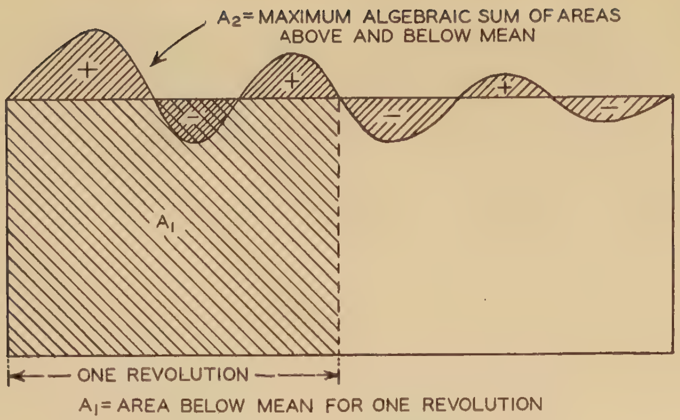
A2= MAXIMUM ALGEBRAIC SUM OF AREAS ABOVE AND BELOW MEAN
+
+
+
−
−
−
A1
ONE REVOLUTION
A1= AREA BELOW MEAN FOR ONE REVOLUTION

Figure 1. Tangential effort diagram

since sometimes it is one and sometimes the other which requires the greater flywheel effect. It is interesting to note that according to equation 1 the flywheel effect varies inversely as the square of the speed, according to equation 2 inversely as the cube of the speed, and according to equation 3 inversely as the fourth power of the speed. Hence the limiting feature depends to a considerable extent upon the speed of the unit. Also from equation 3 it is seen that the flywheel effect necessary on account of parallel operation varies directly with the frequency of the system. Hence parallel operation is relatively less important, and operation alone (which means irregularity factor) more important on 25 cycles than on 60 cycles.

Appendix I. Symbols

WR^2 = flywheel effect of unit in (pound-feet)^2 (i.e., pounds weight multiplied by the radius of gyration in feet squared)
kw = rated output of the unit in kilowatts
T_1 = time in seconds required by the unit when operating at full load to develop energy equal to the kinetic energy of the rotor at full speed. This is equal to the commonly accepted inertia constant H divided by rated power factor
rpm = rated speed in revolutions per minute
$\frac{1}{Z}$ = irregularity factor = ratio of maximum variation in speed during a power cycle to normal speed
A_1 = area in tangential effort diagram below mean for *one revolution*
A_2 = maximum algebraic sum of small areas in tangential effort diagram above and below mean (see figure 1)
P_r = synchronizing power in kilowatts per electrical radian as defined in AIEE rules
f = frequency of system in cycles per second
C = engine factor = $9.25\left(\frac{\text{rpm}}{F}\right)^2$
F = frequency of oscillation of unit on an infinite system in cycles per *minute*
y = $100 + \Sigma y_n \text{ sine } (n\omega t + \phi_n)$ = Fourier analysis of tangential effort curve, in which
n = order of harmonic
y_n = amplitude of nth harmonic expressed as a percentage of rated output of unit
ω_n = $0.1045\ n$ rpm for 2-cycle engines, or $0.05225\ n$ rpm for 4-cycle engines
t = time in seconds
ϕ_n = phase angle of nth harmonic
P = power output of generator expressed as a percentage of the rating
Y = power pulsation of generator expressed as a percentage of the rating in kilowatts

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General Electric Photo

This athletic field at LaSalle Institute, Troy, N. Y., shown here in use for an evening entertainment, is floodlighted by 40 1,500-watt lamps which provide an average illumination of 12 foot-candles

Coupling Between Parallel Earth-Return Circuits Under D-C Transient Conditions

By K. E. GOULD
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Introduction

IN TESTS conducted in connection with several d-c railway electrifications, the induced voltages recorded in paralleling communication circuits at times of short circuit on the railway have shown marked divergences from values computed on the basis of uniform earth resistivity and a rate of change of earth current determined from measurements in trolley and rail circuits. Due to the numerous factors which might contribute to these divergences, such as nonuniform division of transient current along the tracks and associated return conductors, the presence of shielding conductors along or near the right-of-way, etc., it was felt that a better understanding of the problem of induction under d-c transient conditions could be obtained by experimental studies of the transient coupling between parallel earth-return circuits, free from the effects of shielding conductors, and with concentrated, rather than distributed, grounds. The study described in this paper was undertaken for this purpose.

The locations for the tests were selected to provide a reasonably large range of earth resistivity; also, at one location it was known that the earth structure departed substantially from uniformity. At each of these locations d-c transient coupling tests were performed in which transient currents, approximately of the form encountered during faults on d-c railway electrifications, were produced in an earth-return circuit, herein referred to as the primary, and measurements were made of the resultant voltages in earth-return circuits, herein called secondary circuits, parallel to and at separations from the primary circuit of from 50 or 60 to 2,000 feet. In addition to the d-c transient tests, measurements were made at each location of the steady state a-c coupling, in magnitude and phase angle, over a range of frequencies from 20 or 30 cycles to 3,200 cycles. From these a-c measurements the transient voltages were computed for a number of cases by evaluating the Fourier integral. The results of the a-c coupling tests were useful also in helping to explain, in a general way, the departures of the measured transient voltages from the voltages computed for uniform earth resistivity.

The measured transient voltages and voltages computed (1) from the a-c coupling measurements and (2) on the basis of a uniform earth resistivity, are shown for several representative cases in figures accompanying the paper.

Test Conditions

Three sets of tests were performed, near Savannah, Ga., Macon, Ga., and Jennings, La., respectively. The primary circuit was nearly straight in each case and was installed along a telephone line. The length of the primary circuit was 5.65 miles, 9.3 miles, and 4.7 miles at the respective locations, these lengths being great enough that end effects were not large even under the d-c transient conditions. The secondary circuits, 4,200 feet in length, were laid out parallel to and near the middle of the primary circuit, and at radial separations therefrom of 50 feet (Macon) or 60 feet (Savannah and Jennings), 200 feet, 500 feet, 1,000 feet, and 2,000 feet.

By means of a 500-volt gasoline-engine-driven d-c generator, and with the use of an air-core choke with taps giving inductances of roughly 1, $^3/_4$, $^1/_2$, or $^1/_4$ henry, it was possible to produce primary currents approximately of the form $I(1-e^{-at})$, with a from about 40 to 110 for t in seconds, and with I somewhat less than 10 amperes. Conductors were brought in from each secondary so that the voltages produced in the secondaries under transient conditions could be recorded by a movie-type oscillograph, on 70-millimeter film moving at a speed of approximately 7 feet per second. Direct-coupled amplifiers were used to obtain the requisite sensitivity for the oscillograph, the over-all response of the measuring equipment being nearly constant for frequencies from zero to 4,000 cycles. The rate of change of primary current was recorded, simultaneously with the secondary voltages, by means of a search coil external to and coaxial with the primary choke.

For the a-c coupling tests a power amplifier driven by a variable frequency oscillator was used to energize the primary circuit and the voltages produced in the secondary circuits were measured with an a-c potentiometer, so that both components of the coupling could be determined. The continuity of the telephone conductors on the pole line carrying the primary circuit was interrupted during both the d-c transient tests and the measurements of a-c coupling to eliminate any possible shielding effects from these conductors.

Typical Results

In figures 1 to 6, inclusive, are shown curves selected from the large amount of data collected as typical of the results of this study. The cases chosen for these illustrations are those in which the time constant of the pri-

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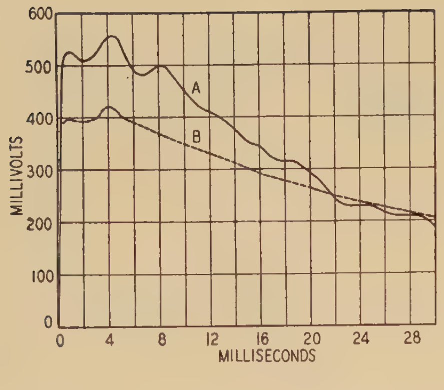

Figure 1. Savannah tests—60-foot separation; rate of change of primary current approximately $440e^{-46t}$, amperes per second

Curve *A*—Measured voltage
Curve *B*—Computed voltage for uniform earth resistivity of 100 meter-ohms

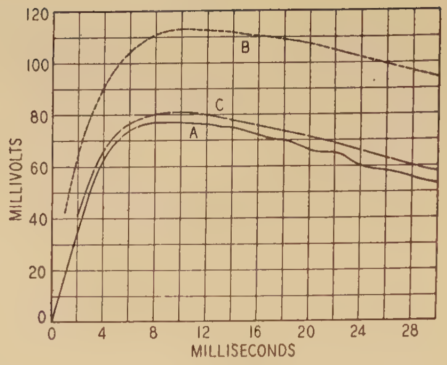

Figure 2. Savannah tests—2,000-foot separation; rate of change of primary current approximately $440e^{-46t}$, amperes per second

Curve *A*—Measured voltage
Curve *B*—Computed voltage for uniform earth resistivity of 100 meter-ohms
Curve *C*—Computed voltage from a-c coupling measurements

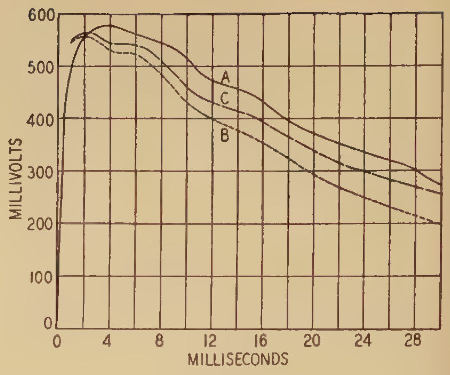

Figure 3. Macon tests—50-foot separation; rate of change of primary current approximately $490e^{-60t}$, amperes per second

Curve *A*—Measured voltage
Curve *B*—Computed voltage for uniform earth resistivity of 500 meter-ohms
Curve *C*—Computed voltage as for curve B, but with ground potentials corresponding to resistivity of 1,500 meter-ohms

mary circuit was most nearly representative for d-c railway electrifications, being about $^1/_{46}$, $^1/_{60}$, and $^1/_{45}$ second for the respective locations. The closest and widest separations were chosen for examples, and although the examples of the transient voltages computed from the a-c coupling measurements are thus limited to the cases of the 2,000-foot separation alone, the value of the illustrations is not significantly impaired thereby. As explained in a later section of the paper, it was not possible to compute the transient voltages for the closest separation from the a-c coupling measurements. Figures 1 to 6 illustrate the fact that with transient earth currents of the type considered here, the induced voltages not only decrease with increased separation, as in the steady-state a-c case, but also attain their maximum values later.

Figure 1 shows the measured transient voltage for the 60-foot separation in the Savannah tests and the computed voltage for a uniform earth resistivity of 100 meter-ohms, using the measured rate of change of primary current. This value of earth resistivity, chosen on the basis of the measured a-c coupling, was found to be about the best one as regards over-all agreement between the measured transient voltages and voltages computed on the basis of uniform earth resistivity.

Figure 2, for the 2,000-foot separation in the Savannah

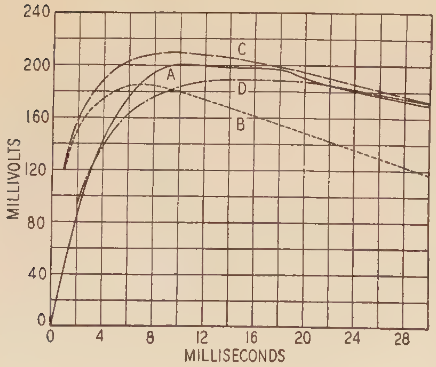

Figure 4. Macon tests—2,000-foot separation; rate of change of primary current approximately $490e^{-60t}$, amperes per second

Curve *A*—Measured voltage
Curve *B*—Computed voltage for uniform earth resistivity of 500 meter-ohms
Curve *C*—Computed voltage as for curve B, but with ground potentials corresponding to resistivity of 1,500 meter-ohms
Curve D—Computed voltage from a-c coupling measurements

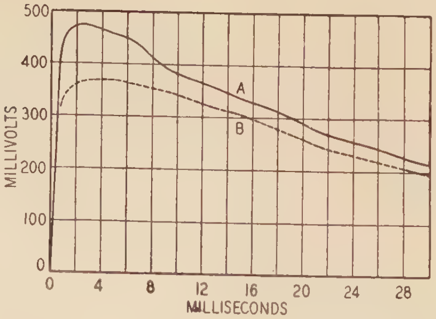

Figure 5. Jennings tests—60-foot separation; rate of change of primary current approximately $535e^{-45t}$, amperes per second

Curve *A*—Measured voltage
Curve *B*—Computed voltage for uniform earth resistivity of 10 meter-ohms

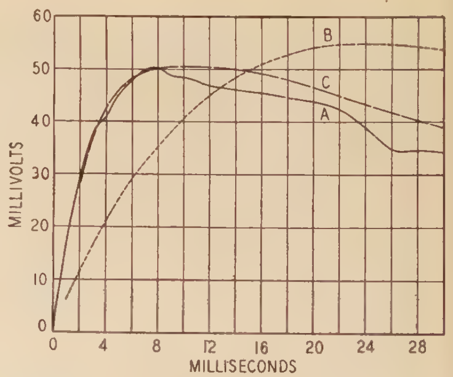

Figure 6. Jennings tests—2,000-foot separation; rate of change of primary current approximately $535e^{-45t}$, amperes per second

Curve *A*—Measured voltage
Curve *B*—Computed voltage for uniform earth resistivity of 10 meter-ohms
Curve *C*—Computed voltage from a-c coupling measurements

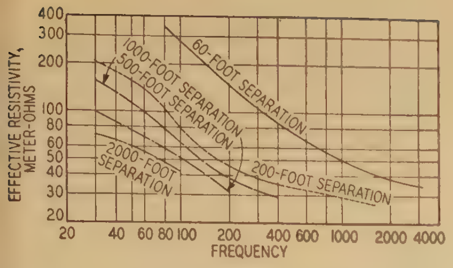

Figure 7. Savannah tests; effective earth resistivity corresponding to measured quadrature coupling

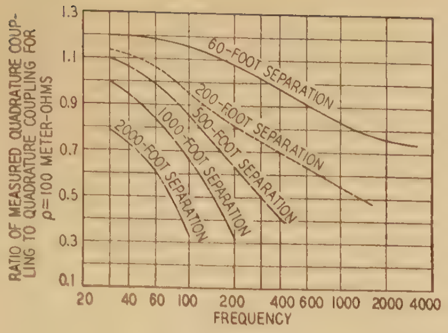

Figure 8. Savannah tests; ratio of measured quadrature coupling to quadrature coupling for uniform earth resistivity of 100 meter-ohms

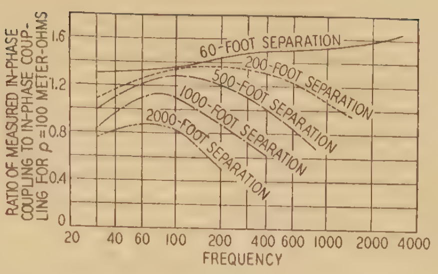

Figure 9. Savannah tests; ratio of measured in-phase coupling to in-phase coupling for uniform earth resistivity of 100 meter-ohms

tests, is similar to figure 1, except that the computed voltage from the a-c coupling measurements, using the measured rate of change of primary current, has been plotted also.

Figure 3, for the 50-foot separation in the Macon tests, is similar to figure 1, except that 2 computed curves are shown, one for a uniform earth resistivity of 500 meter-ohms, about the best single value which could be chosen, and the other for the same resistivity but with the ground potentials taken as corresponding to a resistivity of 1,500 meter-ohms. This was found to result in a fairly good approximation for the substantial increase observed in the effective earth resistivity at the low frequencies.

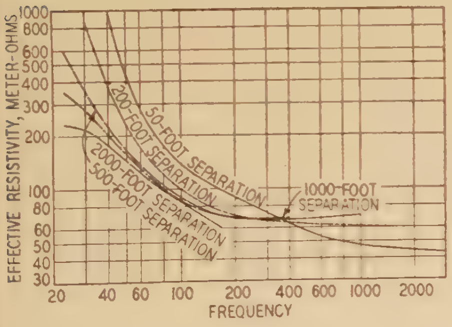

Figure 10. Macon tests; effective earth resistivity corresponding to measured quadrature coupling

Figure 4, for the 2,000-foot separation in the Macon tests, is similar to figure 2, except that computed curves are shown for the 2 conditions represented in figure 3. Figures 3 and 4 illustrate the fact that ground potentials were of greater relative importance at the wider separations.

Figure 5, for the 60-foot separation in the Jennings tests, is similar to figure 1, except that an earth resistivity of 10 meter-ohms was used for the computed voltage. Figure 6, for the 2,000-foot separation, is similar to figure 2, except for the change noted in the uniform resistivity used.

Methods Used in Computing Transient Voltages

Two methods were used in computing the transient voltages for uniform earth resistivity, one a rigorous method, the other an approximate and simpler method giving sufficiently accurate results for the cases involved here. In the simpler method, which was used to obtain the curves in figures 1 to 6 for uniform earth resistivity, the transient voltage was computed for a primary circuit infinite in length, and to this voltage was added a term equal to the product of the primary current and the d-c mutual resistance, for uniform earth resistivity, of the actual primary and secondary circuits. For the case of an infinite length of primary circuit, the simple formal expression for $h(t)$, the voltage produced in the secondary circuit by unit step current (0 for $t < 0$, one ampere for $t > 0$) in the primary circuit, is given by[1]

$$h(t) = \frac{10^4\rho}{9.6x^2}\left[1 - \exp\left(\frac{-2.92\cdot 10^{-8}x^2}{\rho t}\right)\right] \text{ volts per kilofoot}$$

where

ρ = earth resistivity, meter-ohms
x = separation, feet
t = time, seconds

and thus a great part of the labor involved in exact computations for circuits of finite length is avoided.

A rigorous solution for $h(t)$, for circuits of finite length and with uniform earth resistivity, is available,[2] $h(t)$ so computed being but little different, in the present cases, from $h(t)$ for a primary circuit of infinite length, and only for large values of (t). However, for the rigorous method, it appeared easier to compute $h(t)$ for the finite circuits involved by plotting the in-phase component, $a(\omega)$, of the coupling from curves based on the mutual impedance formulas,[1] in the form $a(\omega)/\omega$ and evaluating the Fourier integral[4] numerically, since previous computations made this process a simple one.

The voltage $V(t)$ produced in a secondary circuit by a primary current $I(t)$, the rate of change of which is $I'(t)$, is given by one form of the extension theorem[5] as

$$V(t) = \int_0^t h(t-\lambda)I'(\lambda)d\lambda$$

From this formula, using the measured rate of change of primary current, the transient voltage was computed for uniform earth resistivity.

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[1] For all numbered references, see list at end of paper.

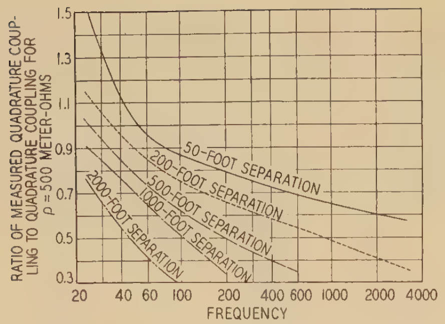


Figure 11. Macon tests; ratio of measured quadrature coupling to quadrature coupling for uniform earth resistivity of 500 meter-ohms

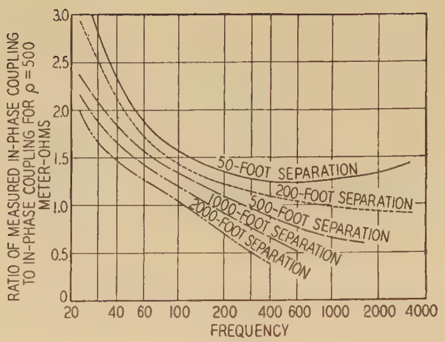


Figure 12. Macon tests; ratio of measured in-phase coupling to in-phase coupling for uniform earth resistivity of 500 meter-ohms

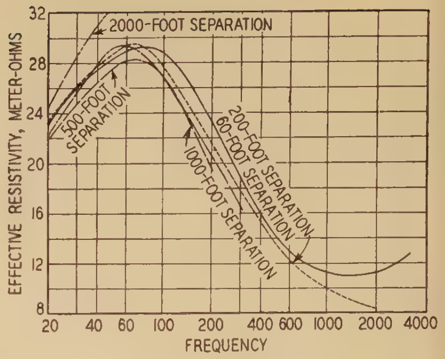


Figure 13. Jennings tests; effective earth resistivity corresponding to measured quadrature coupling

In computing the transient voltages from the in-phase and quadrature components, $a(\omega)$ and $b(\omega)$, of the measured a-c coupling, $a(\omega)/\omega$ and $b(\omega)/\omega$ were plotted from the experimental data and the Fourier integral[4]

$$h(t) = \frac{2}{\pi}\int_0^\infty \frac{a(\omega)}{\omega}\sin\omega t\cdot d\omega$$

$$= a(0) + \frac{2}{\pi}\int_0^\infty \frac{b(\omega)}{\omega}\cos\omega t\cdot d\omega$$

was evaluated numerically. The measured coupling was extrapolated to $\omega = 0$ on the assumption of the uniform resistivity used in computing the transient voltages. For the closest separation these computations could not be made with any significant degree of accuracy, largely because $b(\omega)/\omega$ did not converge with sufficient rapidity toward zero with increasing ω, but for the other separations $h(t)$ could be computed with satisfactory accuracy from $b(\omega)$ for small values of t and from $a(\omega)$ for larger values of t, up to about 10 milliseconds. The results from $a(\omega)$ and from $b(\omega)$ agreed within perhaps 10 per cent or less over a considerable range of t, thus providing a fairly conclusive check of the computations. For values of t from 10 milliseconds to 30 milliseconds, the upper limit it seemed advisable to adopt, it was assumed that $[h(t) - a(0)]$ varied inversely with t. When end effects are negligible and the earth homogeneous, this relation is true for large values of t and apparently represented a sufficiently accurate approximation here. The extension theorem was applied, as for the case of uniform resistivity, to obtain the transient voltages corresponding to the measured rate of change of primary current.

Correlation of Results With Measured A-C Coupling

Figure 7 shows, for each separation in the Savannah tests, and as a function of the frequency, the effective earth resistivity corresponding to the quadrature component of the measured a-c coupling, that is, the value of uniform earth resistivity for which the quadrature component of the coupling, as determined from formulas for mutual impedance of grounded circuits of finite lengths,[3] would be equal to the measured quadrature coupling. The quadrature component was used in this connection because the effective earth resistivity could be determined more satisfactorily from it than from the in-phase component, although even from the quadrature component the effective resistivity could be determined satisfactorily only for the ranges of separation and frequency for which the resistivity is plotted in figure 7.

Figures 8 and 9 have been prepared to show the coupling data from the Savannah tests in a convenient form for correlating the behavior of the transient voltages with the behavior of the a-c coupling. Figure 8 shows the ratio of the measured quadrature coupling to the quadrature coupling corresponding to a uniform earth resistivity of 100 meter-ohms. Figure 9 shows the corresponding ratio for the in-phase coupling. The fact that the computed transient voltage for a resistivity of 100 meter-ohms was smaller than the measured voltage in the case of the 60-foot separation seems reasonable in view of the curves of figures 8 and 9 for this separation, since the ratios in this case are generally greater than unity, more than offsetting the smaller quadrature ratio at the higher frequencies. That the computed voltage for a resistivity of 100 meter-ohms was larger than the measured voltage in the case of the 2,000-foot separation is in accord with the fact that the ratios in figures 8 and 9 for this separation are considerably less than unity for the entire frequency range.

Figures 10, 11, and 12, for the Macon tests, are similar to figures 7, 8, and 9, respectively. Here the principal condition, illustrated in figures 3 and 4, to be correlated with the measured a-c coupling is that, for both separations, the computed voltages for a resistivity of 500 meter-ohms were greater than the measured voltages at small values of time and smaller at large values of time. The computations involving the Fourier integral showed clearly that at small values of time the transient voltages are closely related to the quadrature coupling at the higher frequencies, especially if the earth resistivity is not too low, whereas at large values of time the in-phase coupling at low frequencies is of most significance in predicting the transient voltages. Thus the above mentioned condition regarding computed and measured voltages is seen to be consistent with the curves of figures 11 and 12. The high ratios shown in figure 12 for the low frequencies tend to substantiate the pragmatic approximation of taking the ground potentials to correspond to an earth resistivity of 1,500 meter-ohms.

Figures 13, 14, and 15, from the Jennings tests, are similar to figures 7, 8, and 9, respectively. On the basis of figure 13, it would appear that the best single earth resistivity to use in computing the transient voltages would be some 25 meter-ohms. However, as the computations involving the Fourier integral showed, the in-phase coupling at the low frequencies is of principal significance with regard to the maximum values of the transient voltages at wide separations, especially where the earth resistivity is as low as that indicated here, and an earth resistivity of 10 meter-ohms is seen, from figure 15, to be more nearly consistent with this criterion. A resistivity of 10 meter-ohms was found to result in about as good an over-all agreement as could be obtained between the measured voltages and voltages computed on the basis of uniform earth resistivity.

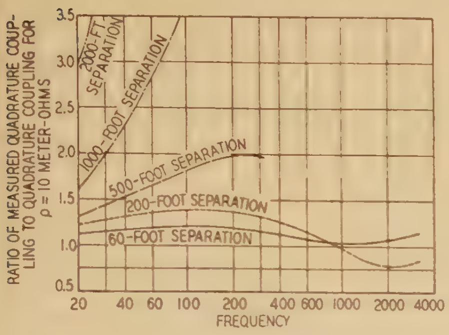


Figure 14. Jennings tests; ratio of measured quadrature coupling to quadrature coupling for uniform earth resistivity of 10 meter-ohms

As illustrated in figures 5 and 6, the use of an earth resistivity of 10 meter-ohms results in a maximum value for the computed voltage smaller than the maximum value of the measured voltage at the closest separation and greater than the maximum value of the measured voltage at the widest separation. Moreover, the computed voltages increased more slowly than the measured voltages, subsequent to the beginning of the transient, attained their maximum values later, and decreased more slowly at large values of time. These conditions are consistent with the curves of figures 14 and 15, since (1) for the close separations the transient voltages at small values of time can be predicted only from the measured quadrature coupling at the higher frequencies, whereas at large values of time the in-phase coupling at the lower frequencies is of most significance and (2) for the wide separations the transient voltages are related closely to the behavior of the measured in-phase coupling at low frequencies.

The computed voltages for the 2,000-foot separation, based on the measured a-c coupling, will be seen from figures 2, 4, and 6 to agree quite satisfactorily with the measured voltages and, although in some cases the agreement was not quite as good as that shown by these examples, it was found that in cases where the nonhomogeneity of the earth caused the transient voltages to depart considerably from the voltages which would have occurred with uniform earth resistivity, the computed voltages from the a-c coupling measurements took the effects of such nonhomogeneity into account fairly accurately.

Summary

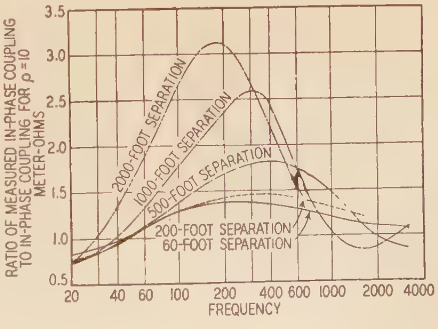
Widely different types of earth structure were indicated by the a-c coupling measurements at the 3 locations where these tests were performed, and by the behavior of the transient induced voltages. The departures of the measured transient voltages from the values which would have occurred with a uniform earth resistivity could generally be correlated satisfactorily with the measured a-c coupling.

At Savannah, Ga., a value of 100 meter-ohms was found to give about the best over-all agreement between the measured transient voltages and the voltages computed on the basis of uniform earth resistivity, this value being a reasonable one to choose from the standpoint of the measured a-c coupling. The voltages computed for this uniform resistivity were considerably lower than the measured voltages at the closest separation and, with increasing separation, the computed voltages gradually increased in comparison with the measured voltages, until at the widest separation they were substantially higher than the latter.

At Macon, Ga., the earth structure was apparently highly irregular, but with a reasonable value of uniform earth resistivity, 500 meter-ohms, as indicated by the a-c coupling measurements, the maximum values of the computed transient voltages agreed roughly with the maximum values of the measured voltages. However, these computed voltages increased more rapidly than did the measured voltages, subsequent to the beginning of the transient, and decreased more rapidly after the maximum values were attained. It was found that the approximation of taking the ground potentials to correspond to a resistivity of 1,500 meter-ohms improved the agreement between these computed voltages and the measured voltages. This constituted a crude way of taking into account the large in-phase coupling measured at the low frequencies.

At Jennings, La., the departures of the earth structure from homogeneity affected seriously the transient induced voltages, and careful examination of both components of the measured coupling was required to arrive at the

Figure 15. Jennings tests; ratio of measured in-phase coupling to in-phase coupling for uniform earth resistivity of 10 meter-ohms



value of earth resistivity, 10 meter-ohms, to give the best over-all agreement between the measured transient voltages and the voltages computed on the basis of uniform earth resistivity. Although the measured voltages and computed voltages for a resistivity of 10 meter-ohms were fairly well in accord as regarded maximum values, the wave shapes were considerably different. The computed voltages increased more slowly, subsequent to the beginning of the transient, than did the measured voltages, and decreased more slowly after the maximum values were attained.

The transient voltages computed from the measured a-c coupling were in satisfactory agreement with the measured voltages, indicating that in this method of computing transient voltages the effects of nonhomogeneities of the earth were taken into account where such effects were of importance.

References

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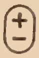
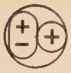
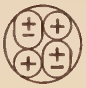
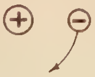
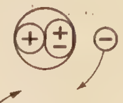
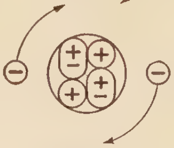
2. Transients in Grounded Wires Lying on the Earth's Surface, John Riordan. *Bell System Technical Journal*, volume 10, July 1931, pages 420–31.

3. Mutual Impedance of Grounded Wires Lying on the Surface of the Earth, R. M. Foster. *Bell System Technical Journal*, volume 10, July 1931, pages 408–19.

4. Electric Circuit Theory and the Operational Calculus (a book), J. R. Carson. Chapter 11.

5. Reference 4, Chapter 2.

Nuclear Physics Chart[1]

Name	Picture Representation	Probable Composition	Charge	Approximate Mass	Remarks
Neutrino *	(±)	Unknown	Zero	1/2000 Unit	Undiscovered but theoretically necessary for conservation of mass energy in many nuclear transformations.
Electron	⊖	Elementary	1 Unit Neg.	1/2000 Unit	Carrier of electricity in wire conduction and vacuum tubes.
Positron **	⊕	Elementary	1 Unit Pos.	1/2000 Unit	Discovered recently. Very unstable. Probably unites with electron and both disappear. ** Particle ejected during breakdown of a large number of artificially prepared radio-active substances.
Proton	⊕	Elementary	1 Unit Pos.	1 Unit	Ordinary hydrogen nucleus or ionized hydrogen atom.
Neutron		Proton Electron	Zero	1 Unit	Discovered recently. Electron and proton are closely bound so there is no external electric charge. This enables it to penetrate matter easily until it unties with a nucleus forming an isotope usually radio-active.
Deuteron Heavy Hydrogen Nucleus Diplon		Neutron Proton	1 Unit Pos.	2 Units	Discovered recently.
Alpha Particle Helium Neucleus		2 Neutrons 2 Protons	2 units	4 Units	Given off in ordinary radioactivity.
Protium Hydrogen Atom		Proton Electron	Zero	1 Unit	Electron is separated from proton and loosely bound to it. May be ionized in discharges to form proton and free electron.
Deuterium *** Heavy Hydrogen Atom Diplogen Hydrogen Isotope		Proton Neutron Electron	Zero	2 Units	Discovered recently. Occurs one part in ten thousand of ordinary hydrogen. May be easily ionized to form a deuteron.
Helium Atom		2 Protons 2 Neutrons 2 Electrons	Zero	4 Units	Electrons loosely bound. May be ionized in discharges to form alpha particles and two free electrons.

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[1] Compiled by Charles M. Lack, Westinghouse Electric and Manufacturing Company, and republished by permission from page 33 of *Electronics* for August 1937.

* In certain nuclear transformations it is necessary to postulate the ejection of small chargeless particles of 2 varieties or else abandon the laws of conservation of mass energy. One of these particles (neutrino) has a variable mass usually less than that of the electron and the other (anti-neutrino) a negative mass of the same amount.

** Modern physics believes mass and energy are different forms of the same thing and under certain conditions can be directly transformed one into the other. The mass of an electron, for instance, is equal to the energy of an electron or proton after being accelerated through 500 kv. When an electron and positron combine and disappear, 2 gamma rays are produced by the 500-kv type.

*** An isotope of an element is a form of the element having the same chemical properties but different mass. The chemical properties of all elements are determined by the number of outer loosely bound electrons. Thus to form an isotope it is necessary to increase or decrease the number of neutrons in the nucleus.

Carbon Brushes for Steel-Mill Equipment

By W. C. KALB
MEMBER AIEE

The problems associated with brush application on commutating-type equipment in the iron and steel industry are discussed in relation to type of equipment and conditions encountered in service.

The paper also discusses a newly defined measure of brush performance, termed "commutation factor," which provides an improved basis for control of the performance characteristics of brushes.

Mention is made of the development of electrographitic grades having mild polishing action, designed to prevent the formation of troublesome surface film on commutators exposed to the contaminating atmospheric conditions frequently encountered in steel mill applications.

THE PROBLEMS associated with brush application in the iron and steel industry are fundamentally the same as those encountered wherever electrical generating equipment and motive power are used. However, service conditions in this particular industry are in some respects more severe than those usually encountered and, in certain instances, definitely unique. Discussion of the problems of brush application as related to this particular industry would therefore seem to be merited.

For the purpose of this discussion it is convenient to divide the types of equipment considered into 3 classes; first, industrial-type motors having shunt characteristics; second, series-type motors; third, generators and motors of large capacity used on mill drives. The application of brushes to slip rings will not be discussed since the number so used is small in comparison with those used on commutating equipment and the problems of application essentially the same as in other fields of service.

Undercutting of commutator mica has become an almost universal practice in steel mills. This practice removes the necessity for using brushes possessing decided abrasive characteristics. It also constitutes a valid reason for avoiding the use of so-called "self lubricating" brushes, i.e., brushes which have been impregnated with an oil or wax to maintain a film of lubrication on the commutator surface. The presence of such material tends to fill the slots of an undercut commutator with a conducting deposit which increases danger of commutator failure. Nearly all grades of brushes now manufactured have some graphite in their composition, either as a component of the original mix or imparted later by the process of graphitization. This graphite provides ample lubrication for the service conditions under which the brush is ordinarily used.

Industrial type motors of shunt characteristics present few serious problems in brush application. Apart from greater liability to troubles from conducting or abrasive dust and contaminated atmosphere, conditions of service are much the same in steel mills as in other industries. Where provision is made to insure cleanliness within the motor frame, nonabrasive electrographitic brushes give excellent service on this class of equipment. Atmospheric contamination of the commutator surface may require brushes possessing some cleaning action, the amount depending on local conditions. Under such conditions carbon-graphite brushes or special graphite brushes having a moderate cleaning action are often used. Because of variation in the amount of polishing effect required to maintain a clean commutator surface, a brush which works well on one motor may not give satisfactory performance on an identical motor in another part of the same mill. It is not advisable to attempt to offset the presence of abrasive or conducting dust by brush selection. The only sure remedy for troubles arising from this source is to prevent the dust entering the motor. This is best accomplished by piping clean, outside air to the interior of the motor frame. The dust cannot enter against an outward current of air from all openings in the frame.

Series type motors often encounter very severe operating conditions in steel-mill service. Some of the principal applications for this type of motor are on cranes, open-hearth chargers, mill screw-down, side guards, and table rolls. Heat and scale dust are almost always present in the vicinity of these motors, while oil vapor and other atmospheric contamination adds further complication on some applications. Piping in outside air will provide protection against heat, dust, and contaminated atmosphere for those machines which have a fixed location but this protection cannot be given to crane motors or others which are carried on moving equipment. In addition to the heavy load peaks, normally characteristic of the service on which series type motors are applied, the practice of "plugging" further increases the severity of operating conditions. By plugging is meant reversal of the controller before the motor comes to rest. Designers can build motors which will withstand the mechanical stresses resulting from this practice but it is impossible to provide the electrical compensation necessary to maintain good commutating conditions when direction of rotation and impressed voltage are reversed from their normal relationship. As a result, the burden of controlling commutation is thrown almost entirely onto the brushes.

Electrographitic brushes of the harder grades find extensive application on series-type motors in the steel mills. Brushes of this type, in general, have good commutating characteristics and, while not definitely abrasive, possess sufficient mass hardness to provide considerable cleaning action on the commutator. They are also highly resistant to the disintegrating effect of the sparking which inevitably occurs when motors are called upon to meet such extreme demands. In some instances, however, the

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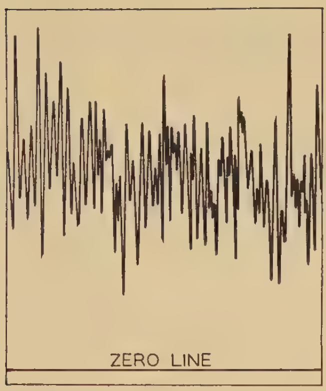

Figure 1. Oscillographic record of contact drop on commutator with flat spot

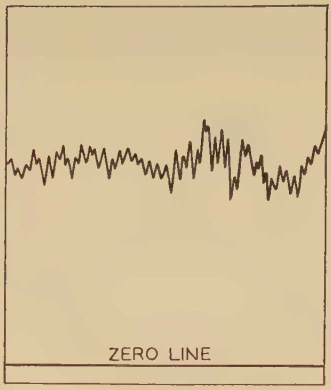

Figure 2. Oscillographic record of contact drop on smooth commutator

mass hardness of the electrographitic brush is not enough to overcome the burning of the commutator surface. The use of a carbon-graphite brush, having some abrasive action, is then indicated. While this abrasive action does result in some commutator wear, selection of a grade with the correct degree of abrasiveness will result in less commutator depreciation and lower maintenance expense than will be experienced if burned bars and flat spots are allowed to develop on the commutator. The latter condition may necessitate shutting down the mill for removal of the armature, an expense item of considerable moment. It may also result in more rapid loss of commutator capacity from returning than would be experienced from wear under a set of properly selected abrasive brushes.

The generators and large capacity motors used on modern steel mill drives constitute the third class of equipment on which brush application is considered in this discussion. Brushes of relatively high carrying capacity are required for this service since the motors are sometimes loaded almost to the stalling point and the current density in the brushes on both the motor and the generator rises to a very high value. In view of the sudden and extreme fluctuations of current resulting from the load conditions encountered it is also essential that brushes having exceptionally good commutating characteristics be applied. These requirements have led to the almost universal adoption of electrographitic brushes for motors as well as generators in this field of application.

Due to the industrial importance of heavy-duty electric drives in the steel mills and the fact that the demands there made upon the brushes are among the most severe encountered in heavy duty d-c power service, a thorough investigation of the brush requirements for this class of equipment has been carried on over the past few years. As a first step in this investigation, thousands of brush samples were obtained from service, together with accurate records of performance. Wherever possible these brushes were obtained in complete sets, with each brush identified as to polarity and location on the machine. A large proportion of these samples were taken from heavy duty steel mill drives, although brushes were also obtained from other heavy duty applications. Investigation was concentrated on brushes of the electrographitic type because of their predominance in this class of service. These brushes were then carefully studied in order to isolate some property common to those having good performance records and lacking in those which had failed in service.

It has long been recognized that the tests commonly applied to brush materials are not closely enough identified with the performance characteristics of the brush to provide a reliable basis for control. The tests referred to are specific resistance (electrical resistance per unit cube) hardness (as measured by the Schore scleroscope or other means) and apparent density (weight per unit of volume). These tests are simple to make and, since they do not destroy or impair the stock, they can be applied to each block of brush material. Unfortunately, however, they do not possess a well-defined relationship to the commutating properties of the brush. Contact drop and coefficient of friction are properties closely related to brush performance but tests of these characteristics require a great deal of time and it is impracticable to apply them to each block of brush material. Furthermore, the results obtained are influenced to a large extent by factors independent of the brush itself which affect the surface of the ring or commutator on which the test is being made.[1] Time, temperature, humidity, and atmospheric contamination are some of the factors which may influence the results obtained from tests of these properties. Brush manufacturers, as a class, have tried to control the performance characteristics of brushes by holding specific resistance, hardness, and apparent density within close limits but this plan has not been successful. Frequently 2 brushes which are almost identical in respect to these 3 properties will show entirely different performance characteristics.

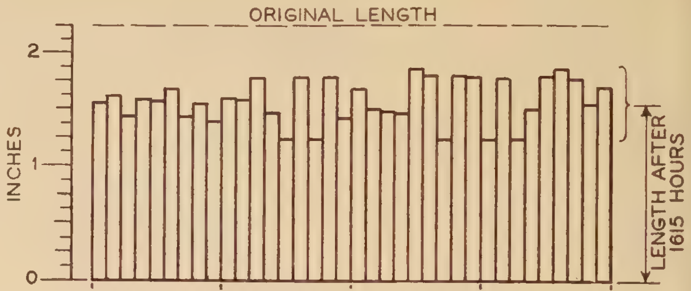

Figure 3. Wear on brushes manufactured under customary basis of control, after 1,615 hours

It is a well-known fact that one requisite to good brush performance is the maintenance of intimate contact between brush and commutator. Slight irregularities in commutator surface tend to impair this intimacy of contact thus causing sparking and resultant damage to commutator surface. Figure 1 is traced from an oscillographic record of the contact drop between a brush and a commutator on which a slight flat spot had developed. Figure 2 is a similar record from the same brush and commutator after the flat spot had been removed. Of course, the condition illustrated in figure 1 was accompanied by considerable sparking but no sparking occurred after removal of the flat spot. It is known that irregularities in commutator surface as small as 0.0001 inch may seriously impair intimacy of contact with some grades of brushes while other grades are capable of maintaining firm contact in the presence of surface irregularities of considerably greater magnitude. In the study of this problem it soon became evident that commutating performance is closely associated with the ability of a brush to ride smoothly on the commutator and that property, in turn, with the elastic or, conversely, the shock absorbing properties of the brush.

A means was found to measure the shock-absorbing capacity of a brush and, as established by subsequent service tests, this also proved to be a measure of commutating performance. The test applied is essentially a measure of the elastic properties of the brush material. It disclosed that brushes having exceptionally good commutating properties possess a certain degree of elasticity while brushes less satisfactory in commutation show a different degree of elasticity. The characteristic so measured has been termed "commutation factor," indicating its close relationship to commutating performance. For practical purposes arbitrary numbers are assigned to indicate various commutation factors or differing degrees of elasticity. The test for this characteristic can be made quickly and does not injure the sample tested. It therefore affords an excellent basis for inspection, applicable to every block of brush material passing through production.

A practicable measure of brush performance having been found, methods were developed for the accurate control of materials and processes of production so that the required commutation factor can be imparted in accurately controlled degree to the finished product. By the methods developed it is possible to produce a series of several grades, similar in general characteristics but each having a different and sharply defined performance characteristic in respect to commutation.

This newly defined characteristic serves, not only as a measure of commutating performance, but also as a basis for obtaining improved uniformity in performance by making possible close control of one of the most important physical properties affecting performance. This is an accomplishment of major importance on applications as exacting as heavy-duty steel-mill drives. The superiority of control methods based on the use of "commutation factor" as the controlling property over methods designed to control resistance, hardness, and density, is shown by figures 3 and 4. Figure 3 shows the variation in wear among the brushes from 4 studs of a large synchronous converter after 1,615 hours of service. These brushes were of a well-known electrographitic grade and were all within close limits of tolerance in respect to resistance, hardness, and density. They did not, however, show corresponding uniformity when the test for commutation factor was applied. The brushes represented by figure 4 were manufactured under the commutation factor basis of control. They were placed on the same synchronous converter immediately after removal of the brushes represented by figure 3 and the record of wear was taken from the same 4 studs after 3,082 hours of service. The improvement in commutating performance was as marked as the improvement in uniformity of brush life which these charts indicate. A large number of carefully observed service tests confirm the evidence here presented that commutation factor is a dependable measure of brush performance and provides a reliable basis for production control.

This extensive comparison of brush performance and related physical properties also disclosed a reciprocal relationship between commutation factor and resistance to mechanical wear. Under mild commutating conditions, where electrical wear at the brush face is slight, a brush with comparatively low commutation factor will, as a rule, give longer life than one having high commutation factor. On the other hand, under severe commutating conditions the electrical wear of the brush with lower commutation factor may be so much more than that of one with high commutation factor that the latter will be superior in life as well as in commutating performance.

Atmospheric contamination is perhaps a greater source of difficulty on heavy-duty mill drive equipment than it is on the smaller series and industrial-type motors to which reference has been made in earlier paragraphs. In some mills elaborate air conditioning apparatus has been installed to remove acid fumes, oil vapor, and gaseous contamination which might cause the development of troublesome surface film on the commutators of mill-drive generators and motors. Most electrographitic brush grades are so free from all abrasive properties that they are incapable of polishing off the surface film resulting from contamination of the surrounding air. As a result, on some installations it is frequently necessary to resurface the commutators, an operation which may require shutting down the mill for an appreciable period of time. Naturally, this involves considerable expense and, if of frequent occurrence, may be a source of serious depreciation in commutator capacity. To better meet the needs of such operating conditions there has recently been developed a series of electrographitic grades possessing a mild polishing action. These have been tried out on a number of mill-drive installations and have proved capable of maintaining a clean, well-polished surface on many commutators previously troubled by film deposition when entirely nonabrasive brushes were used. The abrasive effect of these brushes is not great enough to be a source of appreciable commutator wear so that commutator de-

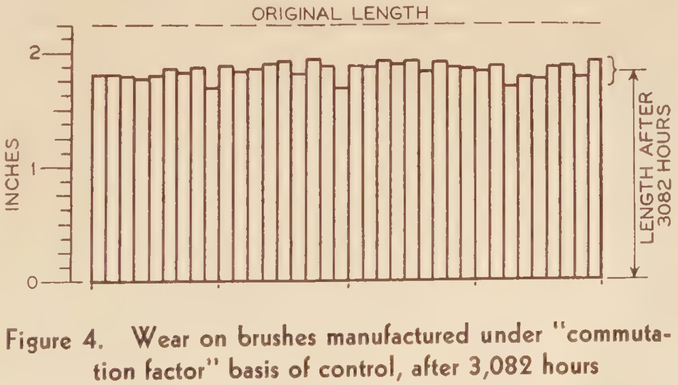

Figure 4. Wear on brushes manufactured under "commutation factor" basis of control, after 3,082 hours

preciation is less than that resulting from the necessity for frequent resurfacing. Furthermore, the liability to expensive shut downs and delays is reduced. Because their abrasive effect is so slight these brushes cannot be depended upon to remove existing surface faults such as burned bar edges or flat spots. Except for their slight polishing action these grades have the same general properties as the nonabrasive electrographitic grades and, like the latter, are produced with graduated and accurately controlled commutation factor.

Just why commutation factor should be a measure of the commutating performance of a brush is a point on which there may be diverse opinions. The relationship seems to be associated, at least in part, with the greater shock absorbing property possessed by a material showing high commutation factor and the resulting improvement in intimacy of contact when such a brush is subject to the impact of slight irregularities on the rapidly moving commutator surface. Whatever the explanation of this relationship may be, the fact of its existence has been demonstrated by service as well as laboratory tests. The discovery of this relationship represents another step of progress toward the solution of the many perplexing problems associated with commutation on equipment subject to such exacting conditions of service as are today encountered in the iron and steel industry.

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Large Steel Pipe Welded by Improved Methods

IN preparation for completion of the Colorado River aqueduct, now scheduled for the winter of 1938–39, construction has been under way for some time on the distribution system which the Metropolitan Water District of Southern California is building for delivery of aqueduct water to member cities. The 242-mile main aqueduct from the Colorado River has its western terminus in Cajalco reservoir. Some 150 miles of distribution lines will be required to convey the water to the 13 cities of the District, located on the coastal plain between Cajalco and the sea. These distribution lines are the equivalent of a major water system in themselves, including a large mileage of tunnels, precast concrete pipe up to $12^1/_2$ feet in diameter, and steel pipe in diameters up to $11^1/_2$ feet. Because of the large diameter of the steel line, particular interest attaches to methods of fabrication in the shop and in welding metal as thick as $^{31}/_{32}$ inches under trench conditions, that is, where the pipe cannot be rotated during the welding process.

The steel pipe sections are all 33 feet 4 inches in length; diameters range from 10 feet 4 inches to $11^1/_2$ feet except on the bridge across the Santa Ana River where pipe of 9 feet 8 inches diameter is used. The thickness varies, according to the head, from $^{17}/_{32}$ to $^{31}/_{32}$ inch. Correspondingly, the weight ranges from 18 to 28 tons per section.

Shop Welding. Each section is made up of 4 steel plates rolled to the proper curve and requiring only a single longitudinal weld in each plate to form a cylinder of the pipe diameter. The completed section is made up by putting together 4 of these cylinders and welding the 3 circumferential joints. In assembling a section, longitudinal welds are staggered. Circumferential welds are made in a large automatic electric welding machine which completes the weld, even in the $^{31}/_{32}$-inch thickness, with a single pass.

Two types of mounting are used for the welding machines: For longitudinal seams the welding equipment is on a carriage which moves lengthwise of the pipe as the weld progresses; a larger machine, used for making the circumferential welds, is stationary and the pipe is rotated as the weld progresses. Both welding machines operate on top of the pipe.

The adjoining edges of the plates to be welded are beveled so the molten metal flows in a V-shaped groove filled in advance with a flux. The flux is a silicious material applied in a powdered form resembling ground glass. The heat of the electric current, fusing the flux, forms a covering over the molten metal closely resembling glass in texture and brittleness. This glass-like covering is so brittle it is easily broken off with a hammer after the weld has cooled. The automatic welding machines work at about 40 volts and use a current of 1,100 or 1,200 amperes. In the longitudinal welds a pair of plates with a V groove similar to that in the joint itself are tack welded at the end of the joint where welding starts, thus affording a few inches for the welding operation to get well under way before reaching the joint in the pipe. Test coupons, attached at the end of the longitudinal weld, are taken every 800 linear feet of weld. All pipes with metal thicknesses exceeding $^5/_6$ inch are annealed, after welding, at temperatures of 1,100 degrees Fahrenheit after which they are cooled slowly.

Welding Field Joints. When a joint is fully entered ready for welding, it is tacked with welds 4 or 5 inches long spaced as required. This is done to prevent any tendency to creep as welding progresses around the circumference. The tack welding is done by one man on the inside and two on the outside.

Completion of the weld involves a number of passes varying with the thickness of the metal. With 7 passes for the thickest metal, the $11^1/_2$-foot pipe requires 40 man-hours per joint or the equivalent of one man's time for a full week on the inside and outside of a single joint. With a crew of about 40 welders, joints in the trench are completed, inside and outside, at the rate of 8 sections per day.

All welds, in shop and field, are done electrically. In the field, current is supplied by portable motor-generator sets equipped for remote voltage control. The welder sets the control for any voltage he prefers from 40 to 80; the amperage is about 250.

From "Steel Pipe $11^1/_2$ Ft. in Diameter Welded by Improved Methods" published in *Engineering News-Record* for May 20, 1937, pages 747–51.

Matrices in Engineering

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I. Summary

IT IS THE purpose of this paper to develop in a concise and simple manner the theory of matrix algebra from the foundations, and to emphasize the use of this powerful method in engineering problems.

It appears not to be a well-known fact among engineers that the matrix is not only a convenient notation that summarizes in a natural and convenient form whole groups of operations but that it is actually possible to solve the sets of differential or algebraic equations written in matrix form in a most convenient manner.

The method by which this is done is one of the utmost simplicity but makes use of some fundamental theorems on matrices not usually found in texts. The theory is scattered throughout the mathematical literature. It has, therefore, been considered worth while in the interest of unity and simplicity to build the subject from the foundations. It thus requires no previous knowledge of matrix algebra for the understanding of this paper.

The recent work of Kron, Sah, and others using the notation of tensors and dyadics in studying the behavior of circuits and machinery suggests the power and flexibility of more advanced mathematical methods in expressing the behavior of physical systems.

A perusal of this paper will illustrate that the matrix appears to be a natural tool to employ in the solution of certain classes of problems.

In the matrix notation we can actually solve the differential or algebraic equations involved without first unscrambling the variables and then proceeding in the conventional manner. The only restrictions on the differential equations is that they be linear and homogeneous. However, they may have variable or constant coefficients.

II. The Fundamentals of Matrix Algebra

1. Definition of a Matrix

By a matrix is meant a system of mn quantities arranged in a rectangular array of m rows and n columns. A matrix is usually denoted as:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = [a]$$

The subscript m refers to the row and the subscript n refers to the column.

The quantities a_{ij} are called the elements of the matrix, they may be constants or functions of a variable.

This arrangement appears similar to that of a determinant. However, a matrix is not a determinant. A determinant has a certain definite value. A matrix, on the other hand, as we shall see, represents a set of operations that involve the elements of the matrix.

We note here that n may differ from m. If $m = n$, we have a square matrix of order n. The square matrix is, as we shall see, the fundamental matrix. A rectangular matrix may be regarded as a square matrix whose missing rows or columns may be filled with elements whose value is zero.

2. Equality of Matrices

The question now arises, when may it be said that the matrix $[a] = [b]$? It is usual to define 2 things as equal when they cannot be distinguished from each other. We adopt this procedure in this case and define 2 matrices as equal when they have the same number of rows and of columns and if they have corresponding elements identical. That is, if $a_{ij} = b_{ij}$ then, by definition $[a] = [b]$.

3. The Zero and Unit Matrices

We define a matrix $[a]$ to be a zero matrix denoted by $[0]$ if all of its elements $a_{ij} = 0$. That is, by definition if $a_{ij} = 0$ then $[a] = [0]$.

We define the square matrix $[I]$ all of whose elements are zero except those in the principal diagonal which are all unity as the unit matrix. As an example, for a third order matrix:

$$[I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We shall see that the unit matrix plays the same rôle in matrix algebra that unity plays in ordinary algebra.

4. Square Matrix, the Fundamental Matrix

Suppose we have a matrix $[a]$ of m rows and n columns. For preciseness, let us say that $m > n$. If this condition holds, then our matrix $[a]$ is a rectangular matrix. We can, however, consider $[a]$ as a square matrix whose lacking columns are filled with elements of value zero.

As an example consider $m = 3$, $n = 2$, then:

$$[a] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 0 \end{bmatrix}$$

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1. For all numbered references, see list at end of paper.

We can, therefore, confine our discussion to square matrices. If $m < n$, the lacking rows would be filled with zeros.

5. The Fundamental Operations

A. Addition and Subtraction

By definition $[a] + [b] = [c]$ provided that $c_{ij} = a_{ij} + b_{ij}$.

That is, the sum of the matrices $[a]$ and $[b]$ is defined as a new matrix $[c]$ whose elements are the sums of the corresponding elements of $[a]$ and $[b]$.

Similarly by definition $[a] - [b] = [c]$ provided that $c_{ij} = a_{ij} - b_{ij}$. Subtraction is merely a special case of addition.

B. Multiplication

In matrix algebra an ordinary number or function is called a scalar in order to distinguish it from a matrix. If $[a]$ is a matrix and k is some scalar quantity, we define $k[a] = [b]$ if $b_{ij} = ka_{ij}$.

That is, multiplication of a matrix by a scalar yields a new matrix whose elements are k times the elements of the original matrix.

We have now to define the multiplication of one matrix by another. It must be realized that we are at liberty to define multiplication of matrices in any manner we please. However, of all possible methods of defining multiplication we select one because of its usefulness, as we shall see.

We define the multiplication of the two matrices $[a]$ and $[b]$ as being a matrix $[c]$ whose elements c_{ij} are specified in a certain manner.

That is

$$[a][b] = [c]$$

if

$$c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$$

That is, if

$$[a] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

and

$$[b] = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

then

$$[a][b] = \begin{bmatrix} (a_{11}b_{11} + a_{12}b_{21}) & (a_{11}b_{12} + a_{12}b_{22}) \\ (a_{21}b_{11} + a_{22}b_{21}) & (a_{21}b_{12} + a_{22}b_{22}) \end{bmatrix}$$

The rule is very simple. In words: "The product $[a]$ $[b]$ of 2 square matrices is a matrix $[c]$ in which the element which lies in the ith row and jth column is obtained by multiplying each element of the ith row of $[a]$ by the corresponding element of the jth column of $[b]$ and adding the results.

This rule is not as formidable as it looks. It can best be grasped by studying the example above. As a special case multiplication by the unit matrix $[I]$ gives $[a]$ $[I]$ = $[I]$ $[a]$. From this it is seen that the behavior of the unit matrix in multiplication is as if it were the number one in ordinary algebra.

Multiplication is not commutative in general. That is, $[a]$ $[b] \neq [b]$ $[a]$ in general.

This is obvious because

$$\sum_{n=1}^{n} b_{in}a_{nj} \neq \sum_{n=1}^{n} a_{in}b_{nj}$$

in general.

The Singular Matrix

A matrix is said to be singular if the determinant of its elements is zero. If the determinant formed from the elements of a matrix does not vanish, the matrix is said to be a nonsingular matrix.

C. The Inverse of a Matrix

If $[a]$ $[b] = [I]$, then, by definition, $[a]$ is called the inverse of $[b]$, and similarly $[b]$ the inverse of $[a]$. It is usual to denote the inverse of a matrix $[a]$ by $[a]^{-1}$. That is $[a]$ $[a]^{-1} = [I]$ and $[a]^{-1}$ $[a] = [I]$, the unit matrix. By actually multiplying out we arrive at the following rule for determining the elements of the inverse of a matrix $[a]$. The rule is analytically as follows:

Let $[b] = [a]^{-1}$, then the elements b_{ij} of the matrix $[b]$ are given by $b_{ij} = A_{ji}/|A|$ where $|A|$ is the determinant formed from the elements of the matrix $[a]$ and A_{ij} is the cofactor, or the minor with the appropriate sign of the element a_{ij} in the determinant $|A|$. Since in order for b_{ij} to have any values which are finite it is necessary that $|A| \neq 0$, it can be seen that only a nonsingular matrix has an inverse.

D. Division of Matrices

Division is defined as the inverse of multiplication. However, since multiplication is not commutative, there are 2 kinds of division. That is, if $[c]$ $[b] = [b]$ $[d]$, we cannot infer that in general $[d] = [c]$. From the principles discussed so far, we can prove the following theorem: If $[a]$ is any matrix and $[b]$ is any nonsingular matrix, there exists only one matrix $[x]$ which satisfies $[a] = [b]$ $[x]$. The proof of this is as follows: Let us multiply both sides of the equation $[a] = [b]$ $[x]$ by $[b]^{-1}$ the inverse of $[b]$. Then we have $[b]^{-1}$ $[a] = [b]^{-1}$ $[b]$ $[x] = [I]$ $[x] = [x]$ since multiplication by the unit matrix always gives us back our matrix. We have thus found $[x] = [b]^{-1}$ $[a]$. However, if we had started with the relation $[x]$ $[b] = [a]$ then a similar process would have given $[x] = [a]$ $[b]^{-1}$. Since, in general $[a]$ $[b]^{-1} \neq [b]^{-1}$ $[a]$, we see that from $[c]$ $[b] = [b]$ $[d]$ we cannot infer that $[c] = [d]$. The order of multiplication in matrix algebra cannot be interchanged in general. In the case where it can be interchanged, that is if $[a]$ $[b] = [b]$ $[a]$, the matrices are called symmetric.

E. The Vanishing of a Product of Matrices

If $[a] \cdot [b] = [0]$ that is, if the product of 2 matrices yields the zero matrix there are 3 possibilities. Either the matrix $[a] = [0]$ or $[b] = [0]$ or if neither the matrix $[a]$ or $[b]$ is a zero matrix, the rule for multiplication tells us that

$$\sum_{n=1}^{n} a_{in} b_{nj} = 0$$

for all values of i and j.

By expanding this set of equations out and making some elementary manipulations it is easily shown that the elements of the various rows of $[a]$ or of $[b]$ are proportional to the elements of the top row. As a special case let us consider $[a]$ and $[b]$ as matrices of the third order then on the supposition that neither is a zero matrix we can say that

$$[a] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ \beta a_{11} & \beta a_{12} & \beta a_{13} \\ \omega a_{11} & \omega a_{12} & \omega a_{13} \end{bmatrix}$$

or

$$[b] = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ m b_{11} & m b_{12} & m b_{13} \\ n b_{11} & n b_{12} & n b_{13} \end{bmatrix}$$

where β, ω, m, and n are constants. That is, the elements of the lower rows are proportional to the elements of the top row.

This result, as we shall see, is of extreme usefulness.

F. Powers of Matrices

We define powers of matrices in the following manner

Positive integral powers $[a]^p = [a][a] \ldots [a]$ to p factors

Negative integral powers $[a]^{-p} = ([a]^{-1})^p$

$[a]^0 = [I] =$ the unit matrix

6. The Laws of Matrix Algebra

We see now with the exception that multiplication is not in general commutative, that is:

$$[a][b] \neq [b][a]$$

That our matrix algebra is similar to ordinary algebra, that is:

$$[a] + [b] = [b] + [a]$$
$$[a] + ([b] + [c]) + [c] = ([a] + [b]) + [c]$$

If k is a scalar and l is a scalar

$$k[a] + k[b] = k([a] + [b])$$
$$k[a] + l[a] = (k + l)[a]$$

If $[c]$ is a matrix and $[d]$ is a matrix

$$[c][a] + [c][b] = [c]([a] + [b])$$
$$[c][a] + [d][a] = ([c] + [d])[a]$$

The only precaution is that $[a]\ [b] \neq [b]\ [a]$ in general

7. The Characteristic Matrix of a Matrix

Let $[a]$ be a matrix whose elements are constants.

Then the matrix $[A] = \mu[I] - [a]$, where μ is a parameter and $[I] =$ the unit matrix, is called the characteristic matrix of $[a]$

8. The Characteristic Function of a Matrix [a]

Let $[A]$ be the characteristic matrix of $[a]$, then the determinant of the elements of the matrix $[A]$ $|A|$ is called the characteristic function of $[a]$

9. The Latent Roots of a Matrix [a]

Let $|A| =$ the characteristic function of $[a]$

$$\therefore |A| = \alpha(\mu)$$

Then the equation $|A| = 0$ is the characteristic equation of the matrix $[a]$

Now since

$$[A] = \mu[I] - [a]$$

The characteristic equation $|A| = 0$ is in general an equation of the nth degree in μ where n is the degree of the matrix $[a]$.

The n roots $\mu_1, \mu_2, \ldots \mu_n$ of the characteristic equation are called the latent roots of the matrix $[a]$.

As an example consider

$$[a] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

The characteristic matrix is $[A] = \mu[I] - [a]$ or

$$[A] = \begin{bmatrix} (\mu - a_{11}) & -a_{12} \\ -a_{21} & (\mu - a_{22}) \end{bmatrix} = \text{characteristic matrix of } [a]$$

$$\begin{vmatrix} (\mu - a_{11}) & -a_{12} \\ -a_{21} & (\mu - a_{22}) \end{vmatrix} = 0$$

is the characteristic equation of $[a]$ or

$$\mu^2 - \mu(a_{11} + a_{22}) + (a_{11}a_{22} - a_{21}a_{12}) = 0$$

The roots of the equation

$$\mu_1 = \alpha_1 + jB$$
$$\mu_2 = \alpha_1 - jB$$

are the latent roots of $[a]$

10. The Cayley-Hamilton Theorem

This theorem, as we shall see, provides us with our most potent weapon in the solution of differential equations and circuit problems. The theorem: If $[a]$ is a matrix of constant elements and $\alpha(\mu) = 0$ is the characteristic equation of $[a]$, then $\alpha([a]) = [0]$ where $[0]$ is the zero matrix. In other words a matrix satisfies its own characteristic equation in a matrix sense. Example: As we have just seen if

$$[a] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

then

$$\alpha(\mu) = \mu^2 - \mu(a_{11} + a_{22}) + (a_{11}a_{22} - a_{21}a_{12}) = 0$$

is the characteristic equation of $[a]$. Then the Cayley-Hamilton theorem states that

$$\alpha([a]) = [a]^2 - [a](a_{11} + a_{22}) + [I](a_{11}a_{22} - a_{21}a_{12}) = 0$$

For a proof of this powerful theorem, see reference 2.

11. Functions of Matrices

How are we to define functions of the matrix $[a]$? In analysis, it is customary to define a function of Z such as $\log Z$, e^z, sine Z, etc., in certain regions of the complex Z plane by their power series expansions. In matrix algebra we shall adopt a similar procedure.

Consider a function of Z $\alpha(Z)$ such as e^z, $\log Z$, sine Z, etc., which is definable as a power series in Z. That is:

$$\alpha(Z) = \sum_{n=0}^{n} C_n Z^n$$

Similarly we define a function of a matrix $[a]$ as

$$\alpha([a]) = \sum_{n=0}^{n} C_n [a]^n$$

with the proviso that $C_0 = C_0[I]$. From the fundamental definition, it is possible by repeated applications of the Cayley-Hamilton theorem to arrive at a simpler form for $\alpha([a])$.

It may be shown, see Duncan and Collar "Solution of Oscillation Problems by Matrices," *Philosophical Magazine*, volume 17, series 7, page 900, that the above definition reduces to the following one:

$$\alpha[a] = \sum_{r=1}^{n} \alpha(\mu_r)[Z_r]$$

where

$$[Z_r] = \frac{\prod_{s \neq r} (\mu_s[I] - [a])}{\prod_{s \neq r} (\mu_s - \mu_r)}$$

where $\mu_1 \ldots \mu_2 \ldots \mu_n$ are the latent roots of the matrix $[a]$.

This theorem is valid only if $[a]$ is a matrix of constant elements.

Professor H. F. Baker has given an alternative form which is sometimes more convenient in practice.[5] Baker's form is:

$$\alpha[a] = \frac{D_{n-1}}{D}[a]^{n-1} + \frac{D_{n-2}}{D} + [a]^{n-2} + \ldots + \frac{D_1}{D}[a] + \frac{D_0}{D}[I]$$

where $\mu_1, \mu_2, \ldots \mu_n$ are the latent roots of $[a]$

$$D = \begin{vmatrix} 1 & 1 & \ldots & 1 \\ \mu_1 & \mu_2 & \ldots & \mu_n \\ \ldots & \ldots & \ldots & \ldots \\ \mu_1^{n-1} & \mu_2^{n-1} & \ldots & \mu_n^{n-1} \end{vmatrix}$$

and D_{r-1} is the determinant derived from D by replacing the elements of the rth row by

$\alpha(\mu_1) \ldots \alpha(\mu_2) \ldots \alpha(\mu_n)$

Both forms are useful, as we shall see.

Example: Let it be required to find

$$e^{[n]} \text{ where } [n] = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

The characteristic equation of $[n]$ is

$$\alpha(\mu) = \begin{vmatrix} (\mu - 1) & 0 \\ 0 & (\mu - 2) \end{vmatrix} = (\mu - 1)(\mu - 2) = \mu^2 - 3\mu + 2 = 0$$

The latent roots, are therefore, $\mu_1 = 1$, $\mu_2 = 2$

Now

$$D = \begin{vmatrix} 1 & 1 \\ \mu_1 & \mu_2 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1$$

$$D_1 = \begin{vmatrix} 1 & 1 \\ e & e^2 \end{vmatrix} = e^2 - e$$

$$D_0 = \begin{vmatrix} e & e^2 \\ 1 & 2 \end{vmatrix} = 2e - e^2$$

$$e^{[n]} = \frac{D_1}{D}[n] + \frac{D_0}{D}[I] = (e^2 - e)\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} +$$

$$(2e - e^2)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} e & 0 \\ 0 & e^2 \end{bmatrix} \qquad e^{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}} = \begin{bmatrix} e & 0 \\ 0 & e^2 \end{bmatrix}$$

another matrix.

12. The Limiting Form of a Matrix When Raised to a Very High Power

A very useful result may be obtained from the above formulas if $\mu([a])$ is placed equal to $[a]^m$.

Let us substitute this in the above, then

$$[a]^m = \sum_{r=1}^{n} \mu_r^m [Z_r]$$

where $\mu_1 \ldots \mu_n$ are the latent roots of $[a]$ arranged in descending order of magnitude. Let us suppose all roots are unequal. Now

$$[Z_r] = \frac{\prod_{s \neq r} (\mu_s[I] - [a])}{\prod_{s \neq r} (\mu_s - \mu_r)}$$

We see from this that $[Z_r]$ is independent of m.

Now if m is very large and μ_1 is the largest root, then

$$\lim_{m \to \infty} \sum_{r=1}^{n} \mu_1^m [Z_1] = \mu_1^m [Z_r]$$

This is true because μ_1, being the largest of the roots, when it is raised to a very high power the term containing μ_1^m is much greater than the other terms and hence the other terms may be neglected in comparison with it. Therefore

$$\lim_{m \to \infty} [a]^m = \frac{\mu_1^m(\mu_2[I] - [a])(\mu_3[I] - [a]) \ldots (\mu_n[I] - [a])}{(\mu_2 - \mu_1)(\mu_3 - \mu_1) \ldots (\mu_n - \mu_1)}$$

Example: What is

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} ?$$

Here as before $\mu_1 = 2$, $\mu_1 = 1$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}^{238} = \frac{2^{238}([I] - [a])}{-1}$$

$$= -2^{238}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\right)$$

$$= -2^{238}\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \qquad = 2^{238}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

This theorem is of extreme importance in raising matrices to high powers. It is valid only when the latent roots are different. If the difference between the latent roots is great, the formula gives accurate answers even for rather small powers of a matrix.

13. The Differentiation of a Matrix

Suppose we have a matrix $[u]$ whose elements are functions of the variable t; that is, $u_{ij} = u_{ij}(t)$. We define

$$\frac{d}{dt}[u] = \begin{bmatrix} \frac{du_{11}}{dt} & \frac{du_{12}}{dt} & \dots & \frac{du_{1n}}{dt} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \frac{du_{n1}}{dt} & \dots & \dots & \frac{du_{nn}}{dt} \end{bmatrix}$$

That is, if we let $D = \frac{d}{dt}$, then

$$D[u] = [Du]$$

14. Integration of a Matrix

Let $[u]$ be a matrix whose elements are functions of the variable t, that is, $\mu_{ij} = \mu_{ij}(t)$

Let

$$Q = \int_{t_0}^{t} (\ )dt$$

stand for the integral operator. We define

$$\int_{t_0}^{t} [u]dt = Q[u] = \begin{bmatrix} Qu_{11} & \dots & Qu_{1n} \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ Qu_{n1} & \dots & Qu_{nn} \end{bmatrix}$$

that is, $Q[u] = [Qu]$ or the integral of a matrix is a matrix whose elements are the integrals of the elements of the original matrix.

If u_{ij} are constants, then

$$Q[u] = (t - t_0)[u]$$

15. Linear Transformation

Consider a set of variables $(e_1 \dots e_n)$ related linearly to a set $(i_1 \dots i_n)$ by the equations.

$$\begin{aligned} e_1 &= Z_{11}i_1 + Z_{12}i_2 + \dots + Z_{1n}i_n \\ e_2 &= Z_{21}i_1 + Z_{22}i_2 + \dots + Z_{2n}i_n \\ &\dots\dots\dots\dots\dots\dots \\ &\dots\dots\dots\dots\dots\dots \\ e_n &= Z_{n1}i_1 + Z_{n2}i_2 + \dots + Z_{nn}i_n \end{aligned}$$

In matrix notation

$$[e] = [Z][i]$$

where

$$[e] = \begin{bmatrix} e_1 \\ \dots \\ \dots \\ \dots \\ e_n \end{bmatrix} \qquad [i] = \begin{bmatrix} i_1 \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ i_n \end{bmatrix}$$

$$[Z] = \begin{bmatrix} Z_{11} \dots Z_{1n} \\ \dots\dots\dots \\ \dots\dots\dots \\ Z_{n1} \dots Z_{nn} \end{bmatrix}$$

That is, by our matrix notation, we may concisely write the above system of equations in the matrix form.

III. The Most General Linear Network

If in the above linear transformation we allow the variables $(e_1 \dots e_n)$ to represent the alternating voltages impressed in the various meshes of an n-mesh network and $[Z]$ and $[i]$ represent the impedance and current matrices, respectively, of the network, then the equation $[e] = [Z]\ [i]$ represents the matrix equation characterizing the behavior of the most general possible linear network.

If the voltages are alternating of angular frequency ω, then

$$Z_{ij} = J\omega L_{ij} + R_{ij} - \frac{J}{\omega C_{ij}} \qquad J = \sqrt{-1}$$

As we shall see, in case we desire the solution for the transient state we may still use matrices.

In general $Z_{ij} = Z_{ji}$. Let us solve the matrix equation for the current. Let us multiply $[Z]\ [i] = [e]$ by $[Z]^{-1}$ the inverse of $[Z]$

$$\begin{aligned} [Z]^{-1}[Z][i] &= [Z]^{-1}[e] \\ [i] &= [Z]^{-1}[e] \end{aligned}$$

where $[Z]^{-1} = [Y]$ the admittance matrix from II part 5c. The elements of $[Y]$ are given by

$$Y_{ij} = \frac{A_{ji}}{|Z|}$$

A_{ij} is the cofactor of the element Z_{ij} of the matrix $[Z]$.

Having determined the elements of the admittance matrix $[Y]$, we calculate the matrix

$$[i] = [Y][e]$$

The various elements of the current matrix $[i]$ give the various currents in the several branches.

Because $Z_{ij} = Z_{ji}$, therefore $A_{ij} = A_{ji}$, consequently, we have immediately the familiar reciprocity theorem as well as the superposition theorem directly from the equation

$$[i] = [Y][e]$$

IV. Traveling Waves on Multiconductors

Suppose we have n parallel conductors with electrostatic and electromagnetic coupling between them.

We may write the 2 matrix equations for the currents and voltages

$$-\frac{\partial}{\partial x}[e] = [Z][i] \quad \text{and} \quad -\frac{\partial}{\partial x}[i] = [Y][e]$$

Where the elements of the matrix $[e]$ represent the various voltages between the lines and the elements of the matrix $[i]$ represent the currents in the n lines. The matrix

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots & Z_{2n} \\ \dots & \dots & \dots & \dots \\ Z_{n1} & \dots & \dots & Z_{nn} \end{bmatrix}$$

$$Z_{ii} = R_{ii} + L_{ii}\frac{d}{dt} \qquad Z_{ij} = M_{ij}\frac{d}{dt} \qquad i \neq j$$

R_{ii} represents the resistances per unit length of the conductors, L_{ii} the inductances per unit length of the conductors, and M_{ij} represents the mutual inductance per unit length of the various conductors. The matrix

$$[Y] = \begin{bmatrix} Y_{11} & \dots & Y_{1n} \\ \dots & \dots & \dots \\ Y_{n1} & \dots & Y_{nn} \end{bmatrix}$$

$$Y_{ij} = G_{ij} + C_{ij}\frac{d}{dt} \qquad i \neq j$$

G_{ij} = the leakage coefficient per unit length

C_{ij} = mutual capacitance per unit length

$$\frac{-\delta^2[e]}{\delta x^2} = \frac{\delta}{\delta x}[Z][i] = [Z]\frac{\delta}{\delta x}[i]$$

Since $[Z]$ is a constant matrix

$$\frac{-\delta^2}{\delta x^2}[i] = [Y]\frac{\delta}{\delta x}[e]$$

Substituting in the original equation we obtain the fundamental equations for the voltages and currents

$$\frac{\delta^2}{\delta x^2}[e] = [Z][Y][e]$$

$$\frac{\delta^2}{\delta x^2}[i] = [Y][Z][i]$$

As we shall see, we can proceed to the solution of the actual differential equations in matrix form.[9]

V. The 4-Terminal Network

From the general theory discussed in part III, it can be easily shown that for the case of a linear, passive, bilateral 4-terminal network shown diagrammatically as in figure 1.

$$E_1 = E_2A + BI_2$$
$$I_1 = E_2C + DI_2$$

where $AD - BC = 1$ or in the matrix form

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} E_2 \\ I_2 \end{bmatrix}$$

Assume that 2 such circuits are connected in series as in figure 2.

If the 2 networks are identical, we have,

$$\begin{bmatrix} E_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} E_3 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} E_3 \\ I_3 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^2 \begin{bmatrix} E_3 \\ I_3 \end{bmatrix}$$

If there are n such circuits in series,

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^n \begin{bmatrix} E_{n+1} \\ I_{n+1} \end{bmatrix}$$

If we wish to get the $(n + 1)$ currents and voltages in terms of the initial values, we have

$$E_2 = DE_1 - BI_1$$
$$I_2 = -CE_1 + AI_1$$

or

$$\begin{bmatrix} E_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \cdot \begin{bmatrix} E_1 \\ I_1 \end{bmatrix}$$

If we have n identical networks in series then

$$\begin{bmatrix} E_{n+1} \\ I_{n+1} \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}^n \cdot \begin{bmatrix} E_1 \\ I_1 \end{bmatrix}$$

If n is very large, we may use the formula for raising a matrix to a very high power.

Because of the natural way in which matrix algebra enters into linear transformations, their application to circuit theory are seen to be unlimited.

VI. The Solution of Differential Equations

The brief examples which are given above and obvious other applications are rather well known to some communication engineers. However, the powerful use which may be made of matrices in the actual *solution* of linear differential equations is not well known.

1. The Differential Equation in Matrix Form

In modern electrical and mechanical engineering there are a great class of problems in circuits, vibrations, etc., which lead either to linear equations of high order or to systems of linear equations. These equations may have either variable or constant coefficients.

Any differential equation or a set of differential equations can be changed into another involving only first order differential coefficients by writing the highest order coefficient as the differential coefficient of a new dependent variable.

All the other differential coefficients are regarded as separate dependent variables, and, for each new variable introduced, another equation must be added to the set.

These added equations are merely the mathematical statement that each new variable is the differential coefficient of the variable that represents the next differential coefficient of lower order. In this way the most general homogeneous linear equation or set of linear homogeneous equations with either constant or variable coefficients may be written as:

$$\frac{dx_1}{dt} = u_{11}x_1 + u_{12}x_2 + \dots + u_{1n}x_n$$
$$\frac{dx_2}{dt} = u_{21}x_1 \dots\dots\dots\dots + u_{2n}x_n$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\frac{dx_n}{dt} = u_{n1}x_1 \dots\dots\dots\dots + u_{nn}x_1$$

In matrix form: if $D = \dfrac{d}{dt}$

$$[Dx] = [u][x]$$

or

$$\frac{d}{dt}[x] = [u][x]$$

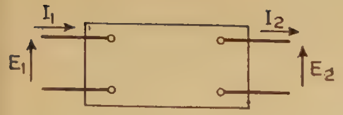


Figure 1

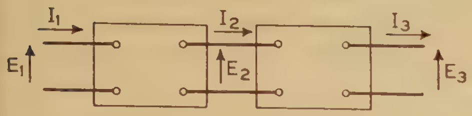


Figure 2

As a preliminary to the integration of this matrix differential equation let us consider the integration of the ordinary differential equation

$$\frac{dy}{dt} = \mu y \qquad \mu = \mu(t)$$

subject to $y = y_0$ at $t = t_0$ the boundary condition.

If we integrate, we obtain, on introducing the boundary conditions.

$$y = y_0 + \int_{t_0}^{t} \mu(t)\, y dt$$

This is an integral equation. We will solve it by successive approximations. Let Q be the operator defined by

$$Q = \int_{t_0}^{t} (\ )\, dt$$

Then

$$y = y_0 + Q\mu y$$

Substitute y in the second member and we obtain

$$y = y_0 + Q\mu[y_0 + Q\mu y] \qquad = y_0 + y_0 Q\mu + Q\mu Q\mu y$$

Repeating the process,

$$y = y_0(1 + Q\mu + Q\mu Q\mu + Q\mu Q\mu Q\mu + \ldots)$$

This series is very rapidly convergent.

THE MATRIX EQUATION

We return now to our matrix equation

$$\frac{d}{dt}[x] = [u][x]$$

Let us require as a boundary condition that $[x] = [x^0]$ at $t = t_0$

Then carrying out the above process on the matrix equation, we obtain the matrix integral equation

$$[x(t)] = [x^0] + \int_{t_0}^{t} [u(t)][x(t)]dt$$

This may be solved by successive approximations.

$$[x(t)] = ([I] + Q[u] + Q[u]Q[u] + \ldots + \ldots)[x_0]$$

$[I]$ = unit matrix

$$Q = \int_{t_0}^{t} (\ )dt \text{ operator}$$

2. The Matrizant

We denote

$$\Omega^{t_0,t}[u] = \left([I] + \int_{t_0}^{t} [u]dt + \int_{t_0}^{t} [u]dt \int_{t_0}^{t} [u]dt + \ldots\right)$$

and the solution of the equation is

$$[x(t)] = \Omega^{t_0,t}[u][x^0]$$

by the name of the "matrizant" of $[u]$.[3] [4]

FUNDAMENTAL PROPERTIES OF THE MATRIZANT $\Omega^{t_0,t}[u]$

1. $\Omega^{t_0,t_0}[u] = [I]$
2. $\frac{d}{dt}\Omega^{t_0,t}[u] = [u]\Omega^{t_0,t}[u]$
3. If $[u]$ is a matrix of constant elements, then

$$\Omega^{t_0,t}[u] = [I]e^{[u](t-t_0)}$$

or, if we let $T = t - t_0$

$[I]$ = unit matrix

then,

$$\Omega^{t_0-t}[u] = [I]e^{[u]T}$$

e = base of natural logarithms

These fundamental properties may all easily be deduced from the above definition of

$$\Omega^{t_0-t}[u] = \left([I] + \int_{t_0}^{t} [u]dt + \int_{t_0}^{t} [u]dt \int_{t_0}^{t} [u]dt + +\right)$$

For a further set of properties, the above paper by Doctor Baker may be consulted.

3. Linear Equations With Constant Coefficients

As an example, let us consider the system:

$$dx_1/dt = u_{11}x_1 + u_{12}x_2$$
$$dx_2/dt = u_{21}x_1 + u_{22}x_2$$

where u_{ij} are constants

Or in the short matrix form:

$$D[x] = [u][x]$$

Let the solution be subject to the boundary conditions that:

$$x_1 = x_1^0 \text{ and } x_2 = x_2^0 \text{ at } t = t_0$$

(Note: The superscripts here must not be confused with exponents, they are merely a convenient notation used to denote the initial values of x_1 and x_2 and, since the place for subscripts is already taken it is convenient to use superscripts.) By the foregoing theory, the solution under these initial conditions is:

$$[x] = \Omega^{t_0,t}[u][x^0]$$

In this case, since $[u]$ is a matrix of constants,

$$\Omega^{t_0,t}[u] = [I]e^{[u]T}$$

where $T = t - t_0$.

We must now compute the matrix function $e^{[u]T}$ by the rule given in part II, section 11. Following the procedure given there, we first compute the latent roots of $[u]$. The characteristic equation is:

$$|\mu I - u| = 0 \quad \text{or} \quad \begin{vmatrix} (\mu - u_{11}) & -u_{12} \\ (-u_{21}) & (\mu - u_{22}) \end{vmatrix} = 0$$

Or, expanding the determinant,

$$\mu^2 - \mu(u_{11} + u_{22}) + (u_{11}u_{22} - u_{12}u_{21}) = 0$$

Let us call the 2 roots of this equation μ_1 and μ_2. In general, the 2 roots will be complex numbers.
We can, therefore, in general denote the 2 roots as:

$$\mu_1 = \alpha + \beta \quad \text{and} \quad \mu_2 = \alpha - \beta$$

Computing $e^{[u]t}$ by Baker's rule (part II, section 11) we first must compute:

$$D = \begin{vmatrix} 1 & 1 \\ (\alpha + \beta) & (\alpha - \beta) \end{vmatrix} = (\alpha - \beta) - (\alpha + \beta) = -2\beta$$

$$D_1 = \begin{vmatrix} 1 & 1 \\ e\alpha T+\beta T & e\alpha T-\beta T \end{vmatrix} = -2e^{\alpha T} \sinh \beta T$$

$$D_0 = \begin{vmatrix} e^{\alpha L+\beta T} & e^{\alpha T-\beta T} \\ \alpha + \beta & \alpha - \beta \end{vmatrix} = 2e^{\alpha T}(\alpha \sinh \beta T - \beta \cosh \beta T)$$

We note that, by definition $[u]^0 = [I]$. We have, therefore, by Baker's rule:

$$e^{[u]T} = e^{\alpha T} \sinh \beta T \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} - e^{\alpha T}/\beta\, (\alpha \sinh \beta T - \beta \cosh \beta T)[I]$$

Or, therefore, the matrizant, in this case is:

$$\Omega^{t_0,t}[u] = e^{\alpha T} \begin{bmatrix} (u_{11} - \alpha)/\beta \sinh \beta T + \cosh \beta T) & (u_{12}/\beta \sinh \beta T) \\ (u_{21}/\beta \sinh \beta T) & \left(\frac{u_{22} - \alpha}{\beta}\right) \sinh \beta T + \cosh \beta T) \end{bmatrix}$$

Therefore, the equation has the solution:

$$[x] = \Omega^{t_0,t}[u][x^0]$$

If we write

$$\Omega^{t_0,t}[u] = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}$$

$$\Omega^{t_0,t}[u] = e^{\alpha T} \begin{bmatrix} \left(\frac{u_{11} - \alpha}{\beta} \sinh \beta T + \cosh \beta T\right) & \left(\frac{u_{12}}{\beta} \sinh \beta T\right) \\ \left(\frac{u_{21}}{\beta} \sinh \beta T\right) & \left(\frac{u_{22} - \alpha}{\beta} \sinh \beta T + \cosh \beta T\right) \end{bmatrix}$$

where

$$\Omega_{11} = \left\{\frac{(u_{11} - \alpha)}{\beta} \sinh \beta T + \cosh \beta T\right\} e^{\alpha T}$$

$$\Omega_{12} = (u_{12}/\beta e)^{\alpha T} \sinh \beta T$$

$$\Omega_{21} = (u_{21}/\beta e)^{\alpha T} \sinh \beta T$$

$$\Omega_{22} = [(u_{22} - \alpha)/\beta \sinh]\ \beta T + \cosh \beta T)e^{\alpha T}$$

$$T = t - t_0$$

If β is imaginary, $\sinh \beta T$ and $\cosh \beta T$ must be replaced by sine βT and $\cos \beta T$, respectively. If $\beta = 0$, we must obtain the limiting forms from these equations by the usual L'Hospital's rule of differentiation. We have now, the dependent variables in terms of the initial conditions.

4. Linear Equations With Variable Coefficients

The method of matrices is particularly well adapted to the solution of equations whose coefficients are functions of the independent variable t. The only restrictions on the variables is that they be finite and single valued functions of t throughout the range of integration. This limitation is met by physical equations.

Let us consider the most general homogeneous linear differential equation with variable coefficients. This equation may be written:

$$Dx^n + a_1(t)Dx^{n-1} + \ldots + a_{n-1}(t)x = 0$$

where D is the operator d/dt and the coefficients $a_i(t)$ are single-valued analytic functions of t. That is, they do not become infinite in any part of the range for which we desire a solution.

As in the previous case with constant coefficients, we first break up the equation into a set of n homogeneous equations of the first order. This is done by calling successive derivatives new variables. That is, we let $x = x_1$, $Dx = x_2$ and so forth. Our equation is then replaced by:

$$dx_1/dt = u_{11}x_1 + u_{12}x_2 + \ldots\ldots\ldots + u_{1n}x_n$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$dx_n/dt = u_{n1}x_1 + \ldots\ldots\ldots\ldots + u_{nn}x_n$$

where now the coefficients of u_{ij} are functions of t.
In matrix form, we write this set of equations as before: $D[x] = [u][x]$. If we know the initial value of x and the initial value of $(n - \text{one})$th derivatives we denote them by x_1^0, x_2^0, etc., these initial values at $t = t_0$. Our solution is as before:

$$[x] = \Omega^{t_0-t}[u][x^0]$$

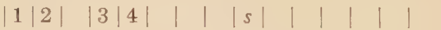
Where, unfortunately $\Omega^{t_0-t}[u]$ does not have the simple form it had when the elements of the matrix $[u]$ were constants instead of functions of t. In this case

$$\Omega^{t_0-t}[u] = \left([I] + \int_{t_0}^{t} [u]dt + \int_{t_0}^{t} [u]dt \int_{t_0}^{t} [u]dt + +\right)$$

from the fundamental defining equation. It is most laborious to use this form to compute the matrizant. However we note that if $[u]$ is a matrix of constants, then

$$\Omega^{t_0-t}[u] = [I]e^{[u]T}$$

where e is the base of the natural logarithms. This suggests that the following procedure be followed. Let the interval of time over which the equation is to be solved be broken up into s small time intervals. The procedure may be more easily followed by the consideration of the following diagram:



Let us assume that during the time interval from t_0 to t_1 the elements of the matrix $[u]$ are approximately constant and have the average value that they assume throughout the interval. Let us denote this matrix by $[u(1)]$. Let in general $[u(s)]$ represent the matrix whose elements are given the average values which they assume in the sth interval from $t = t_{s-1}$ to $t = t_s$. We may then proceed to the solution of our actual equation with variable coefficients by a sort of chain process. That is, assuming that throughout the first interval the elements of the matrix $[u]$ are constant and denoting the matrix of these constant values by $[u(1)]$, we may say that at the end of the first interval the solution is:

$$[x(t_1)] = [I]e^{[u]T}[x^0]$$

where

$$T = t_1 - t_0$$

This is the value that the variables have assumed at the end of the first interval, and hence these values provide the initial conditions for the second interval. That is, the initial conditions for the second interval are $[x(t_1)]$ instead of $[x^0]$. If the intervals are all equal let us call them T; that is

$$t_s - t_{s-1} = T$$

then

$$[x(t_1)] = [I]e^{[u(1)]T}[x^0]$$

and, at the end of the second interval, the solution is:

$$[x(t_2)] = [I]e^{[u(2)]T}[x(t_1)]$$

Since, however, we know $[x(t_1)]$, we can compute $[x(t_2)]$ by a matrix multiplication. The solution for the sth interval can be obtained similarly by a repeated chain of multiplication of the various values of the preceding intervals. The procedure appears laborious, but it is in reality quite simple. It may be noted that the intervals do not have to be of the same length T. It is usually more convenient to choose short intervals where the variations of the elements of the matrix $[u]$ is great and to choose larger intervals where the elements of $[u]$ are more or less constant.

ILLUSTRATION

This method may perhaps be brought out more clearly by means of an illustration. The example worked out by Frazer[6] will be given. It is particularly good as an illustration since it tests the method to the utmost in that the variables fluctuate widely and the values obtained may be compared with the exact solution.

Let us consider the motion of a pendulum of variable stiffness whose equation of motion may be taken as:

$$D^2\alpha + (16\pi^2 e^{-2t} - 1/4)\,\alpha = 0$$

It is easy to solve this equation in exact terms.

The exact solution is:

$$\theta = e^{t/2}\cos(4\pi e^{-t})$$

for the boundary conditions that $\alpha = 1.0$ and $d\alpha/dt = 0.5$ at $t = 0$. To solve this equation in the matrix form we place $\alpha = x_1$ and $d\alpha/dt = x_2$. Then the matrix equation is:

$$D[x]\ \left\|\begin{matrix} 0 & 1 \\ u_{21} & 0 \end{matrix}\right\|\ [x] = [u][x]$$

where

$$u_{21} = 1/4 - 16\pi^2 e^{-2t}$$

Let $u_{21}(s)$ be the average value of $^1/_4 - 16\pi^2 e^{-2t}$ for the sth interval from $t = t_{s-1}$ to $t = t_s$. Then, for that interval:

$$D[x] = \begin{bmatrix} 0 & 1 \\ u_{21}(s) & 0 \end{bmatrix} \cdot [x]$$

The characteristic equation is:

$$\begin{vmatrix} \mu & -1 \\ -u_{21}(s) & \mu \end{vmatrix} = 0$$

and the latent roots are:

$$\alpha_s = 0 \qquad \beta_s = \sqrt{u_{21}(S)}$$

The matrizant relation is as in VI part 3.

$$[x] = \begin{bmatrix} \cosh(\beta_s T) & \sinh(\beta_s T) \\ \dfrac{u_{21}(s)\sinh(\beta_s T)}{\beta_s} & \cosh(\beta_s T) \end{bmatrix} [x^{s-1}]$$

if β_s is real. If, however, β_s is imaginary, then circular functions must replace the above hyperbolic functions. The numerical values of the elements of Ω^{t_{s-1},t_s} are given in the table I.

The matrizant $\Omega^{t_{s-1},t_s}[u]$ is formed for each interval from the values given in the above table. From the value of the matrizant for the first interval and the initial conditions, the values of the dependent variables at the end of that interval and hence the initial conditions for the next interval can be found.

For example, the product $\Omega^{t_0,t_1}[x^0]$ may be carried out in table form. That is, the square matrix $\Omega t_0,t_1$ is multiplied by the line matrix $[x^0]$. This gives a line matrix

Table I

Start of Interval t=	u_{21}(s)	β	cos βT cosh βT	sin βT sinh βT	sin βT/β sinh βT/β	u_{21}(s)sin βT/β u_{21}(s)sinh βT/β
$t = 0$	−129.9	11.39j	−0.6504	0.7590	0.0666	−8.650
$t = 0.2$	−86.99	9.327j	−0.2900	0.9570	0.1026	−8.926
$t = 0.4$	−58.23	7.631j	+0.0448	0.9990	0.1309	−7.623
$t = 0.6$	−38.95	6.241j	0.3172	0.9483	0.1519	−5.918
$t = 0.8$	−26.02	5.101j	0.5233	0.8521	0.1670	−4.346
$t = 1.0$	−13.51	3.675j	−0.2636	0.9650	0.2626	−3.546
$t = 1.5$	−4.469	2.114j	+0.4915	0.8709	0.4120	−1.841
$t = 2.0$	−1.000	1.000j	0.5403	0.8415	0.8415	−0.8415
$t = 3.0$	+0.0808	0.284	1.0405	0.2880	1.015	+0.0819

Table II

Initial Value of t	The Matrizant Ω^{t_{s-1},t_s}		Initial Conditions α and Dα	
			Stepped Solution	Exact Solution
0	−0 6504	0.0666	1.0000	1.0000
	−8.650	−0.6504	0.5000	0.5000
0.2	−0.2900	0.1026	−0.6171	−0.725
	−8.926	−0.2900	−8.975	−9.243
0.4	0.0448	0.1309	−0.7419	−0.657
	−7.623	0.0448	+8.111	+9.008
0.6	0.3172	0.1519	1.0285	1.11
	−5.918	−0.3172	6.019	5.855
0.8	0.5233	0.1670	1.2405	1.20
	−4.346	0.5233	−4.1775	−4.43
1.0	−0.2636	0.2626	−0.0487	−0.172
	−3 546	−0.2636	−7.577	−7.646
1.5	−0.4915	0.4120	−1.977	−1.99
	−1.841	0.4915	+2.170	+3.005
2.0	0.5403	0.8415	−0.0776	−0.35
	−0.8415	0.5403	+4.706	+4.745
3.0	1.0405	1.015	3.918	3.64
	0.0819	1.0405	2.608	3.46
4.0			6 724	7.19
			3.035	3.984

which provides the initial conditions for the next interval. The same procedure is carried out for the next interval and repeated to the end of the range in t.

The process may be written symbolically as follows:

$$[x(t_n)] = \Omega^{t_{n-1},\,t_n}\Omega^{t_{n-2},t_{n-1}} \ldots \Omega^{t_0,t_1}[x^0]$$

or more briefly:

$$[x(t_n)] = \Omega^{t_{r-1},t_r}[x^0]$$

This is shown in table II.

It may be remarked that the approximate stepped solution differs quite considerably from the exact solution. However, in the example the variable stiffness fluctuates tremendously during the first interval.

The time intervals at the beginning of the solution should have been chosen shorter. Where the coefficients of the matrix $[u]$ vary considerably shorter time intervals must be taken.

The method here presented is only an approximate one and will give good results only in case the number of intervals during which the variable coefficients are to be considered constant is taken to be great. In the case of second order systems this method provides a rather quick method of arriving at the trend of the solution. In the example given here it is of course needless to calculate the process to 4 significant figures as has been done. It must be realized that the coefficient $(16\pi^2e^{-2t} - {}^1/_4)$ changes tremendously in the neighborhood of $t = 0$, and more accurate results would have been obtained had the number of intervals at the start of the solution been chosen in a different manner.

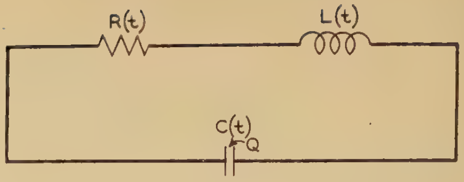
The method here presented is of particular value in second order differential equations whose coefficients do not vary greatly. It is true that when t becomes large, errors will accumulate. Since, however, in most problems the variation of the dependent variable for small values of the independent variable t, is sought this method gives rather quickly the required information.

VII. Variable Networks

The above-mentioned method enables one to obtain the solution for the free oscillations of a network whose parameters vary with the time. The solution may be made as accurate as desired by the correct choice of the time intervals $t_s - t_{s-1}$.

It is now proposed to discuss the transient solution of networks whose parameters vary with the time in a periodic manner. An example of this type of network is the alternator with the mutual inductance varying as a sine function of t. Another example is the telephone transmitter with internal resistance that varies periodically with the time. The condenser microphone is another example of this class of circuit. There are a host of important technical problems of this class. In the field of mechanical engineering the situation is the same. The torsional vibrations of the side-rods of electric locomotives and even the noisy vibrations of small synchronous clock motors may be brought under the heading of systems with periodically varying parameters. For other mechanical problems of this class the reader is referred to "Mechanical Vibrations," a book by Den Hartog. A most extensive bibliography of this class of problems is given by Van der Pol and Strutt.[8]

Figure 3



1. Free Oscillations of a Variable Network

The method will be illustrated by the use of a special case which is of great importance in electrical and mechanical engineering.

For definiteness, let us consider the most general possible series circuit containing a variable resistance, inductance, and capacitance. The circuit is shown in figure 3.

Suppose that at $t = t_0$ a charge is placed on the condenser and we inquire the subsequent behavior of the system. We specify that all the variable elements are periodic in time with the same frequency. That is,

$$R(t + T) = R(t)$$
$$C(T + t) = C(t)$$

and

$$L(t + T) = L(t)$$

where T is the period of the variation. Let

$$R(t) = R_0 + r(t)$$
$$L(t) = L_0 + l(t)$$
$$R/C(t) = S(t) = S_0 + s(t)$$

where $S(t)$, the reciprocal of the capacitance is the elastance. R_0, L_0, and S_0 are the fixed constant resistance, inductance, and elastance of the circuit about which the circuit varies. $r(t)$, $l(t)$, and $s(t)$, are the variable elements of resistance, inductance, and elastance of the circuit. Let us suppose that the variations in the various elements is always smaller than the fixed portion of the various elements. That is,

$$|s| < R_0 \qquad |l| < L_0 \qquad \text{and} \qquad |s| < S_0$$

that is, the absolute values of the variable elements are always less than the fixed components. This is usually the case in actual physical systems.

The fundamental equation of the circuit is:

$$\frac{d^2Q}{dt^2} + \frac{dQ}{dt}\left(R + \frac{dL}{dt}\right)\frac{1}{L} + \frac{S(t)}{L(t)}\quad Q = 0$$

where

$$D = d/dt$$

and Q is the charge on the condenser in coulombs. Let

$$A(t) = \left(R + \frac{dL}{dt}\right)\frac{1}{L_0 + l(t)} \quad \text{and } B(t) = \frac{S_0 + s(t)}{L_0 + l(t)}$$

In the above equation let us substitute

$$Q = e^{-1/2\int A(t)dt}y(t) \text{ for } Q$$

On making this substitution for Q in the above differential equation we find that $y(t)$ must satisfy the equation $D^2y + (B - {}^1/_2DA - {}^1/_4A^2)y = 0$, if we call $(B - {}^1/_2DA - {}^1/_4A^2) = b^2 + \alpha(t)$; that is, we separate this expression into 2 parts, one a constant term and the other a variable term. Our problem now reduces to the solution of $D^2y + [b^2 + \alpha(t)]y = 0$

It may be here noted that because of the periodicity of the parameters we must have: $\alpha(t + T) = \alpha(t)$

The above equation in y is the equation of Hill. It has received extensive treatment particularly in astronomical connections. All the methods published deal with the solution of the equation in series or contour integrals. These solutions are most unsatisfactory from the practical point of view. A theorem will now be established which will show the ease with which this equation may be solved numerically by the matrix method.

2. The Stability of Hill's Equation

The following analysis is a general discussion on the determination of the stability of the physical system whose behavior is described by Hill's equation.

Consider Hill's equation:

$$D^2y + [b^2 + \alpha(t)]y = 0$$

Now let $g_1(t)$ and $g_2(t)$ be 2 linearly independent solutions of the differential equation. The general solution will then be a linear combination of g_1 and g_2 and may be written as:

$$y = c_1g_1 + c_2g_2$$

where c_1 and c_2 are constants.

Let us write $y = x_1$ and $dy/dt = x_2$. We may then write the matrix:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} g_1 & g_2 \\ Dg_1 & Dg_2 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

where $D = d/dt$ at $t = t_0$. Let

$$[x] = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [x^0]$$

If we now let

$$[A(t)] = \begin{bmatrix} g_1(t) & g_2(t) \\ Dg_1(t) & Dg_2(t) \end{bmatrix}$$

and

$$[c] = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

we may write the matrix relation $[x] = [A(t)]\,[c]$. Now to determine the arbitrary constants we have,

$$[x^0] = [A(t_0)][c]$$

and hence

$$[c] = [A(t_0)]^{-1}[x^0]$$

and hence the solution becomes:

$$[x] = [A(t)][A(t_0)]^{-1}[x^0]$$

now because of the periodicity of the variable coefficient $\alpha(t)$, that is, $\alpha(t + T) = \alpha(t)$, we may assume $[x(t_0 + T)] = k[x^0]$, where k is some constant to be determined. But $[x(t_0 + T)] = [A(t_0 + T)]\,[A(t_0)]^{-1}[x^0]$, and hence substituting into the above equation we obtain:

$$k[x^0] = [A(t_0 + T)][A(t_0)]^{-1}[x^0]$$

If we let $[M] = [A(t_0 + T)]\,[A(t_0)]^{-1}$, then we have:

$$k[x^0] = [M][x^0]$$

or

$$[kI - M] = [0]$$

and the constant k depends upon the 2 characteristic roots of the matrix $[M]$. That is if,

$$[M] = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

then k is given by the 2 roots of the equation

$$k^2 - k(M_{11} + M_{22}) + (M_{11}M_{22} - M_{12}M_{21}) = 0$$

Now if $|k_1| < 0$ and $|k_2| < 0$ the motion is stable while if $|k_1| > 0$ or $|k_2| > 0$, the motion is unstable.

By subdividing the interval from t_0 to $t_0 + T$ into small intervals throughout which the variable $v(t)$ may be assumed to have its mean value it is possible to arrive at a numerical value for the matrix $[M]$ and hence to compute the stability of the system. This method has recently been employed by the author to a simple system. The results are to appear in the *Journal* of the Franklin Institute shortly.

3. The Matrix Solution of Hill's Equation

It would be easy to give a numerical example. However, it is best to consider the general case.

We are to solve $D^2y + [b^2 + \alpha(t)]y = 0$ with the initial conditions of y and dy/dt known. Let $y = x_1$ and $dy/dt = x_2$. Our equation reduces to the system.

$$Dx_1 = x_2$$
$$Dx_2 = -[b^2 + \alpha(t)]x_1$$

Let $F(t) = -[b^2 + \alpha(t)]$

The equation in matrix form becomes,

$$D[x] = \begin{bmatrix} 0 & 1 \\ u_{21} & 0 \end{bmatrix} \cdot [x] \qquad u_{21} = F(t)$$

The solution is carried out as in the example of section VI, part 4. The time T for one cycle of variation of $\alpha(t)$ is divided into several intervals of length Δt and the various matrizants calculated.

The solution at the nth interval is given by

$$[x(t_n)] = \prod_{r=1}^{n} \Omega^{t_{r-1},t_r}[x^0]$$

where, as before, $[x^0]$ is the matrix of the initial values of y and dy/dt. If at the end of time $t = T$ or at the end of the completion of one cycle of variation of $\alpha(t)$ the variables x_1 and x_2 have increased to kx_1^0 and kx_2^0 or, in matrix form $[x(T)] = k[x^0]$, then we have solved the equation for any value of t by the use of the formula $y(t + T) = ky(t)$. The important thing particularly in mechanical problems is that we have determined the stability coefficient k and can now predict whether the motion will increase and hence be unstable or die out or keep on without any increase or decrease. The author is in the process of applying this method to several special cases.

Having determined y, Q, the charge on the condenser is obtained from

$$Q = e^{-1/2\int(A_t)dt}y(t)$$

Whether Q builds up or not depends on the relative magnitudes of the decrement factor

$$e^{-1/2\int A(t)dt}$$

and the value of k. In the absence of resistance, $A(t) = 0$. The behavior of Q depends entirely on k in such a case.

VIII. The Solution of Oscillatory Networks

If in the most general linear bilateral network with lumped parameters discussed in part III, we place the resistance elements $R_{ij} = 0$. That is, the resistance in the various branches is negligible, we call the network an oscillatory network. In the conventional solution of such a network for the resonance frequencies and amplitudes it is necessary to solve a set of algebraic equations of high order. A method will now be given by which through the employment of matrices we do not have to solve any algebraic equations whatsoever. The method to be given here is a modification of a method developed by Duncan and Collar in treating mechanical problems.[7]

The fundamental equations for such a circuit in matrix form are:

$$[e] = [Z][i] \qquad \text{(See part III)}$$

where

$$Z_{ij} = J(\omega L_{ij} - 1/\omega C_{ij}) \qquad J = \sqrt{-1}$$

putting $R_{ij} = 0$.

Now, if we are interested in the various resonant frequencies, we place $[e] = [0]$.

Our matrix equation reduces to: $[0] = [Z]\ [i]$ or $[0] = (\omega[L] - 1/\omega[1/C])\ [i]$. The matrix

$$[L] = \begin{bmatrix} L_{11}\ldots L_{12}\ldots\ldots\ldots L_{1n} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ L_{n1}\ldots L_{n2}\ldots\ldots\ldots L_{nn} \end{bmatrix}$$

may be termed the inductance matrix. The inductance matrix includes all the self-inductances ($L_{11}, L_{22}, \ldots L_{jj}$) and all the mutual inductances (L_{ij} where $i \neq j$). The matrix

$$[1/C] = \begin{bmatrix} 1/C_{11}\ldots 1/C_{12} \quad \ldots\ldots 1/C_{1n} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 1/C_{n1}\ldots 1/C_{n2}\ldots\ldots\ldots 1/C_{nn} \end{bmatrix}$$

may be termed the elastance matrix. It includes the self- and mutual-elastances of the system.

If we multiply both sides of our above matrix equation by the scalar ω, the angular frequency, we obtain:

$$(\omega^2[L] - [1/C])[i] = [0]$$

Multiplying by $[L]^{-1}$, the inverse of $[L]$ gives us,

$$(\omega^2[I] - [L]^{-1}[1/C])[i] = [0]$$

where $[I]$ is the unit matrix.

This may be considered as the generalization of the equation

$$(\omega^2 - 1/LC)i = 0$$

for the resonant frequency of a simple series circuit having an inductance L and a capacitance C. In this simple case, the well-known solution for the angular resonant frequency is $\omega = 1/\sqrt{LC}$.

3. The Solution of the Matrix Equation

We must now solve the matrix equation:

$$(\omega^2[I] - [L]^{-1}[1/C])[i] = 0$$

For simplicity, let $\mu = \omega^2$ and $[M] = [L]^{-1}[1/C]$. Our equation now reduces to $[\ I - M]\ [I] = [0]$. We note here (part II, section 7) that the matrix $[\mu I - M]$ is the characteristic matrix of the matrix $[M]$.

The latent roots μ_j of the matrix $[M]$ are related therefore, to the resonant frequencies by the equation

$$f_j = 1/2\pi\sqrt{\mu_j}$$

where the f_j are the resonant frequencies.

We could, of course, evaluate these roots by finding the roots of the characteristic function

$$|\mu I - M| = 0$$

however, this is essentially the conventional procedure which one employs without the use of matrices. We are then confronted with an equation of degree n for μ. It is now proposed to give another method which avoids the solution of the algebraic equation.

Let us consider the matrix $[M]$ raised to a very high power. See part II, section 12. We have the equation

$$\lim_{n\to\infty} [M]^n = \frac{\mu_1{}^n\ [M_1]}{(\mu_2 - \mu_1)(\mu_3 - \mu_1)\ldots(\mu_n - \mu_1)}$$

where μ_1 is the greatest of the latent roots of $[M]$, and $[M_1] = [\mu_2 - M]\ [\mu_3 - M]\ \ldots\ldots[\mu_n - M]$. We note that $[M_1]$ is the product in the Cayley-Hamilton relation with $[\mu_1 - M]$ absent. Therefore, by the Cayley-Hamilton theorem (part II, section 10), we have $[\mu_1 - M]\ [M_1] = [0]$

We now make use of the theorem on the vanishing of a product of matrices (part II, section e). Therefore

$$[M_1] = \begin{bmatrix} a_1A_1\ldots a_2A_2\ldots\ldots\ldots a_1A_n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_nA_1 \quad a_nA_2\ldots\ldots\ldots a_nA_n \end{bmatrix}$$

That is, the matrix $[M_1]$ is such that the elements of any row are proportional to the corresponding elements of the top row of $[M_1]$. Therefore

$$\lim [M]^n = \frac{\mu_1{}^n}{\prod\limits_{r\neq 1}(\mu_r - \mu_1)} \begin{bmatrix} a_1A_1\ldots\ldots\ldots a_1A_n \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_nA_1\ldots\ldots\ldots a_nA_n \end{bmatrix}$$

Now consider *any* line matrix $[X]$ and form the product

$$[M]^n[X] = \frac{\mu_1{}^n}{\prod\limits_{r\neq 1}(\mu_r - \mu_1)}\begin{bmatrix} a_1A_1 \dots\dots\dots a_1A_n \\ \dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots \\ a_nA_1 \dots\dots\dots a_nA_n \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ \dots \\ X_n \end{bmatrix} = \frac{\mu_1{}^n}{\prod\limits_{r\neq 1}(\mu_r - \mu_1)}\begin{bmatrix} a_1R \\ a_2R \\ \dots \\ a_nR \end{bmatrix}$$

where

$$R = A_1X_1 + A_2X_2 + \dots + A_nX_n$$

If the values of $X_1, X_2 \dots\dots\dots X_n$ are chosen so that R is not zero, it follows that $[M]^{n+1}[X] = \mu_1[M]^n[X]$ that is, raising the power of the matrix $[M]$ from n to $n + 1$, gives a resulting matrix that is the same as before except that it is multiplied by μ_1. Therefore, to obtain the fundamental frequency, any series of values are assumed for $X_1 \dots\dots X_n$ and, by continually multiplying $[M]^{n+1}[X]$ is obtained. After n has reached a certain value, a limiting form is obtained when the ratio of successive multiplications is equal to μ_1. When μ_1 has been obtained, the fundamental frequency is obtained from

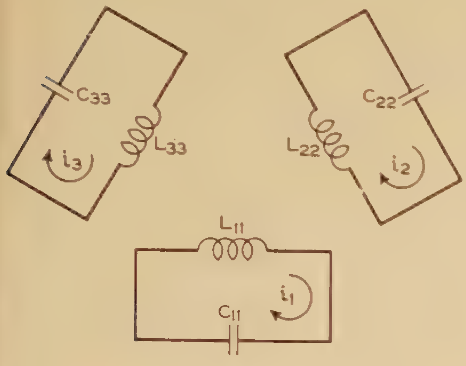


Figure 4

$$f_1 = 1/2\pi\sqrt{\mu_1}$$

It remains now to find the relative amplitude of the currents in the various meshes when the system is oscillating in its fundamental mode. We return to

$$[\mu I - M][i] = [0]$$

and

$$[\mu_1 - M][M_1] = [0]$$

Let $(i_1{}^0\ i_2{}^0 \dots\dots\dots i_n{}^0)$ be the amplitudes of the currents corresponding to the fundamental mode μ_1. Then

$$[\mu_1 - M][i^0] = [0]$$
$$[\mu_1 - M][M_1] = [0]$$

hence $[i^0] = K_1\ [M_1]$, where K_1 is some constant; or, therefore,

$$[i^0] = K_2\begin{bmatrix} a_1R_1 \\ a_2R_2 \\ \dots \\ \dots \\ a_nR \end{bmatrix}$$

Therefore the amplitudes of the various currents are proportional to the quantities $a_1, a_2, \dots\dots\dots a_n$. The method will be illustrated by means of a numerical example.

4. Example, Free Vibrations of a Coupled Circuit

The method will be made clear by the following example. Consider the coupled circuit shown in figure 4. In the circuit, let $C_{11} = C_{22} = C_{33} = 10^{-6}$ farads. Let $C_{ij} = 0$ if $i \neq j$. Let $L_{11} = {}^5\!/_6H$, $L_{22} = {}^1\!/_2H$, $L_{33} = {}^1\!/_6H$, $L_{12} = -{}^1\!/_3H = L_{21}$, $L_{31} = 0 = L_{13}$, $L_{23} = -{}^1\!/_6H = L_{32}$. In matrix form

$$[L] = \begin{bmatrix} 5/6 & -1/3 & 0 \\ -1/3 & 1/2 & -1/6 \\ 0 & -1/6 & 1/6 \end{bmatrix}$$

$$[1/C] = 10^6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 10^6[I]$$

We first calculate $[M] = [L]^{-1}[1/C]$

$$[L] = 1/36\begin{bmatrix} 30 & -12 & 0 \\ -12 & 18 & -6 \\ 0 & -6 & 6 \end{bmatrix}$$

$$[L]^{-1} = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 5 & 5 \\ 2 & 5 & 11 \end{bmatrix}$$

$$[M] = 10^6\begin{bmatrix} 2 & 2 & 2 \\ 2 & 5 & 5 \\ 2 & 5 & 11 \end{bmatrix}$$

We choose

$$[X] = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

for simplicity. Therefore

$$[M][X] = 10^6\begin{bmatrix} 2 & 2 & 2 \\ 2 & 5 & 5 \\ 2 & 5 & 11 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 10^6\begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix} = 18 \times 10^6\begin{bmatrix} 1/3 \\ 2/3 \\ 1 \end{bmatrix}$$

$$[M]^2[X] = 18 \times 10^{12}\begin{bmatrix} 2 & 2 & 2 \\ 2 & 5 & 5 \\ 2 & 5 & 11 \end{bmatrix}\begin{bmatrix} 1/3 \\ 2/3 \\ 1 \end{bmatrix} = (18)(15)(10^{12})\begin{bmatrix} 4/15 \\ 9/15 \\ 1 \end{bmatrix}$$

It may be noted that since we are only interested in the ratio of successive multiplications we may dispense with the numerical factors in front of the matrix. Continuing the process we soon reach:

$$\begin{bmatrix} 4/15 \\ 9/15 \\ 1 \end{bmatrix}\begin{bmatrix} 2 & 2 & 2 \\ 2 & 5 & 5 \\ 2 & 5 & 11 \end{bmatrix} = 14.533\begin{bmatrix} 0.256{,}881 \\ 0.584{,}762 \\ 1 \end{bmatrix}$$

and

$$\begin{bmatrix} 2 & 2 & 2 \\ 2 & 5 & 5 \\ 2 & 5 & 11 \end{bmatrix}\begin{bmatrix} 0.256{,}881 \\ 0.584{,}762 \\ 1 \end{bmatrix} = 14.4495\begin{bmatrix} 0.255{,}238 \\ 0.584{,}762 \\ 1 \end{bmatrix}$$

The ratio is settling down to a number nearly equal to 14.45. By continuing further it is found that the value of $\mu_1 = 14.431 \times 10^6$ and the ratios of the amplitudes of the currents for the fundamental oscillation are about 0.255, 0.584, and 1.000 as given by the final matrix. A similar mechanical problem is discussed by Duncan and Collar in the above-mentioned paper in the *Philosophical Magazine*.

It is seen that the convergency of the process is very high and the calculations are easily made with a comptometer. The results are

$$\mu_1 = 14.431(10^6)$$

hence

$$f_1 = 1/2\pi\sqrt{\mu_1} = 607 \text{ cycles per second}$$

This is the fundamental frequency. When the system is oscillating at the fundamental frequency the ratios of the currents are $i_1 = 0.225$, $i_2 = 0.584$, and $i_3 = 1.000$; that is, with respect to i_3 as unity the other currents have the above values.

For a more complete discussion of this method in similar mechanical problems the reader is referred to "Solution of Oscillation Problems by Matrices" by Duncan and Collar. In this paper a method of obtaining the higher resonant frequencies is also given. The author is working on some more applications of the method.

It must be noted that the method here given while it avoids the direct solution of the detrimental equation makes use of a process which is related to the well-known root-squaring method of Graeffe. Graeffe's method has been shown to be equivalent to the method of Daniel Bernoulli. An excellent discussion and an extension of Bernoulli's method which may be reduced to the method here presented may be found in A. C. Aitken's paper "On Bernoulli's Numerical Solution of Algebraic Equations" Proceedings of the Royal Society, Edinburgh, volume 46, page 289, 1926.

X. Conclusion

It has been the purpose of this paper to show the inherent simplicity and power of the matrix method once the fundamental operations have been grasped.

The theory of matrices as presented in part II has been the result of searching the international literature and it is hoped that the concise presentation may give an impetus to the use of this method. The author is at present attacking the problem of simultaneous partial differential equations by this method and the results thus far show that the matrix method will be more powerful in this connection (See reference 9).

XI. Bibliography

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The Next 50 Years

IN THE 50-year period before us there will be an accelerated advance in the accomplishments of engineering, which, broadly speaking, is the application of science. New discoveries in science, and their use by means of new inventions, will follow one another with ever increasing rapidity. Often one discovery will reveal a score or more of new opportunities which speedily will be developed and made useful to man.

This acceleration in the speed of engineering advancement will be due in part to the increase in the number of workers in this field, made available through the competition of engineering colleges to gain students, and especially through the growing conviction among parents, whether well-founded or not, that an engineering education carries with it an improved economic and social status. The public has become "engineering minded."

No one can foretell with accuracy what specific results will ensue from this intensified development along engineering lines; but a few accomplishments are clearly evident:

1. Space, already so greatly reduced, will, practically speaking, be well nigh eliminated by the conquest of the air.

2. Mass-production will be extended to far more industries.

3. Agriculture will yield more and more to combination-control and mechanization. With the aid of new methods of processing and the discovery of synthetic foods, the necessary agricultural products for a unit of population will be provided with a fraction of the labor and land now used.

4. Automatic control of production and operation will further greatly reduce the need for manual control.

5. New scientific discoveries, new sources of power, and the development of new processes and inventions, will make necessities and many luxuries more generally available.

Such changes will inevitably shorten the work-time required to satisfy the demands of the public. In Canada and the States it has been reduced $^1/_3$ during the last 50 years. Is it not probable that an equal reduction will result in the next 50 years? If so, the required work-time will become only half what it is at present. Probably 20 hours a week is a conservative estimate.

Whatever this reduction may be, it will result largely from the activities of the engineer. Credit for lightening the human burden, however, will be accompanied by liability for any unfortunate results which may ensue, and the public will look to the engineer to provide the means to avoid them.

From an address by the late Harrison P. Eddy, past-president of the American Society of Civil Engineers, prepared for presentation at the recent semicentennial celebration of the Engineering Institute of Canada, upon which occasion Mr. Eddy was to have been made an honorary member of that society. The address was posthumously delivered at a luncheon meeting on June 16, and was published in the August 1937 issue of *Civil Engineering*, pages 545–6.

Short-Circuit Protection of Distribution Networks by the Use of Limiters

By C. P. XENIS
MEMBER AIEE

A LARGE PERCENTAGE of secondary cable faults in distribution networks "burn clear" rapidly. In such cases, the short-circuit current to the fault has not persisted for a sufficiently long period of time to raise the temperature of the cable conductor on either side of the fault to a high enough temperature to fuse, or permanently damage adjacent cable insulation. However, not even the most optimistic treatises on "burnoff" have ever asserted that all secondary cable faults are of this accommodating type. Furthermore, the fault consists of an electric arc between a number of energized copper conductors confined in the small space of the duct, surrounded by masses of molten metal and with water frequently present. Under such a combination of conditions, who can contend that the fault can be depended upon to always clear in time to protect the insulation of cables supplying the fault current?

Companies operating extensive distribution networks have experienced cases of cable failure "spreading" beyond the initial point of fault, and the prevention of extensive damage in such cases depends on the prompt arrival of the operating crews who cut network cables "around" the faulted area. This type of fault is frequently accompanied by fires and explosions for apparent reasons.

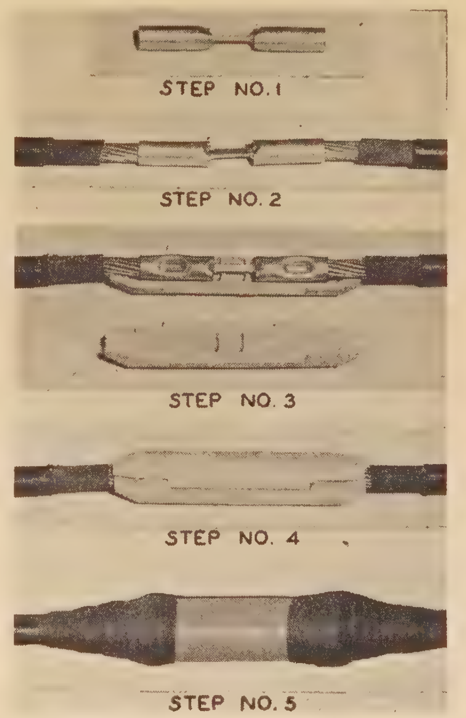
This problem is not new, and the development of noncombustible cable insulation represents one angle of attack. The development of cable joining connectors, lugs, etc., which incorporate fusible elements, so that in addition to their normal splicing functions, they provide adequate fuse protection to the cables to which they are attached, represents another angle of attack to the problem. Undoubtedly other solutions will present themselves in the future.

The Limiter

The limiter consists of a reduced metallic section incorporated in a connector, lug, or other connecting device used in normal cable installation procedure. It has a time-current characteristic calculated to protect the cable to which it is connected over a wide range of short-circuit currents, by interrupting the faulty circuit before the cable insulation reaches the "roasting" point; in other words, the limiter selects with cable insulation. It is so designed as to form an integral part of the joint or cable terminal without the necessity of the additional enclosures, terminal blocks, packing glands, etc. that conventional underground fusing systems require. In short, it is a fuse made into an inconspicuous part of the cable it is intended to protect.

Figure 1 step number 1 shows a connector-type limiter used for protecting number 4/0 nonleaded cables on the distribution systems of the Consolidated Edison Company of New York, Incorporated. This limiter is made from a single piece of copper tubing by flattening the middle portion of the tube and punching out the excess metal, thus leaving the desired fusible section. Step number 2 shows the ends of the number 4/0 cables inserted in the tubular ends of the limiter, while step number 3 shows the assembly after the tubular ends are indented to form a permanent connection between the cables. Step number 4 shows the fireproofing shells assembled around the limiter, while step number 5 shows the limiter installation completed by covering the assembly with a rubber sleeve and taping the ends to the cable insulation. Figure 2 gives the corresponding steps taken in installing a limiter lug on the end of a 500,000-circular-mil nonleaded cable. Note that in this case no rubber covering is provided over the fireproofing shells on the lug since it is used for indoor installations. The limiter lug is also constructed from a single piece of

Figure 1. Installation of connector type limiter



A paper presented at the Edison Electric Institute transmission and distribution committee meeting, Chicago, Ill., May 5, 1937, and recommended for publication by the AIEE committee on power transmission and distribution. Manuscript submitted May 21, 1937; released for publication July 9, 1937.

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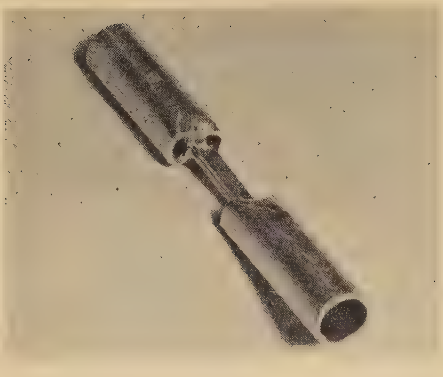

Figure 1A. Connector type limiter

copper tubing, a portion of which has been flattened, bent, and punched out to form the lug and fusible section.

The fireproofing shell (figure 3) plays an important part in the proper operation of the limiter by providing the means of confining the arc developed during the interruption of the circuit. It is molded from a noncombustible dry mixture of asbestos and cement. Note the barriers which further tend to confine the arc and keep it from striking across the large cylindrical portions of the limiter. With the limiter assembly as described, it is possible to interrupt short-circuit currents met in practice without any external indication of arc or flame.

In connecting the above limiters to cables, it should be noted that compression-type mechanical connections were used. Various types of powder driven, hydraulically operated, and screw-type devices are now available which make satisfactory connections. Solder must be avoided in the proximity of limiters because of its low melting point. While in the above illustrations we have indicated the use of limiters on nonleaded cables, their application to rubber-insulated lead-covered cables is equally practical and lead sleeves may be installed over the limiters if desired. However, the use of limiters on paper-insulated lead-covered cables becomes relatively difficult since it is necessary to take additional steps to keep the impregnating oil from entering the fuse chamber of the limiter.

The use of limiters at multiple points of a network system results in the protection of each cable section between such points. The cable fault shown in figure 4 draws its supply of short-circuit current from 3 directions at each multiple point. However, the total short-circuit current from each multiple point must pass through the limiter which is connected to the faulty cable. In other words by installing limiters of the same type at each multiple point of a network system, the selection between limiters is automatically secured, and since the faulty section is equipped with limiters at both extremities, it will be isolated from the rest of the network. This presumes that the limiter is designed so that it will blow at the minimum values of short-circuit current encountered in that network.

Limiter Characteristics

The limiter, like any other type of fuse, has a time current characteristic which defines its performance at various overloads. From previous discussions, it is evident that the ideal characteristic for a limiter is one slightly faster than the insulation damage characteristic of the cable it is intended to protect. In other words, the limiters on the faulted section should fuse before the insulation of the cables feeding the short-circuit reaches a dangerous condition as the result of the total heat generated by all cables in the duct. It therefore follows that limiter design should be preceded by the determination of cable damage characteristics. This has been done experimentally using various rubber-insulated cables, for a single cable in open air and again for a set of main cables installed in concrete duct. Figures 5, 6, and 7 show time-temperature characteristics for various types of cable installations encountered in the network distribution systems of the Consolidated Edison Company of New York, Incorporated. It will be noticed that after the copper conductor reaches a temperature of approximately 265 degrees centigrade the rubber reaches the point of permanent visible damage. Although tests were made on cables insulated with 2 types of rubber base compounds, namely, intermediate rubber compound, ASTM performance rubber compound, some lead-covered, others not, it was found that the slight differences in the results obtained did not justify separate treatment in this report and therefore one set of curves have been plotted. It will also be noted that in cases of rapid temperature rise in the copper conductor, somewhat higher copper temperatures are necessary to cause the same extent of damage to cable insulation. This is the result of the lag in the temperature

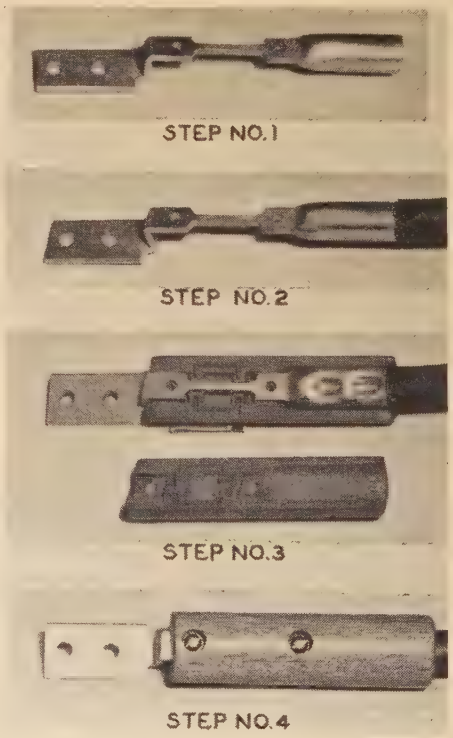

Figure 2. Installation of lug type limiter

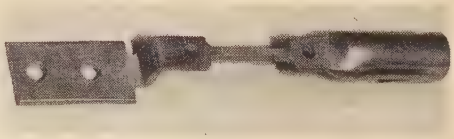

Figure 2A. Lug type limiter

rise of the insulation behind the rise of copper temperature. Figure 8 summarizes the cable damage characteristics for rubber insulation in a more convenient form for comparison with limiter characteristics.

In figures 9 and 10 we have shown cable damage characteristics together with the characteristics of limiters of types illustrated in the first part of this paper. The intersection of the limiter characteristic with the cable damage characteristic indicates the value of current below which the limiter offers no protection to cable insulation. However, if this critical current is below the minimum value of short-circuit current obtainable in the network, this limitation loses all practical importance. A wide range of limiter characteristics is possible by varying the cross section and the length of the fusible section. This phase of the limiter design problem is not much different than that of general fuse link design and therefore, no useful purpose can be served here by going into detail. On figure 9, 2 limiter characteristics have been shown to illustrate possible variations in design.

An additional point to be kept in mind in selecting or designing limiters is that the limiter generates heat at a higher rate than the cable conductor and therefore some of this heat is conducted to the cable to which it is attached. This presents a problem only for currents near the minimum at which the limiter is designed to fuse. Higher currents fuse the limiter before the heat generated has time to affect the adjacent cable insulation. Substantially lower currents (than the minimum fusing values for the limiter) do not generate a sufficient amount of heat to affect the adjacent cable insulation. It is therefore desirable that this particular point be investigated as the last step of design to insure that no overheating of the cable insulation adjacent to the limiter results from heat conducted from the limiter. By judicious change of the length and cross section of the fusible section and by proper selection of the minimum blowing point of the limiter, this condition can be overcome. In this connection the fact that the limiter and adjacent cable insulation are located in the manhole where ambient temperatures are substantially lower than those in the duct in which the overloaded cables are located, assists in arriving at the desired thermal balance. At rated loads the temperature of the fusible section (located within the asbestos shell) is approximately 5 degrees centigrade higher than that of the cable conductor.

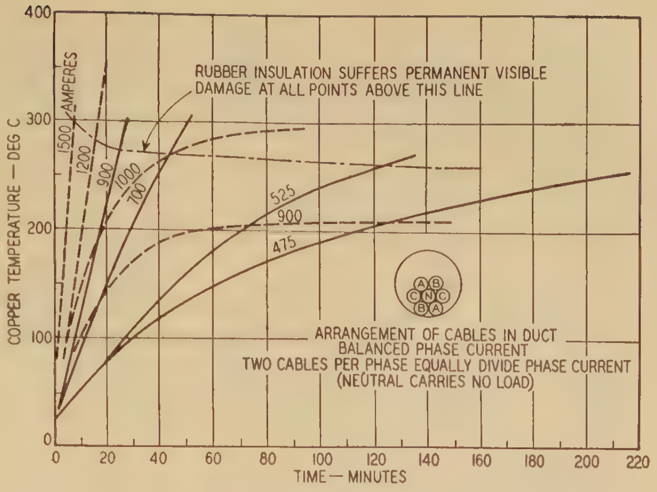

Figure 5. Time-temperature characteristics, rubber-insulated cables

Solid curves—Six number 4/0 cables in duct
Dashed curves—One number 4/0 cable in open air

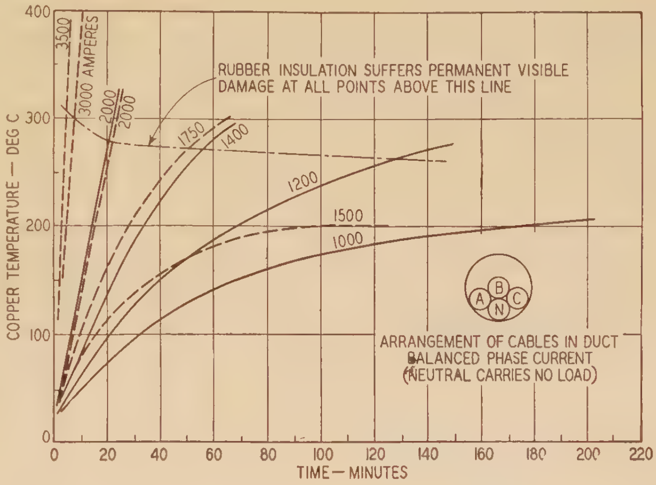

Figure 6. Time-temperature characteristics, rubber-insulated cables

Solid curves—Three 500,000-circular-mil cables in duct
Dashed curves—One 500,000-circular-mil cable in open air

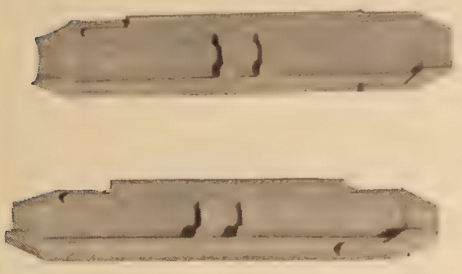

Figure 3. Fireproofing shells for connector type limiter

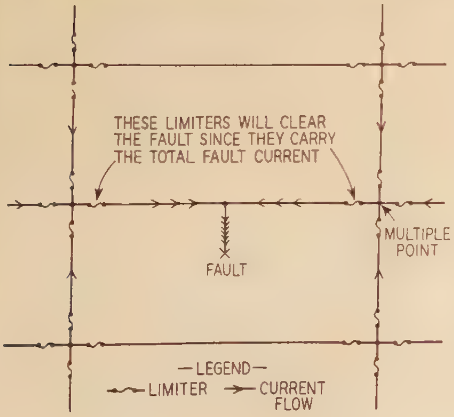
THESE LIMITERS WILL CLEAR THE FAULT SINCE THEY CARRY THE TOTAL FAULT CURRENT
FAULT
MULTIPLE POINT
— LEGEND —
LIMITER
CURRENT FLOW

Figure 4. Schematic current distribution diagram under condition of a secondary network fault

Note: All limiters in this network are of the same type and characteristic

With the aid of the above analysis and reference to the data presented in curve form, it is possible to design limiters which will properly select with the cable insulation. In the a-c distribution system of the Consolidated Edison Company of New York, Incorporated, limiter type *AC-2R* has been used extensively for protecting number 4/0 nonleaded network cables in Manhattan and the

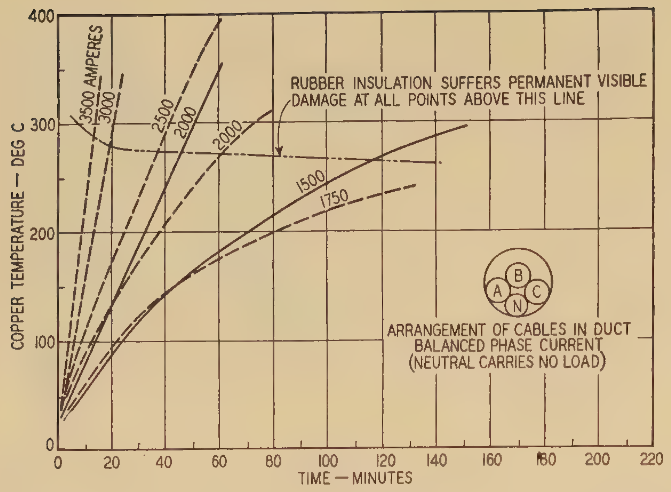

Figure 7. Time-temperature characteristics, rubber-insulated cables

Solid curves—Three 750,000-circular-mil cables in duct
Dashed curves—One 750,000-circular-mil cable in open air

Bronx. Their use is now being extended to other sizes of cables and at other points of the distribution networks.

Selectivity Between Limiters and Network Protector Fuses

Our discussion heretofore has been confined to the proper protection of network cables by means of limiters installed at multiple points. Where the multiple point happens to be also a supply point, 2 additional considerations for the proper design of the system must receive attention:

(*a*) The supply cables (connecting the network protector to the street mains) may be protected by means of limiters.

(*b*) The network protector fuse should properly select with the combination of limiters on the supply cables and those on the network mains at the multiple point.

With reference to consideration (*a*) the desirability of protecting supply cables varies with the type of installation. If, for example, the supply cables represent a very short run between a network protector installed in an underground manhole and the multiple point, the need for protecting the supply cables is questionable. If, however, the supply cables are of considerable length as in the case of transformer banks installed within customer's buildings, the protection of such supply cables and secondary ties is of considerable importance since a serious case of cable failure within the customer's premises is liable to entail unpleasant consequences. This protection can be accomplished in accordance with the methods previously explained. Where the cable terminates at a bus, as would be the case where secondary ties connect to the customer's bus, limiter lugs are used for the purpose. Figure 11 is a schematic diagram representing a network completely equipped with limiters at multiple points and with supply cables protected at both extremities. Figure 12 shows a transformer vault installation with secondary ties, as well as cables connecting the transformer secondary side to the network switch, completely protected by means of limiters. This type of protection has been recently adopted by the Consolidated Edison Company of New York, Incorporated.

Figure 12 also serves to illustrate the problem of obtaining the proper selection between network protector fuse and associated limiters. In the event of a high-voltage feeder failure, if the network protector fails to open because of some defect, it is desirable to have the circuit interrupted by the network protector fuse because of the relative ease of replacement of the latter, in comparison with the replacement of limiters. Hence, it is necessary to consider the characteristic of the network protector fuse in conjunction with that of the associated limiters, taking into consideration the fact that a number

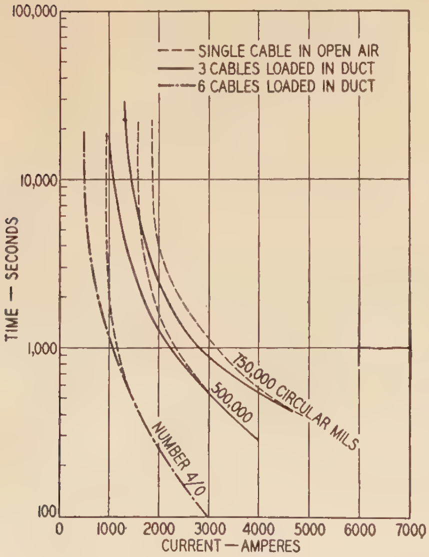

Figure 8. Cable insulation damage characteristics, rubber-insulated cables

Note: Visible damage occurs at points on and above each characteristic curve

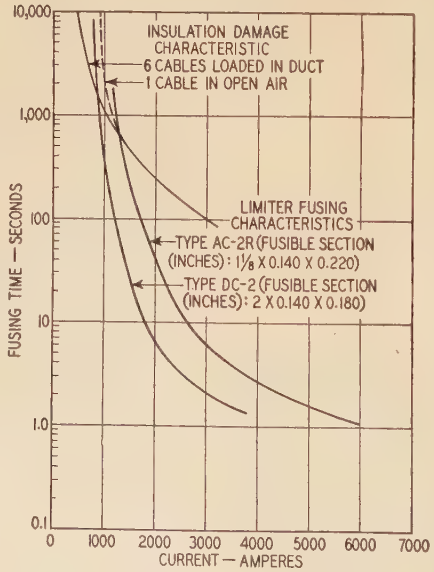

Figure 9. Time-current characteristic, 4/0 cable limiter

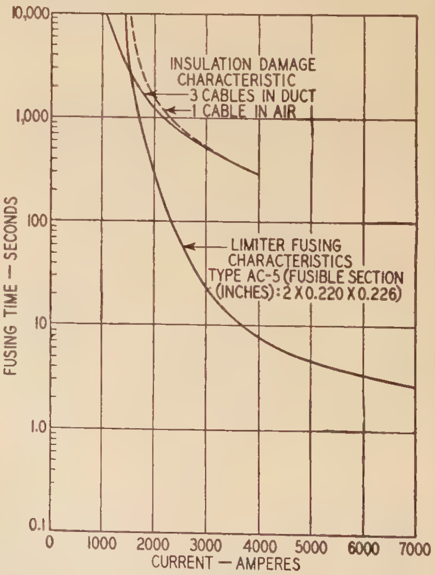

Figure 10. Time-current characteristic, 500,000-circular-mil cable limiter

of limiters in parallel (4 per phase in the case of figure 12) divide the total current flowing through each network protector fuse. Because of differences in length between the street ties and variations in the network layout, it is possible that in the case of backfeed the 4 street ties shown in figure 12 may not equally divide the total backfeed current. It has therefore been decided that the proper selection should be obtained even when one of the 4 secondary ties carries as much as 40 per cent of the total backfeed current. An unbalance of this magnitude is extremely unlikely to occur on the system. Figure 13 shows the characteristic of network protector fuse type *Z*-25 and that of limiter type *AC*-5 when carrying various percentages of the total fuse current. It will be noticed that even when a limiter on one of the secondary ties carries as much as 40 per cent of the total backfeed current, a comfortable margin of selectivity is obtained. The characteristic of the limiter considered on figure 13 (*AC*-5) is shown on figure 10. With a range of selection such as indicated above, the network protector fuse continues to be the weakest link in the chain of network protection.

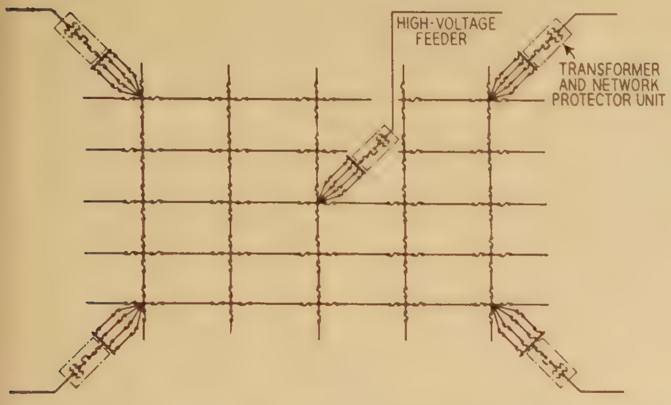

Figure 11. Schematic diagram of distribution network completely equipped with limiters

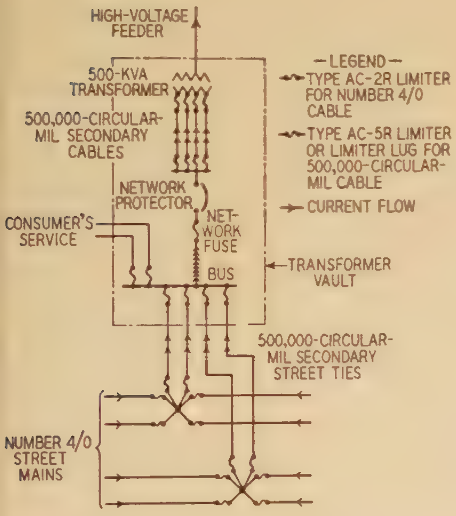
HIGH-VOLTAGE FEEDER
500-KVA TRANSFORMER
500,000-CIRCULAR-MIL SECONDARY CABLES
NETWORK PROTECTOR
CONSUMER'S SERVICE
NET-WORK FUSE
BUS
—LEGEND—
TYPE AC-2R LIMITER FOR NUMBER 4/0 CABLE
TYPE AC-5R LIMITER OR LIMITER LUG FOR 500,000-CIRCULAR-MIL CABLE
CURRENT FLOW
TRANSFORMER VAULT
500,000-CIRCULAR-MIL SECONDARY STREET TIES
NUMBER 4/0 STREET MAINS

Figure 12. Schematic current distribution diagram under condition of backfeed

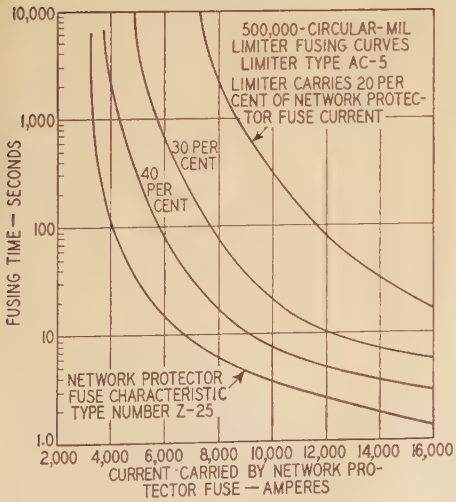

Figure 13. Selectivity between network protector fuse and associated limiters

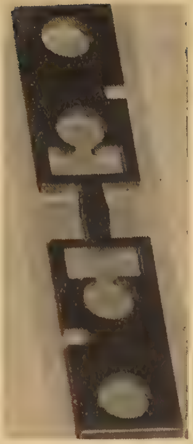

Figure 14 (above). Type Z network protector fuse

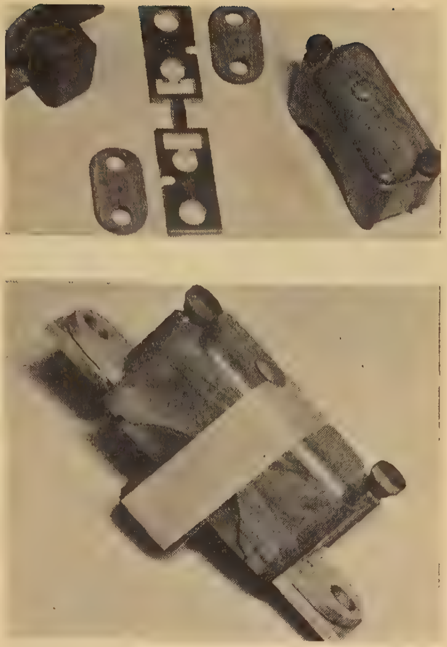

Figure 15 (right). Details of type Z network protector fuse and enclosure (top) and assembly of type Z network protector fuse and enclosure (bottom)

Since the network protector fuse is in series with a group of limiters which are in parallel, the 2 types of devices must necessarily possess time-current characteristics of the same general shape. Hence it was decided to review network fuse design and bring it in line with limiter design so as to obtain the desired selection. The result of this work was a copper fuse, illustrated on figure 14, which provides proper selection with limiters, protects the transformer over a broader range of overload currents than other conventional network protector fuses, and is of a type that can be readily punched out of flat sheet copper. An individual fireproof arc-suppressing enclosure (see figure 15) is provided with each fuse. These enclosures serve to protect men who may happen to be standing in front of an open network protector when the fuse blows, and to prevent damage to equipment and communication of trouble. These fuse assemblies have successfully interrupted currents of the order of 30,000 amperes at full restoration voltage with no restriking, and with the arc sufficiently confined to protect workmen in the immediate vicinity. The fuse characteristic on figure 13 is that of a fuse of this type, and of the size used with a 500-kva bank.

The development outlined in this paper may be briefly summarized as an attempt to revise cable splicing procedure so as to incorporate fuse protection for the large amounts of cable insulation exposed to high copper temperature in the vicinity of short-circuited distribution-network mains. With such a system, those faults which "burn clear" in less time than that required to clear the limiter (see limiter characteristic) will continue to "burn clear" as heretofore. The faults which do not "burn clear" very readily are the ones that endanger insulation on cables supplying the fault current. Such faults will be cleared by limiter action.

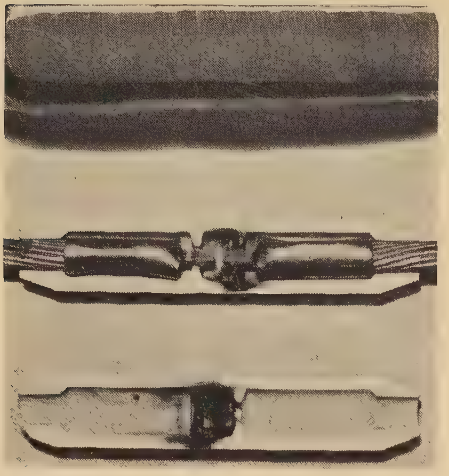

Figure 16. Kerite and friction tape covering (top) and limiter fusing with separation of 1/4 inch (bottom)

The selection of the proper limiter for any distribution system should be based on a study of conditions obtaining in that system, with particular reference to (*a*) insulation damage characteristic of network cables, (*b*) minimum short circuit current, (*c*) selection with network protector fuses. Standardization in limiter design must necessarily be preceded by standardization in distribution network design.

The Consolidated Edison Company of New York, Incorporated, has installed 16,000 limiters in the past year. During this period there have been 2 cases of faults on cables equipped with limiters (35th Street and Sixth Avenue and 40th Street and Sixth Avenue). Both cases were caused by cables being "bull-pointed" by subway construction contractors. In each case the limiters properly cleared and protected the cables to which they were attached. Figure 16 shows a limiter which has blown in the field and cleared a short-circuit.

Pooling Power in a Large Industrial Center

A PROBLEM of The Detroit (Mich.) Edison Company incident to the provision of reserve capacity against the exigencies of power plant operation and against outages occasioned by routine maintenance, and the provision of excess capacity to meet the unpredictable and sudden demands of a vigorously expanding industrial community, was discussed in a paper presented at the semiannual meeting of The American Society of Mechanical Engineers, Detroit, Mich., May 17–21, 1937, by J. W. Parker and R. E. Greene (A'26) respectively, vice-president and member of the engineering division of the company. The paper was published in *Mechanical Engineering* for May 1937, pages 335–9.

Diversity in time of use constitutes the major advantage in pooling the power requirements of any community, inasmuch as the peak load of residential and commercial users does not occur at the same hour as the peak industrial load. If each wholesale user of power operated its own isolated power plant, the installed capacity required would be much greater than is needed if all plants in an area are interconnected. Reserve capacity in a large system can be less than the aggregate reserve capacity in a series of isolated plants for the same load, where one spare unit may be a relatively large part of the total capacity.

In the Detroit system, reserve capacity in machines running and on the line is maintained at least equal to the capacity of the largest machine on the line. In case a machine must be shut down in an emergency, the running reserve should be restored at once, and, therefore, a second large machine is kept standing ready for an immediate start.

Pooling of power resources benefits the community because freedom is allowed in the choice of sites for manufacturing establishments, inasmuch as large quantities of water for condensers may be obtained by locating central stations on rivers, rather than requiring all manufacturers to build plants on the rivers. Such concentration in large stations, however, requires considerable investment in a transmission and distribution system, amounting in the Detroit system to approximately 56.6 per cent of the total property. The territory served is divided into 4 areas, each with a major generating station. Interconnection may be described as "loose linked," for limited-capacity ties are provided which may or may not normally carry load but which are sufficient in capacity to compensate for any losses in generating capacity which can be reasonably anticipated. If an uncontrollable fault develops in one area, the remainder of the system cuts clear.

The authors conclude that: "With all its advantages of both reliability and economy, the central system cannot, under certain circumstances, compete with certain types of load. Combining electric generation with a supply of steam required in some industrial process, produces power as a by-product at a much lower cost than can be attained by the best of condensing turbines. Seldom, however, does the requirement of steam for process work and building heating coincide with the manufacturer's own use of power. Paralleling such by-product generation with the central system may be, and often is, good economy rather than providing spare capacity against breakdown or being compelled to generate power under costly conditions in periods when no corresponding demand for steam exists. In a sense, the central system becomes a broker ready to sell or buy power as the circumstances of the moment may dictate."

Extension of 2-Reaction Theory to Multiphase Synchronous Machines

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An extension of the 2-reaction theory of synchronous machines as developed by R. H. Park is given in this paper for machines with any number of phases.

At THE 1929 AIEE winter convention, Park[1] and Ku[2] presented simultaneously 2 papers on the analysis of 3-phase synchronous machines. While Ku's method was based on Heaviside's operational calculus and Stokvis-Fortescue's symmetrical components, Park generalized the work of Blondel, Dreyfus, Doherty, and Nickle and gave a complete analysis of the salient-pole machine with any number of rotor circuits. Ku's work has since been extended to the salient-pole case with inductive and capacitive load[3] and also to the asynchronous operation[4] of synchronous machines. The 2-reaction theory has been further extended by Park[5] to the consideration of synchronizing and damping torque during continuous and transient oscillations. The 2-reaction operational impedances of machines with field collars are given by Waring and Crary,[6] and, more recently, the effect of condenser load is considered by Crary.[7] The present paper is a further extension of the 2-reaction theory as developed by Park to the general case of n-phase salient-pole synchronous machines. It may be pointed out that the 2-reaction components can also be simply related to the symmetrical components[8,9] and a similar extension based on operational calculus and symmetrical components[10] may serve as a good comparison between the 2 methods of attack.

Fundamental Circuit Equations

Consider an ideal synchronous machine with n phases, and let

$i_A, i_B, i_C \ldots i_N$ = per-unit instantaneous phase currents
$e_A, e_B, e_C \ldots e_N$ = per-unit instantaneous phase voltages
$\psi_A, \psi_B, \psi_C \ldots \psi_N$ = per-unit instantaneous phase linkages
t = time in electrical radians
p = d/dt

Then there is

$$\left.\begin{aligned} e_A &= p\psi_A - ri_A \\ e_B &= p\psi_B - ri_B \\ e_C &= p\psi_C - ri_C \\ &\ldots\ldots \\ e_N &= p\psi_N - ri_N \end{aligned}\right\} \qquad (1)$$

The phase linkages of an n-phase salient-pole synchronous machine may be given as follows (see appendix I):

$$\begin{aligned} \psi_A = {} & I_d\cos\theta - I_q\sin\theta - x_0 i_A + \left[x_0 - \left(\frac{x_d + x_q}{2}\right)\right]\left(\frac{2}{n}\right) \times \\ & [i_A + i_B\cos\theta_0 + i_C\cos 2\theta_0 + \ldots + i_N\cos\{(n-1)\theta_0\}] - \\ & \left(\frac{x_d - x_q}{2}\right)\left(\frac{2}{n}\right)[i_A\cos 2\theta + i_B\cos(2\theta - \theta_0) + \\ & i_C\cos(2\theta - 2\theta_0) + \ldots + i_N\cos\{2\theta - (n-1)\theta_0\}] \\ \psi_B = {} & I_d\cos(\theta - \theta_0) - I_q\sin(\theta - \theta_0) - x_0 i_B + \\ & \left[x_0 - \left(\frac{x_d + x_q}{2}\right)\right]\left(\frac{2}{n}\right)\Big[i_A\cos\theta_0 + i_B + \\ & i_C\cos\theta_0 + \ldots + i_N\cos(n-2)\theta_0\Big] - \left(\frac{x_d - x_q}{2}\right)\left(\frac{2}{n}\right) \times \\ & [i_A\cos(2\theta - \theta_0) + i_B\cos(2\theta - 2\theta_0) + \\ & i_C\cos(2\theta - 3\theta_0) + \ldots + i_N\cos(2\theta)] \\ & \ldots\ldots\ldots\ldots\ldots\ldots \\ \psi_N = {} & I_d\cos\{\theta - (n-1)\theta_0\} - I_q\sin\{\theta - (n-1)\theta_0\} - \\ & x_0 i_N + \left[x_0 - \left(\frac{x_d + x_q}{2}\right)\right]\left(\frac{2}{n}\right)[i_A\cos(n-1)\theta_0 + \\ & i_B\cos(n-2)\theta_0 + i_C\cos(n-3)\theta_0 + \ldots + i_N] - \\ & \left(\frac{x_d - x_q}{2}\right)\left(\frac{2}{n}\right)[i_A\cos\{2\theta - (n-1)\theta_0\} + i_B\cos 2\theta + \\ & i_C\cos\{2\theta - (n+1)\theta_0\} + \ldots + i_N\cos\{2\theta - (n-2)\theta_0\}] \qquad (2) \end{aligned}$$

where θ = position of rotor axis in electrical radians measured from the axis of phase A, and $\theta_0 = 2\pi/n$. Let the transformations from the phase quantities to 2-reaction components be given by the following (see appendix II):

$$\begin{aligned} f_d = {} & \frac{2}{n}[f_A\cos\theta + f_B\cos(\theta - \theta_0) + f_C\cos(\theta - 2\theta_0) + \ldots + \\ & f_N\cos\{\theta - (n-1)\theta_0\}] \qquad (3) \end{aligned}$$

$$\begin{aligned} f_q = {} & -\frac{2}{n}[f_A\sin\theta + f_B\sin(\theta - \theta_0) + \\ & f_C\sin(\theta - 2\theta_0) + \ldots + f_N\cos\{\theta - (n-1)\theta_0\}] \qquad (4) \end{aligned}$$

where f may stand for e, i, or ψ. It may be verified that

$$\psi_d = I_d - x_d i_d \qquad (5)$$

$$\psi_q = I_q - x_q i_q \qquad (6)$$

From previous work of Park,[1] Waring and Crary,[6] and others,[9] we know that I_d can be expressed in terms of E_d and i_d, and, similarly, I_q can be expressed in terms of E_q and i_q. Hence by extending to the n-phase case, we have

$$\psi_d = G_d(p)E_d - x_d(p)i_d \qquad (5a)$$

$$\psi_q = G_q(p)E_q - x_q(p)i_q \qquad (6a)$$

Let also

$$f_0 = \frac{1}{n}[f_A + f_B + f_C + \ldots + f_N] \qquad (7)$$

Then

$$\psi_0 = -x_0 i_0 \qquad (8)$$

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1. For all numbered references, see list at end of paper.

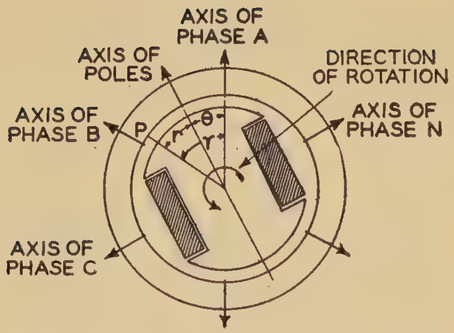

Figure 1

P IS ANY POINT ON ARMATURE SURFACE

For the transformation of other variables, we have in general,

$$f_{u(k)} = \frac{2}{n} [f_A + f_B \cos (k+1)\theta_0 + f_C \cos \{2(k+1)\theta_0\} + \ldots + f_N \cos (k+1)\theta_0] \quad (9)$$

$$f_{v(k)} = \frac{2}{n} [0 + f_B \sin (k+1)\theta_0 + f_C \sin \{2(k+1)\theta_0\} + \ldots - f_N \sin (k+1)\theta_0] \quad (10)$$

where (k) has the values from one to $(n-3)/2$ for n odd and to $(n-4)/2$ for n even. For n even, we have also

$$f_w = \frac{1}{n} [f_A - f_B + f_C - \ldots - f_N] \quad (11)$$

Then it can be shown that

$$\psi_{u(k)} = -x_0 i_{u(k)} \quad (12)$$

$$\psi_{v(k)} = -x_0 i_{v(k)} \quad (13)$$

$$\psi_w = -x_0 i_w \quad (14)$$

The equations for the phase voltages can now be transformed to the following:

$$e_d = p\psi_d - ri_d - \psi_q p\theta \quad (15)$$

$$e_q = p\psi_d - ri_q + \psi_d p\theta \quad (16)$$

$$e_0 = p\psi_0 - ri_0 \quad (17)$$

$$e_{u(k)} = p\psi_{u(k)} - ri_{u(k)} \quad (18)$$

$$e_{v(k)} = p\psi_{v(k)} - ri_{v(k)} \quad (19)$$

$$e_w = p\psi_w - ri_w \quad (20)$$

Notice that since ψ_d and ψ_q only involve the current components i_d and i_q, i_d and i_q can be obtained by solving 15 and 16 simultaneously. Equations 17 to 20 are even simpler as linkages are functions of their own current components, hence each equation only contains one current component corresponding to the voltage component, and the common impedance function is $z_0(p) = r + x_0 p$.

The reverse transformations from the "2-reaction" components to the phase quantities are as follows:

$$f_A = f_d \cos\theta - f_q \sin\theta + f_{u(1)} + 0 + \ldots + f_w$$

$$f_B = f_d \cos(\theta - \theta_0) - f_q \sin(\theta - \theta_0) + f_{u(1)} \cos 2\theta_0 + f_{v(1)} \times \sin 2\theta_0 + \ldots - f_w$$

$$f_C = f_d \cos(\theta - 2\theta_0) - f_q \sin(\theta - 2\theta_0) + f_{u(1)} \cos 4\theta_0 + f_{v(1)} \sin 4\theta_0 + \ldots + f_w$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$f_N = f_d \cos(\theta + \theta_0) - f_q \sin(\theta + \theta_0) + f_{u(1)} \cos 2\theta_0 - f_{v(1)} \sin 2\theta_0 \quad (21)$$

Appendix I. Armature Flux Linkages

The machine will be assumed to be "ideal" as defined by Park.[11] The no-load per-unit space fundamental flux density at the armature surface will be (see figure 1)

$$\beta_0 = I_d \cos\Gamma + I_q \sin\Gamma \quad (1)$$

where

Γ = angle along the armature surface measured in electrical radians from the rotor axis

I_d = per-unit rotor excitation or rotor current in direct axis

I_q = per-unit rotor excitation or rotor current in quadrature axis magnetizing ahead of the poles

The per-unit magnetomotive force due to armature phase currents $i_A, i_B, i_C \ldots i_N$ will be

$$A = -\frac{2}{n} [i_A \cos(\Gamma + \theta) + i_B \cos(\Gamma + \theta - \theta_0) + i_C \cos(\Gamma + \theta - 2\theta_0) + \ldots + i_N \cos(\Gamma + \theta + \theta_0)]$$

$$= -\frac{2}{n} \cos\Gamma [i_A \cos\theta + i_B \cos(\theta - \theta_0) + i_C \cos(\theta - 2\theta_0) + \ldots + i_N \cos(\theta + \theta_0)]$$

$$+ \frac{2}{n} \sin\Gamma [i_A \sin\theta + i_B \sin(\theta - \theta_0) + i_C \sin(\theta - 2\theta_0) \times + \ldots + i_N \sin(\theta + \theta_0)] \quad (2)$$

where θ is the position of rotor axis measured from the axis of phase A, $\theta_0 = 2\pi/n$, n = number of phases.

As the armature linkages for all space harmonics are assumed to be zero for the ideal machine, we have for the wave of per-unit flux,

$$\beta = I_d \cos\Gamma + I_q \sin\Gamma - \frac{2}{n} p_d \cos\Gamma [i_A \cos\theta + i_B \cos(\theta - \theta_0) + i_C \cos(\theta - 2\theta_0) + \ldots + i_N \cos(\theta + \theta_0)] + \frac{2}{n} p_q \sin\Gamma [i_A \sin\theta + i_B \sin(\theta - \theta_0) + i_C \sin(\theta - 2\theta_0) + \ldots + i_N \cos(\theta + \theta_0)] \quad (3)$$

The per-unit linkages in phase A due to the flux are equal to the value of β at $\gamma = \Gamma + \theta = 0$ or $\Gamma = -\theta$, i.e.,

$$\psi_{A(\beta)} = I_d \cos\theta - I_q \sin\theta - \frac{p_d + p_q}{n} [i_A + i_B \cos\theta_0 + i_C \cos 2\theta_0 + \ldots i_N \cos\theta_0] - \frac{p_d - p_q}{n} [i_A \cos 2\theta + i_B \cos(2\theta - \theta_0) + i_C \cos(2\theta - 2\theta_0) + \ldots + i_N \cos(2\theta + \theta_0)] \quad (4)$$

The total per-unit linkages in phase A are then

$$\psi_A = I_d \cos\theta - I_q \sin\theta - x_{l\sigma} i_A - x_m' [i_B \cos\theta_0 + i_C \cos 2\theta_0 + \ldots + i_N \cos\theta_0] - \frac{p_d + p_q}{n} [i_A + i_B \cos\theta_0 + i_C \cos 2\theta_0 + \ldots + i_N \cos\theta_0] - \frac{p_d - p_q}{n} [i_A \cos 2\theta + i_B \cos(2\theta - \theta_0) + i_C \cos(2\theta - 2\theta_0) \times + \ldots + i_N \cos(2\theta + \theta_0)] \quad (5)$$

where

$x_{l\sigma}$ = per-unit self leakage reactance of any phase
x_m' = maximum possible value of per-unit mutual leakage reactance between the phases. When n is even, this is equal to the actual value between diametrically opposite phases. For 3-phase case, this is equal to twice the actual value between any 2 phases

Under balanced conditions of operation with normal armature current flowing, there is

$$\begin{aligned} i_A &= \cos t \\ i_B &= \cos (t - \theta_0) \\ i_C &= \cos (t - 2\theta_0) \\ &\ldots\ldots\ldots \\ i_N &= \cos (t + \theta_0) \\ \theta &= t + t_0 \end{aligned}$$

where t_0 = the value of θ at $t = 0$.
Then, for $I_d = I_q = 0$ there is

$$\psi_A = - \left[x_{l\sigma} + x_m'\left(\frac{n}{2} - 1\right) + \frac{p_d + p_q}{2}\right] \cos t - \left[\frac{p_d - p_q}{2}\right] \cos (t + 2t_0) \quad (6)$$

When $t_0 = 0$ there is

$$\psi_A = - (x_{l\sigma} + x_m + p_d) \cos t = - x_d \cos t \quad (7)$$

where

$$x_m = x_m' \left(\frac{n}{2} - 1\right)$$

x_d = direct synchronous reactance[11,12]

When $t_0 = \pi/2$ there is

$$\psi_A = - (x_{l\sigma} + x_m + p_q) \cos t = - x_q \cos t \quad (8)$$

where x_q = quadrature synchronous reactance

With zero phase-sequence currents impressed there is

$$i_A = i_B = i_C \ldots = i_N$$

$$\psi_A = -(x_{l\sigma} - x_m')i_A = - x_0 i_A \quad (9)$$

where x_0 = zero-sequence reactance

Thus we have obtained the following relations:

$$\begin{aligned} x_{l\sigma} + x_m + p_d &= x_d \\ x_{l\sigma} + x_m + p_q &= x_q \\ x_{l\sigma} - x_m' &= x_0 \\ p_d - p_q &= x_d - x_q \\ p_d + p_q + 2(x_{l\sigma} + x_m) &= x_d + x_q \end{aligned}$$

Equation 5 can now be rewritten in the form

$$\psi_A = I_d \cos \theta - I_q \sin \theta - x_0 i_A - \left[\left(\frac{x_d + x_q}{2}\right) - x_0\right] \times \left(\frac{2}{n}\right) [i_A + i_B \cos \theta_0 + i_C \cos 2\theta_0 + \ldots + i_N \cos \theta_0] - \left(\frac{x_d - x_q}{2}\right) \left(\frac{2}{n}\right) [i_A \cos 2\theta + i_B \cos (2\theta - \theta_0) + i_C \cos (2\theta - 2\theta_0) + \ldots + i_N \cos (2\theta + \theta_0)] \quad (10)$$

Similarly we can write down the equations for the other phases.

Appendix II. Transformation of Variables

The transformations between the phase quantities and the 2-reaction components will be given in the form of matrices as below:

$$C_d^A = \begin{array}{c|c|c|c|c|c|c|} \diagdown & A & B & C & D & \cdots\cdots & N \\ \hline 0 & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \cdots\cdots & \frac{1}{n} \\ \hline d & \frac{2}{n} \cos \theta & \frac{2}{n} \cos (\theta - \theta_0) & \frac{2}{n} \cos (\theta - 2\theta_0) & \frac{2}{n} \cos (\theta - 3\theta_0) & \cdots\cdots & \frac{2}{n} \cos (\theta + \theta_0) \\ \hline q & (-)\frac{2}{n} \sin \theta & (-)\frac{2}{n} \sin (\theta - \theta_0) & (-)\frac{2}{n} \sin (\theta - 2\theta_0) & (-)\frac{2}{n} \sin (\theta - 3\theta_0) & \cdots\cdots & (-)\frac{2}{n} \sin (\theta + \theta_0) \\ \hline u_1 & \frac{2}{n} & \frac{2}{n} \cos 2\theta_0 & \frac{2}{n} \cos 4\theta_0 & \frac{2}{n} \cos 6\theta_0 & \cdots\cdots & \frac{2}{n} \cos 2\theta_0 \\ \hline v_1 & 0 & \frac{2}{n} \sin 2\theta_0 & \frac{2}{n} \sin 4\theta_0 & \frac{2}{n} \sin 6\theta_0 & \cdots\cdots & (-)\frac{2}{n} \sin 2\theta_0 \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \cdots\cdots & \cdots\cdots \\ \hline w & \frac{1}{n} & -\frac{1}{n} & \frac{1}{n} & -\frac{1}{n} & \cdots\cdots & \cdots\cdots \\ \hline \end{array}$$

For example, from the first row, we have $i_0 = \frac{1}{n} (i_A + i_B + i_C + i_D + \ldots + i_N)$

$$C_A^d = \begin{array}{c|c|c|c|c|c|c|c|} \diagdown & 0 & d & q & u_1 & v_1 & \cdots\cdots & w \\ \hline A & 1 & \cos \theta & -\sin \theta & 1 & 0 & \cdots & 1 \\ \hline B & 1 & \cos (\theta - \theta_0) & -\sin (\theta - \theta_0) & \cos 2\theta_0 & \sin 2\theta_0 & \cdots\cdots & -1 \\ \hline C & 1 & \cos (\theta - 2\theta_0) & -\sin (\theta - 2\theta_0) & \cos 4\theta_0 & \sin 4\theta_0 & \cdots\cdots & 1 \\ \hline D & 1 & \cos (\theta - 3\theta_0) & -\sin (\theta - 3\theta_0) & \cos 6\theta_0 & \sin 6\theta_0 & \cdots\cdots & -1 \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots\cdots & \cdots \\ \hline N & 1 & \cos (\theta + \theta_0) & -\sin (\theta + \theta_0) & \cos 2\theta_0 & -\sin 2\theta_0 & \cdots\cdots & \cdots \\ \hline \end{array}$$

For n odd, row w or column w should be omitted.

As a check, we have

$$C_A^d C_d^A = 1$$

The foregoing transformations may be derived in another way as follows:

According to the method of symmetrical components, we have

$$C_1{}^A = \begin{array}{c|cccccc} & A & B & C & D & \cdots & N \\ \hline 0 & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} \\ 1 & \frac{1}{n} & \frac{a}{n} & \frac{a^2}{n} & \frac{a^3}{n} & \cdots & \frac{a^{-1}}{n} \\ 2 & \frac{1}{n} & \frac{a^2}{n} & \frac{a^4}{n} & \frac{a^6}{n} & \cdots & \frac{a^{-2}}{n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \cdots \\ \frac{n}{2} & \frac{1}{n} & -\frac{1}{n} & \frac{1}{n} & -\frac{1}{n} & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \cdots \\ n-2 & \frac{1}{n} & \frac{a^{-2}}{n} & \frac{a^{-4}}{n} & \frac{a^{-6}}{n} & \cdots & \frac{a^{2}}{n} \\ n-1 & \frac{1}{n} & \frac{a^{-1}}{n} & \frac{a^{-2}}{n} & \frac{a^{-3}}{n} & \cdots & \frac{a}{n} \end{array}$$

$a = \epsilon^{j\theta_0}$ where $\theta_0 = \dfrac{2\pi}{n}$

For n odd, row $n/2$ should be omitted.
The reverse transformation is

$$C_A{}^1 = \begin{array}{c|cccccccc} & 0 & 1 & 2 & \cdots & \frac{n}{2} & \cdots & n-2 & n-1 \\ \hline A & 1 & 1 & 1 & \cdots & 1 & \cdots & 1 & 1 \\ B & 1 & a^{-1} & a^{-2} & \cdots & -1 & \cdots & a^2 & a \\ C & 1 & a^{-2} & a^{-4} & \cdots & 1 & \cdots & a^4 & a^2 \\ D & 1 & a^{-3} & a^{-6} & \cdots & -1 & \cdots & a^6 & a^3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ N & 1 & a & a^2 & \cdots & \cdots & \cdots & a^{-2} & a^{-1} \end{array}$$

For n odd, column $n/2$ should be omitted.

The correlation between the symmetrical components and the 2-reaction components are as follows:[9,10]

$$C_d{}^1 = \begin{array}{c|cccccccc} & 0 & 1 & 2 & \cdots\cdot & \frac{n}{2} & \cdots\cdot & n-2 & n-1 \\ \hline 0 & 1 & & & & & & & \\ d & & \epsilon^{-j\theta} & & & & & & \epsilon^{j\theta} \\ q & & -j\epsilon^{-j\theta} & & & & & & j\epsilon^{j\theta} \\ u_1 & & & 1 & & & & 1 & \\ v_1 & & & -j & & & & j & \\ \vdots & & & & \vdots & & \vdots & & \\ w & & & & & 1 & & & \end{array}$$

All the empty spaces should be indicated by 0. For n odd, column $n/2$ and row w should be omitted.

$$C_1{}^d = \begin{array}{c|ccccccc} & 0 & d & q & u_1 & v_1 & \cdots\cdots & w \\ \hline 0 & 1 & & & & & & \\ 1 & & \frac{1}{2}\epsilon^{j\theta} & j\frac{1}{2}\epsilon^{j\theta} & & & & \\ 2 & & & & \frac{1}{2} & j\frac{1}{2} & & \\ \vdots & & & & & & \cdots & \\ \frac{n}{2} & & & & & & & 1 \\ \vdots & & & & & & \cdots & \\ n-2 & & & & \frac{1}{2} & -j\frac{1}{2} & & \\ n-1 & & \frac{1}{2}\epsilon^{-j\theta} & -j\frac{1}{2}\epsilon^{-j\theta} & & & & \end{array}$$

All the empty spaces should be indicated by 0. For n odd, column w and row $n/2$ should be omitted.

Then we can get $C_A{}^d$ and $C_d{}^A$ from the relations:

$$C_A{}^d = C_A{}^1 C_1{}^d$$
$$C_d{}^A = C_d{}^1 C_1{}^A$$

Notice that $C_1{}^d$ and $C_d{}^1$ are of very simple form, and, since $C_A{}^1$ and $C_1{}^A$ are well known from the method of symmetrical components, $C_A{}^d$ and $C_d{}^A$ can be written down very simply without going through the details of matrix algebra.

Appendix III. Derivation of 2-Reaction Flux Linkages

Substituting equation 2 into equations 3 and 4, we can derive the expressions for ψ_d and ψ_q as follows:
Consider ψ_d first. From equation 3, we have

$$\psi_d = \frac{2}{n}[\psi_A \cos\theta + \psi_B \cos(\theta - \theta_0) + \psi_C \cos \times$$
$$(\theta - 2\theta_0) + \ldots + \psi_N \cos\theta - (n-1)\theta_0]$$

Let us examine the different components of the phase flux linkages separately. The first component is $I_d \cos\theta$. Hence

$$\psi_{d(1)} = \frac{2}{n} I_d[\cos\theta\cos\theta + \cos(\theta-\theta_0)\cos(\theta-\theta_0) +$$
$$\cos(\theta-2\theta_0)\cos(\theta-2\theta_0) + \ldots] = I_d$$

Similarly, for the second component $-I_q \sin\theta$, we get

$$\psi_{d_{(2)}} = -\frac{2}{n} I_q[\sin\theta\cos\theta + \sin(\theta-\theta_0)\cos(\theta-\theta_0) +$$
$$\sin(\theta-2\theta_0)\cos(\theta-2\theta_0) + \ldots] = 0$$

For the third component, $-x_0 i_A$, we have

$$\psi_{d_{(3)}} = -\frac{2}{n} x_0 [i_A \cos\theta + i_B\cos(\theta-\theta_0) + i_C\cos(\theta-2\theta_0) + \ldots]$$
$$= -x_0 i_d$$

The fourth component is as follows:

$$\psi_{d_{(4)}} = \frac{2}{n}\left[x_0 - \left(\frac{x_d + x_q}{2}\right)\right]\left(\frac{2}{n}\right)[\{i_A + i_B\cos\theta_0 + i_C\cos 2\theta_0 + ..\}$$
$$\cos\theta + \{i_A\cos\theta_0 + i_B + i_C\cos\theta_0 + \ldots\}\cos(\theta-\theta_0) +$$
$$\{i_A\cos 2\theta_0 + i_B\cos\theta_0 + i_C + \ldots\}\cos(\theta-2\theta_0) + \ldots]$$

Now we may combine the terms into the following groups:

$$\left(\frac{2}{n}\right)[i_A\cos\theta + i_A\cos\theta_0\cos(\theta-\theta_0) + i_A\cos 2\theta_0\cos(\theta-2\theta_0) + \ldots]$$
$$= i_A\cos\theta$$

$$\left(\frac{2}{n}\right)[i_B\cos(\theta-\theta_0) + i_B\cos\theta_0\cos(\theta-2\theta_0) + \ldots + i_B\cos\theta_0\cos\theta]$$
$$= i_B\cos(\theta-\theta_0)$$

So we get

$$\psi_{d_{(4)}} = \frac{2}{n}\left[x_0 - \left(\frac{x_d+x_q}{2}\right)\right][i_A\cos\theta + i_B\cos(\theta-\theta_0) +$$
$$i_C\cos(\theta-2\theta_0) + \ldots]$$
$$= x_0 i_d - \left(\frac{x_d+x_q}{2}\right) i_d$$

For the fifth component, we have

$$\psi_{d_{(5)}} = -\frac{2}{n}\left(\frac{x_d-x_q}{2}\right)\left(\frac{2}{n}\right)[\{i_A\cos 2\theta\, i_B\cos(2\theta-\theta_0) +$$
$$i_C\cos(2\theta-2\theta_0) + \ldots\}\cos\theta$$
$$\{i_A\cos(2\theta-\theta_0) + i_B\cos(2\theta-2\theta_0) + i_C\cos(2\theta-3\theta_0) +$$
$$\ldots + \ldots\}\cos(\theta-\theta_0)$$
$$\{i_A\cos(2\theta-2\theta_0) + i_B\cos(2\theta-3\theta_0) + i_C\cos(2\theta-4\theta_0) +$$
$$\ldots\}\cos(\theta-2\theta_0) + \ldots]$$

Grouping the terms together, there is

$$\left(\frac{2}{n}\right)[i_A \cos 2\theta \cos \theta + i_A \cos (2\theta - \theta_0) \cos (\theta - \theta_0) + i_A \cos (2\theta - 2\theta_0) \cos (\theta - 2\theta_0) + \ldots] = i_A \cos \theta$$

$$\left(\frac{2}{n}\right)[i_B \cos (2\theta - 2\theta_0) \cos (\theta - \theta_0) + i_B \cos (2\theta - 3\theta_0) \times \cos (\theta - 2\theta_0) + \ldots + i_B \cos (2\theta - \theta_0) \cos \theta] = i_B \cos (\theta - \theta_0)$$

So we have

$$\psi_{d(5)} = -\frac{2}{n}\left(\frac{x_d - x_q}{2}\right)[i_A \cos \theta + i_B \cos (\theta - \theta_0) + i_C \cos (\theta - 2\theta_0) + \ldots] = -\left(\frac{x_d - x_q}{2}\right) i_d$$

The sum of 5 components gives

$$\psi_d = I_d - x_d i_d$$

For the quadrature flux linkages, we can derive the expression in a similar manner. The different components will be

$$\psi_{q(1)} = -\frac{2}{n} I_d [\cos \theta \sin \theta + \cos (\theta - \theta_0) \sin (\theta - \theta_0) + \ldots] = 0$$

$$\psi_{q(2)} = \frac{2}{n} I_q [\sin^2 \theta + \sin^2 (\theta - \theta_0) + \ldots] = I_q$$

$$\psi_{q(3)} = \frac{2}{n} x_0 [i_A \sin \theta + i_B \sin (\theta - \theta_0) + \ldots] = -x_0 i_q$$

$$\psi_{q(4)} = -\frac{2}{n}\left[x_0 - \left(\frac{x_d + x_q}{2}\right)\right][i_A \sin \theta + i_B \sin (\theta - \theta_0) + \ldots]$$
$$= x_0 i_q - \left(\frac{x_d + x_q}{2}\right) i_q$$

$$\psi_{q(5)} = -\frac{2}{n}\left(\frac{x_d - x_q}{2}\right)[i_A \sin \theta + i_B \sin (\theta - \theta_0) + \ldots]$$
$$= \left(\frac{x_d - x_q}{2}\right) i_q$$

The sum of the above 5 components will then give

$$\psi_q = I_q - x_q i_q$$

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Fuels of Tomorrow

FUELS of today and tomorrow were discussed by A. C. Fieldner, retiring president, American Society for Testing Materials, in his presidential address at the annual meeting of the society held in New York, N. Y., June 28–July 2, 1937. Doctor Fieldner first covered fuels of today, including such topics as the search for oil, developments in refining petroleum, displacement of coal by fuel oil, rise of natural gas, and coal production and utilization. After discussing fuels of today and outlining national fuel reserves, he discussed fuels of tomorrow, indicating that gas, oil, and coal are available today in abundance. There is coal enough for hundreds, and possibly thousands of years; but natural gas and oil obtainable by present methods may be exhausted in less than 100 years, and a shortage of our domestic supply may begin within 10 or 20 years.

He indicated that coal will continue to be the principal fuel used for the generation of public utility and major industrial power. Technologic improvements and new hydroelectric power will tend to reduce the consumption of coal; on the other hand, an increasing demand for energy and a decreasing supply of cheap residual oil will increase the amount of coal consumed for power purposes. No material change is expected in either direction in the near future, but in 10 or 15 years the trend will favor increased consumption of coal. Tomorrow's power and central heating plant will burn any kind of coal completely and efficiently.

The convenience and uniformity of automatic heating of homes with gas or oil will continue to attract more users, even at higher costs than those prevailing today. The insulation of houses has been improved greatly, and future homes will permit higher unit cost of fuel without increasing the total heating bill.

Referring to railroad and marine fuels he indicated that further improvements in the over-all efficiency of the steam locomotive and a gradual increase of electrification will retain the use of coal for freight traffic throughout the age of oil and natural gas, and that the convenience and economy of Diesel-engine drive for ships and boats is such that its use will continue even after declining production of petroleum requires the production of Diesel fuel from shale or coal.

In discussing the motor fuel supply, one of the most interesting questions, he pointed to the prediction of Snider and Brooks that by 1945 there is the probability of considerable shortage of domestic petroleum. Doctor Fieldner said that from the very beginning of the automobile industry, recurring threats of shortage of gasoline were met—in the field by finding new pools and improving production technique, and in the refinery by increasing yields and making a more efficient product. "The end has not been reached. We are just beginning to use scientific methods in extracting oil from the sands, and catalytic polymerization and hydrogenation eventually will furnish the means for complete conversion of volatile liquids and heavy petroleum to gasoline."

Discussions

Of AIEE Papers—as Recommended for Publication by Technical Committees

ON THIS and the following 2 pages appear all remaining discussions submitted for publication, and approved by the technical committees, on papers presented at the 1937 AIEE winter convention, New York, N. Y., January 25–29. Discussions of papers presented at the sessions on general power applications, vibration and balance, communication and research, power transmission, education, power generation and electrical machinery, instruments and measurements, insulation co-ordination, and lightning protective equipment at the 1937 AIEE summer convention, Milwaukee, Wis., June 21–25, now are being reviewed by the technical committees, and will appear in ELECTRICAL ENGINEERING as they are released for publication.

Members anywhere are encouraged to submit written discussion of any paper published in ELECTRICAL ENGINEERING, which discussion will be reviewed by the proper technical committee and considered for possible publication in a subsequent issue. Discussions of papers scheduled for presentation at any AIEE meeting or convention are closed 2 weeks after presentation.

Watt-Hour Meter Bearings

Discussion and authors' closure of a paper by I. F. Kinnard and J. H. Goss published in the January 1937 issue, pages 129–37, and presented for oral discussion at the instruments and measurements session of the winter convention, New York, N. Y., January 28, 1937.

F. C. Holtz (Sangamo Electric Company, Springfield, Ill.): To me, there are few subjects which offer greater possibilities for scientific adventure and speculation than the study of watt-hour meter bearings. No branch of physics or chemistry seems able to escape being involved in this problem in at least one form or another. It offers a challenge to the best we have in modern scientific technique, in the fine structure and architecture of solids and it plays with stresses seldom considered in modern machinery. Any familiarity with the problem reveals that the great mass of knowledge is still to be gained from carefully planned investigation.

One cannot help but be impressed by the long list of references which is only one indication of the great commercial importance attached to the problem and the desire to further improve what for a time seemed incapable of improvement. Beginning with the work of Shotter and others, it was quite evident that practical improvement lay in the direction of materials rather than of form and the present work is without question a valuable and important contribution.

I trust, in commenting on the present paper, it will not be out of order to draw on the experience of others and to relate certain conclusions which we are able to draw from our own observations.

The success of any bearing system must be judged from the results obtained in the field under all possible conditions of installation and use.

Because of the wide variation in these conditions the application of statistical methods are very helpful in determining the success of any particular arrangement and from this point of view we, as manufacturers, are very grateful for the friendly co-operation and assistance which we received from the public utilities in the form of data on the many millions of meters scattered throughout the country. All experimental data should be examined in its relation to the practical aspects of the problem.

The present paper is for the most part a record of laboratory experiment and from that standpoint is extremely interesting and satisfying to the user of meters in pointing out the care and consideration which is given to the problem before a commercial arrangement is arrived at.

Figure 2 is extremely interesting in showing how the friction varies with pivot radius; however, it should be pointed out that it is also a function of the jewel radius. Strictly speaking, it is probably a function of the order of contact between the 2 surfaces and from this point of view it would be interesting to know how this friction varies as the jewel radius approaches that of the pivot. The values indicated by the authors are very close to what might be considered an average throughout the industry. They are somewhat smaller than the values chosen a few years ago due to the lighter weight moving systems, higher expected values of overload, which together with higher torque gives a greater tendency for the pivot to ride on top of the jewel. While other means are provided to prevent it, the deeper and smaller radius cup is the ideal arrangement.

While the curve of figure 3 is extremely interesting, it is to be noted that in watt-hour meter practice meter pivots are seldom worn at the point. This, as is well known, is due to the fact that both driving and damping torques act to force the pivot up the side of the meter jewel until the forces are balanced, leaving the point of contact at some suitable distance from the axis of the pivot. At light load this is near the stationary position, but in this case the speed of rotation is small, hence no great amount of wear is to be expected. In some respects this is fortunate for if wear results at the higher loads the corresponding friction does not appreciably alter the meter registration; yet during periods of light load the pivot returns to put into contact areas which are little affected and which again has little effect on the initial accuracy. I should like at this time to refer to the curves of figure 7 which, in the light of past experience seems to be an anomalous behavior of materials. It is generally agreed that the pivot-and-cup type of bearing is improved by oiling and that its life is correspondingly increased. The work of Stott, Shotter, and others points very clearly to this conclusion and I am sure that it has been thoroughly supported by field observation. It certainly must be true for a great variety of materials, if not all. Why, then, does the oiled cobalt-tungsten pivot wear as rapidly as dry steel in the beginning and then slow down to that of dry cobalt tungsten at 10,000,000 revolutions, and then less with advancing revolutions?

It would be interesting also to know something of the performance of the oiled steel pivot drawn to the same scale for comparative purposes.

Figure 7 is apt to give the impression that a pivot begins to wear the instant it begins rotation. This may be true of some alloys and even steel in the dry state, but it is not necessarily true of an oiled steel pivot. I have tried to form an opinion based on the great mass of evidence supplied by the operating companies. There is ample evidence in support of the contention that wear begins with some unfortunate accident to the bearing system, the nature of which is, as yet, not fully understood. We know, for example, that some pivots will apparently operate for millions of revolutions without showing more than a trace of wear and then without warning increase rapidly at a fairly uniform rate somewhat as shown in figure 7. It is as if Mr. Kinnard's curves did not all start in the same place but chose to distribute themselves out along the axis of revolutions in some quite arbitrary manner. This type of action is hinted at in Shotter's paper, but so far as I know it has not been recorded in the form of a curve. If this were not true, we could readily predict the exact number of revolutions which a meter should make before changing bearings. I believe that I have the support of the meter man in the above statement based on a systematic and careful examination of meter pivots removed at the time of periodic test. It would be extremely interesting to carefully measure the radius of worn areas on a relatively large number (say 1,000) of meter pivots and plot these results in terms of meter revolutions. This will require that the work be followed in a territory where conditions are fairly uniform both as to loads, methods of handling, etc.

The impact tests are extremely interest-

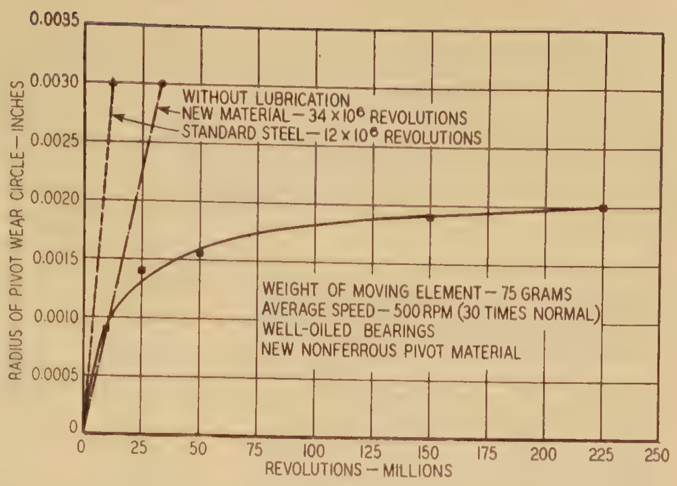


Figure 7. Accelerated life test wear curves

ing in pointing out the manner in which jewels and pivots may be damaged by short circuit conditions. We conducted tests some years ago to determine the relative susceptibility of jewels of various types and orientation to damage by impact. To eliminate certain types of errors and render the results more uniform and dependable, the arrangement provided for applying the impact at the center and thinnest section of the jewel. To do this the jewel was mounted on a relatively massive iron anvil. A small steel ball was placed in the jewel which combination then received the impact from a small hardened steel cylinder dropped through a small tube from varying heights. In this case the impacts were repeated on the same jewel from varying heights until microscopic examination revealed that the jewel had been damaged.

The results show clearly that to obtain consistent results considerable care is necessary in properly mounting the jewel and that the results are influenced by the nature of the back surface of the jewel.

The subject of pivot materials is in itself a most interesting one, and I should like to discuss it according to the 6 qualities laid down by the present authors:

1. *Noncorrosive.* The question of corrosion resistance is, so I am told, tied up with 2 different properties of the metal or alloy. The first assumes that the metals are relatively inert as to forming chemical combinations with the compounds with which they may be in contact. Gold, platinum, osmium, tungsten, and other noble metals belong to this group. They achieve their corrosion resistances through their extreme stability and refusal to readily form compounds.

The second group are those which are relatively active, chemically, and readily form oxide layers over their surfaces which in themselves are chemically inactive. Aluminum and stainless steels belong to this class.

It should be quite obvious that materials of the second class should not be considered acceptable for meter pivots since the abrasive action of the jewel and pivot in contact will remove this oxide covering and expose the under layer to further corrosion. This explains why, under certain conditions a stainless steel pivot will show considerable quantities of red iron oxide after having operated dry in a meter bearing. All such oxides are invariably hard, brittle, and therefore quite abrasive.

2. *Nonabrasive.* A careful examination of the abraded material also raises an interesting question: Are the products of abrasion dependent upon the abrasive qualities of the constituent alloys or do the oxides of such alloys have qualities which are independent of them?

If they depend upon the individual constituents, it would be an easy matter to say which metals should and which should not form a satisfactory combination for a meter pivot. The problem is not so simple for the abrasive qualities of the material removed frequently does not resemble any of its constituents. The former is, nevertheless, a safe guide to follow. Oxides of tungsten and cobalt are each relatively mild abrasives and so far as we know the oxide of the alloy is also. Shotter, in his paper, comments very favorably on tungsten which has been verified by further test, but it requires very careful preparation.

3. *Shock Resistance.* There are some extremely interesting things about the alloys suitable for pivot materials. In addition to the corrosion resistance already referred to, there is the property of hardness.

I am inclined to feel from numerous tests that extreme hardness is essential to abrasion resistance. This follows from a consideration of diamond and sapphire as jewel materials and it is equally true in pivot materials. It may, however, be possible for a pivot material to be so hard as to resist wear, and when used against sapphire, wear the latter.

In most cases the reverse is true, but in some cases such as in alloys containing comparatively large percentages of iridium the pivot will actually cut the jewel, especially when run dry.

Frequently the presence of minute inclusions of foreign matter in the pivot material may damage the jewel. It is extremely necessary, therefore, to watch alloys for the appearance of minute inclusions in any series of experiments to determine a suitable material.

The present paper refers to cobalt-tungsten alloy as being softer than steel. In one sense this is true and in another it is not. Cobalt-tungsten alloy as with other alloys, especially precipitation hardening alloys, comprises a series of hard crystals in a matrix of softer material. For this reason the material is soft from the standpoint of deformation and hard from the standpoint of wear. This property of the structure is shown when determining superficial hardness. A perfectly conical diamond penetrator will frequently leave a triangular shaped depression showing that the hard crystals are moved about as a body in a matrix of softer material.

No comment will be made on the last 2 properties; namely, low friction and workability, since this in itself involves a consideration of many materials. I should, however, remark that certain alloys of osmium, rhodium, and platinum show great promise as pivot materials and it is hoped that the cost can be brought within permissible figures.

In pointing to the possibility of operating the cobalt-tungsten pivot without oil, no mention is made of the relative friction in each case and the variation in friction over long periods of operation. It would be interesting to know something of this.

4. *Lubrication.* With the rapidly growing tendency toward outdoor meter installations, the matter of lubrication is extremely important and the qualities outlined by the authors is an excellent guide in the selection of a suitable lubricant. In general the mineral oils have a somewhat greater tendency to creep but this can be reduced by suitably applying to the surfaces a mixture known to the watchmaking industry as "epilame." Its essential ingredients are steric acid in toluene, and when applied its thickness is of molecular dimensions.

In addition to chemical tests, certain simple mechanical tests are extremely useful in detecting any undesirable qualities. A small quantity of the oil is placed in a hemispherical cup which is alternately rotated and stopped in the presence of warm

The huge Goodyear Zeppelin Air Dock, which may be visited by those attending the forthcoming AIEE Middle Eastern District meeting to be held at Akron, Ohio air or oxygen. Centrifugal action drives the oil over the surface in a thin layer and then allows it to settle to the bottom. This test will quickly reveal any tendency toward gumming and frequently reveals conditions which cannot be detected by chemical tests. It corresponds to an accelerated service test and is therefore of considerable value.

I. F. Kinnard and J. H. Goss: The discussion of the paper, especially by Mr. S. Green, Mr. F. C. Holtz, and Professor M. W. Pullen has brought out much information of interest to users and designers of watt-hour meters. Some of the points raised require further discussion by the authors.

Mr. F. C. Holtz has referred to the effect of the radius of the jewel cup on the value of friction torque between the pivot or ball and the jewel cup. Mr. V. Stott (reference 2 of the bibliography) has shown by a mathematical analysis that a negligibly small error is introduced if the radius of the jewel cup is assumed to be infinite over the small area of contact of a new bearing within the range of radii for pivots, balls, and jewels which are used commercially.

Figure 7, which Mr. Holtz questions, was plotted in error and the authors are pleased to include in this discussion a corrected figure. It will be noted that the oiled cobalt-tungsten bearing follows almost exactly the same wear curve as the dry for the first few million revolutions. A possible explanation for this behavior is that due to the extremely high bearing pressures during this period, little or no lubrication is present under the contact area. As the bearing wears, the pressure drops until the oil film is eventually able to support the load.

There is no question but that many bearings have been operated for long periods without showing appreciable wear. The phenomena of just what initiates wear is difficult to understand. It has been the aim of the authors to present data to permit a logical explanation of the major factors that influence watthour meter bearing life. At this time it appears to be impossible to fully explain every case.

Mr. Holtz suggests it would have been worth while to publish the wear curve for a well oiled steel pivot along with the curve of cobalt-tungsten shown in figure 7. Such a curve was published in a previous article (reference 1 of the bibliography) and for that reason was omitted.

Professor Pullen brings up the point of preforming the radius of the pivot contact area to match the jewel. There has been no change in pivot manufacturing methods to allow the exact matching of the pivot and jewel radius.

The radius of a pivot can be quite accurately ground while somewhat greater variations are found in sapphire jewel cups.

The hardness of boron carbide, figure 5, was plotted in error and should be 19.7 instead of 29.

Figure 10 showing the oxidation of oils should show a total range of acidity of 1.0 per cent instead of 10 per cent. The ordinates should be tenths of a per cent, the decimal points being omitted.

The authors wish to express their appreciation for the favorable discussion of their paper by Doctor H. B. Brooks and by Mr. E. W. Allen.

First Report of Power System Stability

Closure of a report of the AIEE subcommittee on interconnection and stability factors published in the February 1937 issue, pages 261–82, and presented for oral discussion at the power transmission session of the winter convention, New York, N. Y., January 26, 1937. Previous discussions and closure appeared in the May 1937 issue, pages 632–4.

R. D. Evans: Raymond Bailey, H. K. Sels, and Nicholas Stahl have called to our attention the fact that table III contains certain discrepancies as applied to the Pennsylvania-New Jersey interconnection. It is believed that these can best be clarified by republishing part of the table. The accompanying tabulation supersedes item I for the Pennsylvania Power & Light Company and the next 5 columns headed "Pennsylvania and New Jersey Interconnection."

Mr. Bailey has also pointed out certain errors in table III as applying to the Philadelphia Electric Company's system. Under item I for the Conowingo-Plymouth Meeting circuit, line *B*-5 should show "H.T." bus at both sending and receiving ends; and line *E*-2 should show:

Phase: High-speed differential induction impedance.

Ground: High-speed differential and high-speed overcurrent.

Under item II for the Plymouth Meeting-Westmoreland circuit, line *B*-5 should also show "H.T." bus at both sending and receiving ends; and under line *B*-2 should show:

Phase: Induction impedance.
Ground: High-speed directional overcurrent.

Table III. Recent Practices Regarding Stability Studies

Pennsylvania and New Jersey Interconnection

Pennsylvania Power & Light Company
Philadelphia Electric Company
Public Service Electric and Gas Company

	Item I Plymouth Mtg.-Siegfried	Item II Plymouth Mtg.-Roseland	Item III Siegfried-Roseland
A. Transmission			
1. Voltage, kilovolts	220	220	220
2. Distance, miles	48.7	75.8	83.7
3. Frequency, cycles	60	60	60
4. Power, kilowatts sent out	185,000†	185,000†	185,000†
B. Circuit arrangement			
1. Number of circuits	1	1	1
2. Intermediate Switching station	None	None	None
3. Load taken off at intermediate points, kilowatts	None	None	None
4. Synchronous condenser, kva at intermediate points	None	None	None
5. Bussing arrangements			
a. Sending end	H.T.	H.T.	H.T.
b. Receiving end	H.T.	H.T.	H.T.
6. Grounding			
a. Sending end	Solid	Solid	Solid
b. Receiving end	Solid	Solid	Solid
C. Generator			
1. Kilovolt-amperes	Intersystem tie	Intersystem tie	Intersystem tie
2. Short circuit ratio			
3. Transient reactance			
4. Inertia constant			
5. Damper winding			
D. Excitation system			
1. Exciter response			
E. Breakers and relays			
1. Breaker speed, cycles	8–12	3.5–8	3.5–10
2. Relaying type*	Phase: Directional Impedance Ground: Hi-speed Directional Overcurrent	(Same as item I)	(Same as item I)
3. Total time, cycles**	Phase: 9–75 Ground: 9–35 or 70–90	Phase: 9–75 Ground: 9–35 or 50–100	Phase: 12–60 Ground: 12–30 or 60 100
F. Lightning protection			
1. Tower construction	Steel	Steel	Steel
2. Ground wires	2	2	2
3. Counterpoises	None	None except gndg. cable at high-res. tower footings	(Same as item II)
4. Insulators	Susp.: 16 5-3/4" Units Deadend: 18—same	Susp.: 16 or 18 5-3/4" units Deadend: 18 or 20—same	Susp.: 14, 16, or 18 5-3/4" units Deadend: 18 or 20—same
5. Lightning arresters	Plymouth Mtg. only	Plymouth Mtg. only	None

* Note: High-speed directional impedance phase relays installed at Plymouth Meeting end of items I and II.
** Note: In certain cases of sequential tripping, time given applies to first breaker only.
† Note: Based on thermal limitations.

News
Of Institute and Related Activities

Middle Eastern District Meeting and Student Branch Convention

Akron, OHIO, will be host to a 3-day meeting of the Middle Eastern District of the AIEE, October 13–15, 1937. The meeting headquarters will be in the Mayflower Hotel. The local committees have arranged an attractive technical program which consists of 6 sessions and a student session to be held during the mornings and have planned for the afternoons and evenings interesting inspection trips, entertainment, and a banquet. In addition, a special program has been arranged for the women, who are especially urged to attend the meeting.

Akron is called "The City of Opportunity." It is a prosperous, enterprising industrial community with a population of 255,000, ranking thirty-fifth in the United States as to population, but twelfth in the value of its manufactures. It is the world's largest rubber manufacturing center and the dirigible-airship-manufacturing center of the United States. The committee has taken full cognizance of these industries in the arrangement of inspection trips.

Technical Program

A special effort has been made by the meetings and papers committee to arrange a technical program of a sufficiently diversified nature to offer general appeal to those in the electrical engineering profession. To accomplish this, parallel sessions will be held each morning, as shown in the program.

On Friday morning the student technical session will parallel the other 2 meetings. This session will be followed by a luncheon meeting in the Mayflower Hotel of Branch counselors and student chairmen. It is hoped that a minimum of conflict will occur on the part of those attending any particular series of sessions.

Inspection Trips

Those attending the Middle Eastern District meeting will have an opportunity to see a representative cross section of the manufacturing and points of interest in Akron and vicinity. Transportation on all inspection trips by bus will be available at a nominal cost for those who desire it.

In addition to the scheduled inspection trips appearing on the program, a visit to the Goodyear Air Dock and the Goodyear Museum can be arranged for those interested by consulting a member of the registration committee in the hotel lobby.

Trips that are desired should be indicated at the time of registration at the meeting.

Babcock & Wilcox Company

The manufacturing operations of the Barberton plant to be seen will include the complete central station and industrial plant, high- and low-pressure steam boilers, pulverized coal equipment, economizers, air heaters, etc. To those interested in power plant equipment this trip offers an unusual opportunity to see the fabrication processes involved in a wide field of manufacture.

B. F. Goodrich Company

Those choosing this trip will see those manufacturing processes which characterize Akron as being "The Rubber City." Starting with the crude rubber as it is received from the native plantation, it will be followed through the various stages of cutting, plastication, mixing, milling, tubing, and curing to become a finished product in the form of a tire or tube.

Nela Park

An extensive tour has been arranged to be made through Nela Park, Cleveland, for those registered at the District meeting. The laboratory trip is scheduled to begin at 3:00 p.m. and will show the methods of development and manufacture of lamps. Following the tour through the laboratory there will be a discussion on lamp development and lighting research after which an inspection of the General Electric Lighting Institute will be made. Dinner will be given to guests at 6:30 and followed by an evening of popular entertainment.

Because of the many interesting displays of home applications this trip should be of equal interest to the ladies and has been arranged with this in mind.

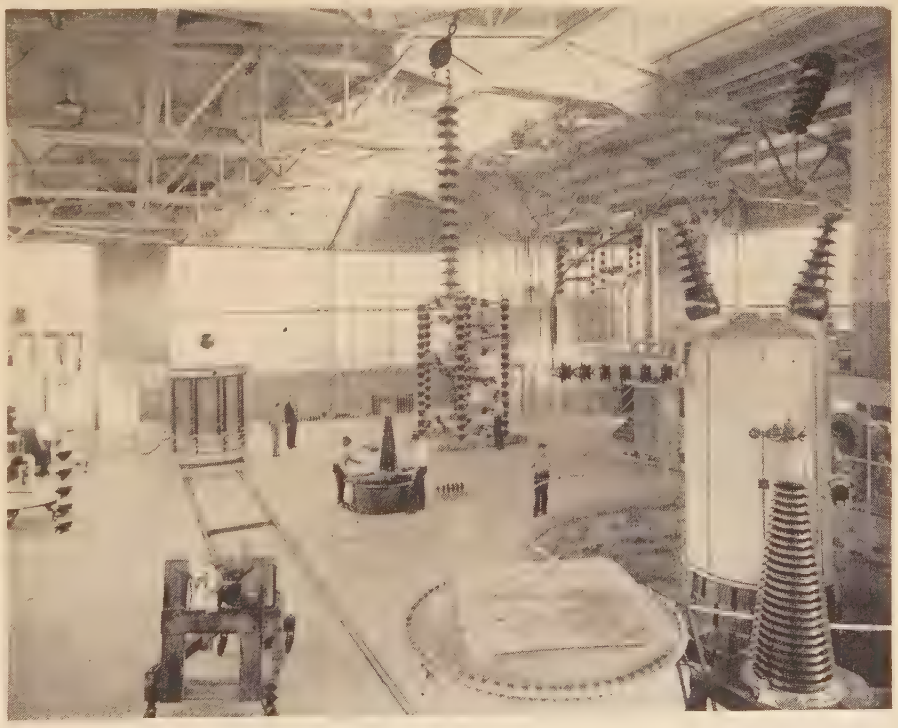
Ohio Brass Company

An inspection of the Barberton plant of the Ohio Brass Company will be made starting from the main office building and will include inspection of the processes, production methods, and routine testing in the manufacture of high-voltage porcelain insulators for both station and line use. The plant inspection will be followed by a demonstration of normal-frequency and impulse tests on various insulator equipment. These tests will be made in the electrical testing laboratory.

Daniel Guggenheim Airship Institute

One of the main test stands of the Daniel Guggenheim Airship Institute is the vertical wind tunnel. It provides a uniform open air stream $6^1/_2$ feet in diameter. Wind velocity can be regulated in small steps from zero to 125 miles per hour. This wind tunnel is driven by a 225-horsepower d-c motor and propeller-type fan, the electrical system being provided with various automatic and hand regulators and safety devices. This wind tunnel was built primarily for lighter-than-air testing, but is now being used also for airplane and various other industrial tests, for example, for testing the wind resistance of buildings and radio towers, the streamlining of trains, ventilating and cooling, etc. Some of the models used in such testing will be demonstrated.

The whirling arm is a unique apparatus, being the only one now in use in the United States for aeronautical testing. It is an arm 32 feet in length, turning at a height of 10 feet above the ground with



Electrical testing laboratory of the Ohio Brass Company, Barberton, a few miles southwest of Akron. An inspection trip to this plant has been included on the program of the AIEE Middle Eastern District meeting to be held in Akron, October 13-15

Program

Eastern Standard Time

In this program, reference to the issue and, in so far as possible, to the page in Electrical Engineering, is given for all papers.

Wednesday, October 13

9:00 a.m.—Registration

9:45 a.m.—Opening of meeting

Address of Welcome: Lee D. Schroy, mayor of Akron

Response: I. Melville Stein, vice-president of Middle Eastern District

10:15 a.m.—Selected Subjects

Presiding: I. Melville Stein

A Review of Radio Interference Investigation, Frank Sanford and Willard Weise, Cincinnati Gas and Electric Company.
Scheduled for October issue

*Notes on Radio Interference, Grover Lapp, Lapp Insulator Company, Inc.

*Radio Disturbance Measurements. W. S. Jennens and C. J. Miller, Jr., Ohio Brass Company.

*Improvements in Mine Ventilation, Theodore Troller, Guggenheim Airship Institute.

10:15 a.m.—Protective Devices

Presiding: Russell Krammes

*Use of Electronic Tubes for Relay Purposes, E. E. George, Tennessee Electric Power Company.

*The Advance in Industrial Switchgear, R. Paxton and F. E. Fairman, General Electric Company.

*The Design and Test of a High-Speed High-Interrupting-Capacity Railway Circuit Breaker, W. F. Skeats and H. E. Strong, General Electric Company.

2:00 p.m.—Inspection Trips

1. Babcock & Wilcox Company, Barberton, or
2. B. F. Goodrich Company, Akron

7:30 p.m.—Inspection Trip

Daniel Guggenheim Airship Institute

Thursday, October 14

9:30 a.m.—Electrical Machinery

Presiding: A. M. MacCutcheon

*Trends in the Design of Power Transformers, L. H. Hill, Allis-Chalmers Manufacturing Company.

Single-Phase Induction-Motor Performance, A. F. Puchstein and T. C. Lloyd, Robbins and Myers, Inc.
Scheduled for October issue

*5,000-Kw. Extraction Turbine, W. S. Kramer, Westinghouse Electric & Manufacturing Company.

*Electrical Application in Rubber Mills, John Grotzinger, R. S. Ferguson, and W. A. Brown, Goodyear Tire and Rubber Company.

9:30 a.m.—Electronics

Presiding: J. R. Martin

Low-Current Ignitors, A. H. Toepfer, Westinghouse Electric & Manufacturing Company.
July, pages 810–12

Regulation of Grid-Controlled Rectifiers, L. A. Kilgore and J. H. Cox, Westinghouse Electric & Manufacturing Company.
September, pages 1134–40

*The Igniter-Type Mercury Arc Rectifier as a Power Conversion Unit, A. Lee Barrett, Pittsburgh Coal Company.

*Mercury Arc Rectifiers in Chlorine Production, L. J. Rimlinger, Pittsburgh Plate Glass Co.

1:30 p.m.—Inspection Trip to Nela Park, Cleveland

6:30 p.m.—Dinner at Nela Park

8:00 p.m.—Popular Entertainment, Nela Park

Friday, October 15

9:30 a.m.—Iron and Steel

Presiding: Wray Dudley

Some High Lights in the Use of Electricity in Steel Mills, E. G. Fox, Freyn Engineering Company.
September, pages 1115–23

Carbon Brushes for Steel Mill Equipment, W. C. Kalb, National Carbon Company, Inc.
September, pages 1165–8

Tension Measurement and Control in Cold Strip Rolling, C. M. Hathaway and F. Mohler, General Electric Company.
September, pages 1141–4

9:30 a.m.—Power Transmission and Distribution

Presiding: F. W. Funk

*Lightning-Proof Transmission Lines, E. Hansson and A. F. Bang, Pennsylvania Water and Power Company.

Lightning Strokes in Field and Laboratory—II, P. L. Bellaschi, Westinghouse Electric and Manufacturing Company.
Scheduled for October issue

*Pennsylvania Railroad Electrification, H. C. Griffith, Pennsylvania Railroad Company.

*Possibilities of Direct Current Power Transmission Using Electrically-Controlled Rectifiers and Inverters, O. K. Marti, Allis-Chalmers Mfg. Co.

*Cross Catenary Transmission Lines for Lightning and Fog Conditions, A. O. Austin, Consulting and Manufacturing Engineer.

9:30 a.m.—Student Technical Session and Conference

Papers to be presented by students from each of the following schools:

Case School of Applied Science
Ohio University
Johns Hopkins University
West Virginia University
The University of Akron

12:00 noon—Luncheon Meeting of Branch Counselors and Student Chairmen

2:00 p.m.—Inspection Trip to Ohio Brass Company, Barberton

7:00 p.m.—Banquet

Presiding: Doctor W. E. Wickenden, president, Case School of Applied Science.

A message from President W. H. Harrison

Lighter-than-air craft will be discussed by Commander Rosendahl of the United States Naval Air Station, Lakehurst, N. J.

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* These papers are scheduled for presentation, but at the time of going to press they have not been accepted for publication.

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a top speed up to 200 miles per hour. Airplane or airship models are attached to the end of this arm and their behavior in curved flight can be investigated and the air forces determined by means of suitable instruments. A wind tunnel used in connection with this whirling arm makes it possible to duplicate the effect of atmospheric gusts on aircraft, when the model attached to the arm is made to traverse the open jet of this tunnel. The air forces in this case are recorded by means of diamonds scratching on glass cylinders.

Another test stand, just now in development, is a water tank, in which the free movement of an airship under the influence of gusts is imitated. Water is used to obtain kinematic and dynamic similarity. The airship model in this tank may be seen describing a path similar to that of an airship in flight in gusty weather.

Among the instruments to be demonstrated is a hot-wire anemometer, which is used for the measurement of rapid fluctuations of air velocity in magnitude and direction. This hot-wire anemometer, consisting essentially of thin platinum wires in a Wheatstone bridge, heated electrically and cooled by the moving air, is an important instrument for laboratory and atmospheric investigations.

Women's Program

A special invitation is extended to the women to attend the District meeting with their husbands. Particular attention has been given the program of activities by the ladies committee and everything will be done to make their visit an enjoyable one.

October 13

Morning—Registration
Luncheon—Mayflower Hotel
Afternoon—Garden pilgrimage, courtesy of Akron Garden Forum, Mrs. T. A. Chittenden, president
Evening—Informal dinner dance

October 14

Morning—Portage Country Club. Golf, games, or bridge
Luncheon—Portage Country Club
Afternoon—Nela Park, Cleveland
Evening—Nela Park, Cleveland

Future AIEE Meetings

Middle Eastern District Meeting
Akron, Ohio, Oct. 13–15, 1937

Winter Convention
New York, N. Y., January 24–28, 1938

North Eastern District Meeting
Pittsfield, Mass., Spring 1938

Summer Convention
Washington, D. C., June 20–24, 1938

Pacific Coast Convention
Portland, Ore., date to be determined

Cuyahoge Valley, Metropolitan Park, and high-level bridge in Akron, Ohio, in which city the forthcoming AIEE Middle Eastern District meeting will be held. The Gorge hydroelectric plant of the Ohio Edison Company may be seen just beyond the bridge, to the right

October 15

Morning—Blimp rides
Luncheon—Y.W.C.A.
Afternoon—Bridge-tea at the home of A. O. Austin, Barberton
Evening—Banquet

STUDENT CONFERENCE

After the student technical session on Friday morning a luncheon conference of the Branch chairmen and counselors will be held in the headquarters hotel.

RULES ON PRESENTING AND DISCUSSING PAPERS

At the technical sessions, papers may be presented in abstract, 10 minutes being allowed for each paper unless otherwise arranged or the presiding officer meets with the authors preceding the session to arrange the order of presentation and allotment of time for papers and discussion.

Any member is free to discuss any paper when the meeting is opened for general discussion. Usually 5 minutes is allowed to each discusser for the discussion of a single paper or of several papers on the same general subject. When a member signifies his desire to discuss several papers not dealing with the same general subject, he may be allowed a somewhat longer time.

It is preferable that a member who wishes to discuss a paper give his name in advance to the presiding officer of the session at which the paper is to be presented. Each discusser is to step to the front of the room and announce, so that all may hear, his name and professional affiliations. Three typewritten copies of discussion prepared in advance should be left with the presiding officer.

Other discussions to be considered for publication should be typewritten (double spaced) and submitted in triplicate to C. S. Rich, secretary of the technical program committee, AIEE headquarters, 33 West 39th St., New York, N. Y., on or before October 29, 1937. Discussion of unpublished addresses and papers need not be submitted.

HOTEL RATES

Members should make room reservations by writing directly to the hotel of their preference. For convenience the rates of the headquarters hotel, the Mayflower, as well as the rates of other hotels are listed in Table I.

ADVANCE REGISTRATION

Members of the District who will attend the meeting should register in advance by returning the addressed advance-registration card which they will receive by mail. This will be helpful to the registration committee and permit badges to be prepared in advance and reduce the work necessary on the opening day to a minimum.

A minimum registration fee of $2.00 will be charged all nonmembers except Enrolled Students and the immediate families of members.

COMMITTEES

The committees handling the various phases of the work in connection with the meeting are as follows:

District Meeting Committee: W. H. Harrison, I. Melville Stein, H. A. Dambly, A. G. Ennis, Paul Frederick, Warren C. Kalb, E E. Kimberly, E. O. Lange, and O. C. Schlemmer.

General Committee: Harold L. Brouse, chairman; A. O. Austin, Ralph Higgins, R. A. Hudson, H. C. Paiste, Harold H. Schroeder, V. W. Shear, and J. T. Walther.

Meetings and Papers Committee: A. O. Austin, chairman; T. F. Brandt, S. C. Henton, and H. L. Rorden.

Entertainment and Reception Committee: R. A. Hudson, chairman; Thomas Dee, R. P. Dunmire, W. C. Fick, Fred Nimmer, R. C. Putnam, A. P. Regal, W. A. Smith, and William H. Tucker.

Hotels and Registration Committee: Verne W Shear, chairman; George G. Clark, H. D. Duncan, F. A. Green, and P. C. Smith.

Transportation and Inspection Committee: H. C. Paiste, chairman; E. B. Crane, T. J. Newman, and N. W. Wing.

Student Sessions Committee: J. T. Walther, chairman; Kenneth Burgan, Carl Delagrange, Robert Miller, and P. D. Remark.

Attendance and Publicity Committee: Harold H. Schroeder, chairman; P. S. Bechtol, R. W. Drushel, and Louis McConaghy.

Finance Committee: Ralph Higgins, chairman; John Grotzinger, R. R. Krammes, and A. L. Richmond.

Women's Committee: Mrs. Ralph Higgins, chairman; Mrs. A. O. Austin, Mrs. J. W. Funk, Mrs. R. A. Hudson, and Mrs. Verne W. Shear.

Table I Hotel Rates

Hotel and Location	Single Without Bath	Single With Bath	Double Without Bath	Double With Bath	Twin
Mayflower (headquarters), 259 S. Main St.	$	$3.00 to 4.50	$	$5.00 to 6.50	$5.50 to 6.50
Portage, 10 N. Main St.	1.75	2.00 to 3.00	2.75	3.00 to 4.50	4.00 to 6.00
Akron , 92 E. Market St.	1.50 to 1.75	2.00 to 2.50	2.50	3.00 to 3.50	3.50
Howe, 11 S. Main St.	....	1.75 to 2.75	....	3.00 to 3.25	4.00
Anthony Wayne, 314 S. Main St.	1.50 to 1.75	2.50 to 2.75	2.50 to 3.00	4.00 to 4.50	....

ASME Fall Meeting. Papers on power, railroads, fuels, and petroleum are planned for the 1937 fall meeting of The American Society of Mechanical Engineers, which will be held at Erie, Pa., October 4–6. Inspection trips to several industrial plants, including that of the General Electric Company at which railroad equipment and refrigerator bodies are made, have been planned.

Additional Awards for 1936 Institute Papers

In addition to the national and District prize awards for papers presented before the Institute during 1936 announced in ELECTRICAL ENGINEERING for June 1937, page 756, the awards for Districts 5 and 8 have now been announced. These awards are:

DISTRICT 5

Prize for Branch paper awarded to Orville A. Becklund for his paper "An Artificial Ear for Sound Measurements," presented at a joint meeting of Minnesota Section and University of Minnesota Branch, April 21, 1937.

DISTRICT 8

Prize for best paper awarded to Joseph S. Carroll (A'24, M'37) and John A. Koontz Jr., (A'11, M'20) for their paper "Cable Vibration—Methods of Measurement," presented at the summer convention, Pasadena, Calif., June 22–26, 1936.

Prize for initial paper awarded to Otto A. Knopp (A'09, M'34) for his paper "Some Applications of Instrument Transformers," presented at the summer convention, Pasadena, Calif., June 22–26, 1936.

ECPD Offers Guides to Engineers

A pamphlet containing suggestions to junior engineers and another presenting a list of university extension facilities throughout the United States have been prepared by Engineers' Council for Professional Development. These pamphlets, together with 5 bibliographical pamphlets described in ELECTRICAL ENGINEERING for August 1937, page 1060, are intended to guide engineers in postcollegiate study.

In "Suggestions to Junior Engineers," a questionnaire is given to aid making a personal appraisal and a reading list to serve as a basis on which the nontechnical culture of the engineer can be continued. The pamphlet giving university extension facilities presents the scope of 54 courses and indicates their availability through class or correspondence study at more than 30 American institutions. The separate pamphlets can be had for 10 cents each, or for 5 cents each in lots of 50 copies, assorted if desired. Requests should be sent to Engineers' Council for Professional Development, 29 West 39th Street, New York, N. Y.

Chemical Industries to Hold Exposition. Plans for The Sixteenth Exposition of Chemical Industries, to be held at Grand Central Palace, New York, N. Y., December 6–11, 1937, are progressing rapidly. A feature of the exposition will be the award of a prize of $250 in cash to the person submitting the "best descriptive expression encompassing the purposes and the benefits redounding to the common good from the activities of the chemical industries." Raw materials, finished products, plant equipment, and machinery and apparatus will constitute major sections of the exhibits. Metals and alloys and the newer synthetic plastics and molded products likewise will be featured. Admission will be without charge, by registration or invitation only, and no tickets will be sold.

Membership—

Mr. Institute Member:

Progress commands respect and admiration. Our Institute has gone forward consistently and the progress made in membership growth is just one of the many reasons for the pride we feel in being identified with this organization. There has been a steady gain in members during the past few years, and each year has had a larger gain than the preceding one. This progress was possible only because of the active assistance and support of the members—not only the officers and committeemen, but you and you. We appreciate your help and want to thank you.

Another year began on August 1. There will be new officers, new committees, and, of course, new members.

We are hopeful that the growth in members in this coming year may be greater than last year's increase. With your help it *will* be greater. We are again counting on your support.

Chairman National Membership Committee

Aluminum Company Plans Expansion

Expenditures which may finally reach $15,000,000 will be necessary to provide added facilities for production of virgin aluminum and for additional fabricating units, electrical installations, housing facilities, and other auxiliary equipment in an expansion program contemplated by the Aluminum Company of America at Alcoa, Tenn. This expansion will require many additional employees with the likelihood that the present force of approximately 5,000 will be doubled when the program is completed. A contract with the Tennessee Valley Authority has been signed whereby the company secures at an early date a large quantity of power to augment the present supply from its own hydroelectric system.

Aluminum Company of America was attracted to this section more than 25 years ago because of the hydroelectric possibilities for an industry which finds cheap power one of the prerequisites for manufacture of its product at a price at which it can compete with other materials. Despite the fact that all the raw materials except power have to be brought into the state, the company found it desirable constantly to expand operations at Alcoa where the annual output now represents about 30 per cent of the virgin aluminum produced in the United States. Fabricating operations at Alcoa have been an important part of the company's manufacturing plants for semifinished and finished aluminum, for many years the largest aluminum sheet and plate mill in the country being located at this point.

To provide power for existing operations, the company has developed 3 large hydroelectric projects on the Little Tennessee River and its tributaries at Calderwood, Cheoah, and Santeetlah. It has several other dam sites for future use, one of which, on the Nantahala River, is already partially developed. Application for the completion of several of these projects is about to be made to governmental authorities.

In addition to the large sums spent in Tennessee and western North Carolina for power projects to serve the Alcoa works, the company has its own hydroelectric plants on the Yadkin River to provide power for an aluminum works at Badin, N. C. Most recent announcement of expansion in that section was the statement within the past month that the company is building a dam at Tuckertown.

Engineering Society of Detroit Receives Gift. Total endowment of The Engineering Society of Detroit, successor to the Detroit Engineering Society, has reached $1,000,000 with a second gift of $500,000 from The Horace H. and Mary A. Rackham Fund. The purpose of this gift is to provide a suitable building to be used by the society as a permanent home; income from the first half-million-dollar gift is to be used for operating expenses. The Engineering Society of Detroit is an entirely new organization with 12 affiliated groups and a membership which numbers nearly 3,000.

Student Training in Air Conditioning

An extensive educational program designed to attract young men into the air-conditioning industry has been begun by the General Electric Company. The plan is modeled after the test courses started by the company in 1888.

The new program, although sponsored by the company itself, will be worked out through its distributors in the field, in the belief that the distributor organization offers wider practical experience and greater possibilities for the employment of the students. It will be open primarily to graduates of colleges, technical schools, and high schools who desire to establish themselves in the industry, but in special instances it may be open to men who have been otherwise employed, provided they demonstrate potential capabilities for the business. Because many phases of air-conditioning work involve engineering problems, the opportunity is generally greatest for those who have had technical training.

Practical manuals, courses, and distributor training activities have been designed by the General Electric air-conditioning department. They will be augmented by various forms of instruction in the field by district managers and factory men. Applications for training should be made to distributors or to the headquarters of the air-conditioning division of the company in Bloomfield, N. J.

Conference Board Elects Officers

At its 21st annual meeting, the National Industrial Conference Board elected officers for the ensuing year. Elon H. Hooker, president of the Hooker Electrochemical Company, was elected chairman, and John Henry Hammond was elected honorary chairman. Doctor Virgil Jordan continues as president and chief executive of the board.

The following vice-chairmen were elected: Irenée du Pont, director, E. I. du Pont de Nemours and Company; E. Kent Hubbard, president, Manufacturers Association of Connecticut; Hon. Walter J. Kohler, chairman, Kohler Company; and W. Gibson Carey, Jr., president, Yale and Towne Manufacturing Company. Fred I. Kent, director of the Bankers Trust Company, was re-elected treasurer. Members of the executive committee elected for the ensuing year are:

Neal Dow Becker, president, Intertype Corporation, Brooklyn, N. Y.
E. R. Behrend, president, Hammermill Paper Company, Erie, Pa.
Philip E. Bliss, president, The Warner & Swasey Company, Cleveland, Ohio
Walter S. Case, president, Case, Pomeroy & Company, New York, N. Y.
S. Bayard Colgate, president, Colgate-Palmolive-Peet Company, Jersey City, N. J.
Arthur M. Collens, president, Phoenix Mutual Life Insurance Company, Hartford, Conn.
John F. Deasy, vice-president, Pennsylvania Railroad Company, Philadelphia
William C. Dickerman, president, American Locomotive Company, New York, N. Y.
James F. Fogarty, president, The North American Company, New York, N. Y.
David M. Goodrich, chairman, The B. F. Goodrich Company, Akron, Ohio
De Forest Grant, chairman, Federal Seaboard Terra Cotta Company New York, N. Y.
R. J. Hamilton, president, American Radiator Company, New York, N. Y.
John Henry Hammond, Hines, Rearick, Dorr & Hammond, New York, N. Y.
Howard Heinz, president, H. J. Heinz Company, Pittsburgh, Pa.
J. Taber Loree, vice-president, The Delaware and Hudson Railroad Corporation, Albany, N. Y.
Thomas Luke, president, West Virginia Pulp and Paper Company, New York, N. Y.
Auguste G. Pratt, president, The Babcock and Wilcox Company, New York, N. Y.
A. W. Robertson (A'27), chairman, Westinghouse Electric & Manufacturing Company, Pittsburgh, Pa.
David Sarnoff (M'23), president, Radio Corporation of America, New York, N. Y.
Malcolm B. Stone, president, Ludlow Manufacturing Associates, Boston, Mass.
J. A. Sweetser, president, Bigelow-Sanford Carpet Company, New York, N. Y.

Conference board councilors include:

Eugene G. Grace, president of the Bethlehem Steel Company
Cornelius F. Kelley, president of the Anaconda Copper Mining Company
L. F. Loree, president of the Delaware and Hudson Railroad Corporation
Hon. Andrew W. Mellon, Pittsburgh, Pa.
George M. Verity, chairman of the American Rolling Mill Company
Owen D. Young, chairman of the General Electric Company

Isle Maligne Plant Reaches Designed Capacity

With the completion of the current installation of the twelfth 33,500-kva 112.5-rpm hydroelectric unit, the Isle Maligne Hydroelectric power station at the head of the Saguenay river about 100 miles north of Quebec will have been brought to the ultimate of its designed capacity of some 402,000 kva.

Described as "the largest single installation in water power development ever undertaken" up to that time, by the late W. S. Lee in his paper "Hydroelectric Development of the Saguenay River," (AIEE Trans., v. 44, 1925, pp. 722–36), the project was authorized in December 1922, and the plant went into operation in 1925 with 8 units installed and space provided for 4 more. Located at the outlet of Lake St. John, the station is authorized to use the storage capacity of that lake between "ordinary low water and ordinary high water." Surveys established the permissible depth variation at 17.5 feet. which corresponded to an area variation from 328 square miles at low water to some 408 square miles at high water, and made available effective storage, of an estimated 187.1 billion cubic feet, adequate to serve effectively for the estimated 30,000-square-mile run-off area. Median operating head is 110 feet.

The magnitude of the project, its far-northerly location and the consequent severe winter weather conditions, and its distance from centers of transportation and civilization, made the project one of especial interest at the time of its construction. As described and discussed in some detail by Mr. Lee, "the design and layout of the construction plant were given as much attention....as the design of the power house."

Ford of Canada to Expand Power Plant

A power plant expansion program to cost about $3,500,000 has been undertaken by the Ford Motor Company of Canada, Ltd., at its Windsor plant, according to a recent announcement. The program provides for increased output, lower production cost, and the use of some of the original equipment without enlarging the present plant building.

The original plant consisted of 3 boilers each having a capacity of 100,000 pounds of steam per hour, each at a pressure of 235 pounds per square inch and a temperature of 580 degrees Fahrenheit; space was provided for a fourth boiler. Two turbogenerators were used, with capacities of 10,000 and 5,000 kw, respectively. When the present program is completed, the generating capacity will have been increased to 35,000 kw and the steam generating capacity to 600,000 pounds per hour. The ultimate development contemplated to be undertaken in the future will have a maximum output of 45,000 kw and a steam generating capacity of 800,000 pounds per hour.

The program now under way will include the installation of 3 new boilers and 2 new turbines, one of which will be a superposed unit exhausting into the existing 10,000-kw turbine. After careful study, a turbine throttle pressure of 800 pounds per square inch and temperature of 800 degrees Fahrenheit were selected as being most economical for this installation.

The 800-pound-pressure turbogenerator will be a 5,000-kw unit and will exhaust at 175 to 200 pounds pressure and 500 to 600 degrees Fahrenheit into the existing low-pressure 10,000-kw condensing turbogenerator, and in addition will supply steam for high-pressure process use. The other turbogenerator to be installed will be a 20,000-kw condensing double-bleeding unit. Both bleed points will be automatically controlled so that the turbine will furnish high-pressure process or low-pressure heating steam.

Electrical Companies to Exhibit at 1939 Fairs

Plans for electrical exhibits at the New York World's Fair and the Golden Gate International Exposition are progressing rapidly, and several large electrical companies already have signed leases for exhibition space in the fair buildings or have contracted for lots on which they will erect their own exhibition structures.

Although no definite plans for individual company exhibits have been made public, the Westinghouse Electric & Manufacturing Company, which has contracted for a floor space of 10,922 square feet in the projected Electrical Production Building (see accompanying illustration) of the New York World's Fair, has announced that its exhibit will include demonstrations of scientific research and electrical applications in air conditioning, transportation, and lighting equipment, radio, television, and X rays. The General Electric Com-

Photograph of a scale model of the Electrical Production Building to be constructed for the 1939 World's Fair at New York

pany, expecting to present an amplification of its "House of Magic," a series of dramatic demonstrations of the achievements of electrical research, has signed a contract for a lot 68,339 square feet in area upon which it will build its own exhibition structure.

The Radio Corporation of America likewise has contracted for a lot space of 81,699 square feet. David Sarnoff (M'23) president, in describing the World's Fair plans of RCA said that the exhibit would show " . . . how radio safeguards the passage of ships at sea, and the flight of ships in the air, . . . how photographs are transmitted across the ocean by radio, . . . the most modern equipment for recording sound in motion-picture studios and reproducing it in theaters." Other attractions will include a radio communications center at the fair, so that visitors may send radiograms to any part of the world, an actual demonstration of what goes on behind the scenes in radiobroadcasting, and a public demonstration of television, whatever its status may be at the time of the exposition.

Other companies that have contracted for building space at the New York fair include: Bell Telephone System, 139,400 square feet (see June 1937 issue, page 766); Consolidated Edison Company of New York, Inc., 75,183 square feet; Johns-Manville, 29,317 square feet; American Gas Association, 122,008 square feet; and United States Steel Corporation subsidiaries, 55,166 square feet.

The Electrical and Communications Exhibits Building at the Golden Gate International Exposition will occupy a ground area of 89,400 square feet near the main portal of the exposition grounds. Westinghouse Electric & Manufacturing Company, General Electric Company, and the Bell Telephone System have signed contracts for exhibition space, and have indicated broadly the nature of their exhibits. The Westinghouse display will occupy nearly 5,000 square feet, and is expected to be a comprehensive showing of products for industry and household, mostly operating demonstrations of a scientific and educational character. The General Electric exhibits will be similar to those at the New York fair. The Bell System exhibit, which will occupy a space of approximately 7,000 square feet in the electrical building, will provide demonstrations in which the visitors themselves will participate, and will present new features in communications devices. The demonstration equipment is being designed by Bell Telephone Laboratories, Inc., will be erected by the American Telephone and Telegraph Company, and will be operated by The Pacific Telephone and Telegraph Company.

Several foreign nations have completed negotiations for sending scientific and commercial exhibits to both fairs, and many others have signified their intentions of being represented.

Swedish Engineers Plan 1,500-Kilometer Line. Leading Swedish electrical engineers recently met in Stockholm to discuss problems of the electrical industry in that country, among which was a consideration of the possibility of transmitting to southern Sweden the energy produced in the rich waterpower sources of the far North. A line from Krångede, in northern Sweden, to Malmö, in the extreme South, a distance of 1,000 kilometers (621 miles) already is under construction, and a 1,500-kilometer (930-mile) line is being considered. Aluminum is being used in increasing amounts in the construction of these long Swedish lines, and the Stockholm convention of engineers paid a visit to a modern aluminum plant producing 2,000 tons of aluminum per year electrolytically, with a power consumption of 40,000,000 kilowatt-hours of electrical energy.

Standard for Welding Electrodes. The first draft of a specification for coated electrodes for structural steel has been issued by the French committee for welding standards. Eleven diameters (0.04 to 0.51 inch) and 4 lengths (6 to 18 inches) are standardized. Four standard colors are given to electrodes according to their mechanical properties. Mechanical properties are determined on all-weld-metal specimens machined from a deposit made in an angle iron 10 inches long. The speed of welding must be such that the corners of the angle iron do not become red. Copies of the draft have been ordered by the American Standards Association for use of its members.

New Generating Unit to Use 2,400-Pound Steam

According to a recent announcement of the American Gas and Electric Company, a new steam-electric generating unit to operate at the record pressure of 2,400 pounds per square inch and temperature of 940 degrees Fahrenheit will be installed in the Twin Branch station of one of its subsidiaries, the Indiana and Michigan Electric Company, Mishawaka, Ind. The decision to use such high pressure and temperature is the outcome of studies in high temperatures and pressures for generating units made by engineers of the company with the object of reducing plant investment and increasing operating efficiency, with full consideration to the financial soundness of investment.

The new unit will have a rating of 62,500 kw and will consist of a 22,500-kw 2,400-pound-pressure turbogenerator, cross-compounded with a new 40,000-kw 400-pound-pressure turbogenerator. This unit, together with the 2,500-pound-pressure boiler and accessory equipment, will entail an expenditure of some $4,000,000.

Westinghouse Creates "New Products" Division

Responsibility for developments either new in the art or not previously undertaken by the company has been centralized by the Westinghouse Electric & Manufacturing Company of East Pittsburgh, Pa. in a newly formed "new products" division, according to a recent announcement. Functioning the same as the other operating units of the company, this division will include separate engineering, manufacturing, and sales departments.

The division will concentrate its facilities in developing and broadening the field of applications of products that have not reached the status of commercial apparatus. Then, after the field for each new product has been determined and the product has passed through the initial plant stage, the product will be taken over by one of the regular divisions. It is stated also that the new division will be responsible for training a staff that will be "adequate for the further promotion of the product."

Technological Trends Studied by Government. New inventions necessitate constant revision of national policies, according to a report of the National Resources Committee on "Technological Trends and National Policy," recently released by Chairman Harold L. Ickes, Secretary of the Interior. The first report on the science resources of the United States deals with new inventions and their social implications and calls attention to the need for planning to meet the new situations caused by technological advance. Prepared by a large group of scientists and engineers, the report "presents a survey of most of the fields of technology and applied science. . . . " The National Resources Committee urges that a series of continuous studies be undertaken by existing planning boards and by appropriate agencies developed by the Federal Government, with the definite function of investigating and reporting on the progress and trends of science and invention, and the possible social and economic effects that may follow them. The report recommends that to study the technical aspects of the question the science committee of the Natural Resources Committee make an investigation of the adequacy of reporting on inventions and discoveries and advise on the feasibility of more balanced coverage, of selecting those more socially significant, and of assembling such data in some central location. Copies of the report are available from the Superintendent of Documents, Washington, D. C.

More Boulder Dam Generators. Contracts for the supply of 2 additional turbo-generator units to increase the generating capacity of the Boulder Dam plant to half of its full projected value of 1,835,000 horsepower recently were awarded. The new turbines, vertical-shaft units rated at 115,000 horsepower each, are to be constructed by the Allis-Chalmers Manufacturing Company. Contracts for the generating equipment were awarded to the Westinghouse Electric & Manufacturing Company. Already serving the city of Los Angeles and other municipalities in southern California are 4 3-phase 60-cycle units, each rated at 82,500 kva, with an output voltage of 16.5 kv. Of these, 2 were manufactured by the General Electric Company and 2 by the Westinghouse Electric & Manufacturing Company; in addition, the General Electric Company now is constructing 2 similar units, which will be completed in 1938. The units just contracted for will be ready for operation in 1939 and 1940.

New York Building Code Now Permits Welding. A new building code for the city of New York, N. Y., to become effective January 1, 1938, permits the use of welding in place of riveting in steel construction work. Previously, welding was prohibited because the existing code was formulated before modern welding methods were evolved. This action is congruous with New York's recent noise-abatement campaign; moreover, welded steelwork is advantageous in several other ways, being lighter, stiffer, creating more compact joints, and effecting an appreciable saving in the amount of steel required.

Power Exposition to Be Held. The Chicago (Ill.) Exposition of Power and Mechanical Engineering, to be held at the New International Amphitheatre, October 4–9, 1937, is expected to be one of the largest events of its kind ever held in the Midwest area, according to plans of its sponsors. Advance reports of machines and materials to be exhibited are said to be comprehensive and of interest to both executives and operators in the power and mechanical-engineering fields.

Standard Symbols for Welding. Realizing that welding cannot take its place as an engineering tool unless suitable means for conveying explicit information from designer to workman are provided, the American Welding Society has derived and published a set of standard welding symbols covering both fusion and resistance welding. The symbols, which are ideographic, have been made as simple as possible, and are based upon a principle of having the most common form of any weld require a minimum of numerical data. In effect the system amounts to a shorthand system with which a large amount of information may be transmitted in a few simple lines and figures. Complete information may be obtained from the American Welding Society, 33 West 39th Street, New York, N. Y.

Current Items From American Engineering Council

Who's Who in Engineering

Engineers and engineering organizations have a new directory and biographical dictionary in the fourth edition of "Who's Who in Engineering," which has just been published by the Lewis Historical Publishing Company, Inc., 8th Avenue and 14th Street, New York, N. Y. It is well done and is a most excellent reference volume. In addition to the biographical dictionary this issue carries a comprehensive list of engineering organizations in the United States and brief descriptions of each of several functional instrumentalities of engineering societies.

Coal Price Fixing Is Here

Twenty-three district boards of code members managed by the National Bituminous Coal Commission in the Investment Building at 15th and K Streets, N. W., Washington, D. C., came into existence on June 21, 1937. All are authorized by the Guffey or Bituminous Coal Act of 1937, entitled "An act to regulate interstate commerce in bituminous coal and for other purposes." The principle objectives are the establishment of marketing rules and regulations, fixing minimum and maximum prices, and the prevention of unfair competition in the bituminous-coal industry.

The Bituminous Coal Act of 1937 also created a Consumers' Counsel to provide protection for all consumers of bituminous coals in the United States. The counsel has established headquarters in the Tower Building at 14th and K streets, N. W., Washington, D. C. "It shall be the duty of the Consumers' Counsel to appear in the interest of consuming public in any proceedings before the Commission, to offer testimony, examine witnesses or parties to the proceedings," and to have the Commission certify to information or conduct investigations of conditions believed to be responsible for unfair consumer prices.

American Engineering Council has been asked to assist Consumers' Counsel and the Commission with the location of engineers and engineer executives experienced in the several phases of the bituminous-coal industry and it has been invited to co-operate in bringing the objectives of these regulatory bodies to the attention of all engineers connected with or interested in bituminous-coal production, distribution, and consumption. Both agencies are seeking engineering advice and assistance with efforts being made to improve the soft-coal situation in the public welfare.

Engineering societies are urged to acquaint individual engineers and committees with the fact that the coal agencies "are now attempting to help establish fair minimum and maximum prices and are sponsoring the promulgation of rules and regulations regarding the production, marketing, and use of bituminous coal which may have far reaching effects on all fuel users as well as all fuel producers, dealers, etc." Results of such efforts are likely to change power and other industrial costs for which certain engineers may be directly or indirectly responsible

Copies of the "Bituminous Coal Act of 1937, the Bituminous Coal Code, Taxes on the Sale or Other Disposal of Bituminous Coal, and Rules of Practice and Procedure before the National Bituminous Coal Commission" may be obtained directly from the agencies named, from the National Coal Association in the Southern Building, or from the Government Printing Office in Washington, D. C. The Counsel and the Commission invite correspondence and co-operation calculated to improve the present bituminous coal situation.

Most of the activities of these soft-coal agencies involve problems of more concern to trade associations than professional societies, but questions of potential importance to engineers employed in industry as well as those in consulting and private practice are having serious consideration. It is, therefore, suggested that engineers who are concerned with any phase of the bituminous-coal industry or the use of soft coal in other industries be urged to keep themselves advised regarding what is being done by these government regulatory bodies with soft-coal prices and the control of production, distribution, and consumption.

Consumers' counsel is John C. Carson of Michigan, former secretary to the late Senator Couzens. His principal assistants are Ellery P. Gordon and W. L. Chandler. The commission is: Chairman, Charles F. Hosford, Jr., of Pennsylvania; Pleas E. Greenlee of Indiana, Thomas S. Hammond of Kentucky, John C. Lewis of Iowa, Walter H. Maloney of Missouri, C. E. Smith of West Virginia, and Percy Tetton of Ohio. Although the National Bituminous Coal Commission and the Consumers' Counsel were created jointly by the Bituminous Coal Act of 1937, they function as independent agencies of the Department of Interior under the administrative control of Secretary Ickes.

Letters to the Editor

CONTRIBUTIONS to these columns are invited from Institute members and subscribers. They should be concise and may deal with technical papers, articles published in previous issues, or other subjects of some general interest and professional importance. ELECTRICAL ENGINEERING will endeavor to publish as many letters as possible, but of necessity reserves the right to publish them in whole or in part, or reject them entirely.

ALL letters submitted for consideration should be the original typewritten copy, double spaced. Any illustrations submitted should be in duplicate, one copy, to be an inked drawing but without lettering, and other to be lettered. Captions should be furnished for all illustrations.

STATEMENTS in these letters are expressly understood to be made by the writers; publication here in no wise constitutes endorsement or recognition by the American Institute of Electrical Engineers.

AIEE Publications

To the Editor:

The recent discussion at the summer convention in Milwaukee, Wis., regarding Institute publications prompts me to send the following comments.

At the outset I want to express my keen appreciation of the excellent work that has been done by the publication committee and of the able manner in which the editorial work of ELECTRICAL ENGINEERING has been conducted. As a regular contributor to Institute publications I have had full co-operation. The work has been efficiently conducted and speedily and accurately executed. Nor is my experience unique in this respect. I believe that a survey will show that other contributors have a similar reaction to Institute publications.

I am also mindful of the excellent services rendered to the Institute by the technical program committee. They have given us excellent technical programs at all Institute conventions. They have introduced technical conferences at which ideas are exchanged freely by authorities in the field, Through their efficient and capable secretary they have given contributors prompt and courteous service which is beyond criticism.

It is not in the administrative aspects of Institute publications that the fault lies, it is rather in the quality of the articles that are submitted for publication.

If one were to classify the Institute membership according to their interests he would find the following classes:

a. The highly-specialized analytically-inclined members. This group comprises a rather small minority of the Institute membership. It is technically strong and is responsible for the majority of the papers published in ELECTRICAL ENGINEERING and the TRANSACTIONS.

b. The practicing engineers whose work is of a technical nature and who like to keep up with the progress of the art but who are not sufficiently specialized or analytically inclined to read and understand the majority of the published work. This group constitutes, by far, the largest minority (if not a majority) of the Institute membership.

c. The student and cadet engineers who have no practical background but who will form the future backbone of the Institute. They constitute a sizeable minority of the membership.

A few would question the fact that Institute publications should be so designed as to meet the needs and requirements of all the membership. The problem is how to modify the publications so as to accomplish this result.

I wish to submit a plan which would seem to solve this problem and which follows closely the general plan presented by the publication committee and has for its object the endorsement of the policies submitted by that committee. The plan consists of making the TRANSACTIONS and ELECTRICAL ENGINEERING 2 entirely distinct publications.

The TRANSACTIONS would comprise exclusively highly technical and highly specialized papers which contain new contributions to the art. Such papers would be submitted to the various technical committees of the Institute and would be earmarked by these committees as worthy of presentation at Institute conventions and of publication in the TRANSACTIONS. The TRANSACTIONS would thus become a publication which satisfies the needs of group *a* of the membership.

The journal ELECTRICAL ENGINEERING, since it is received by all the members of the Institute, would comprise articles of interest to the membership at large (groups *a*, *b*, and *c*). Papers published in ELECTRICAL ENGINEERING would also be submitted to the various technical committees but would be earmarked by these committees for publication in the journal. Such articles need not contain new contributions. The following types of articles might properly be published in ELECTRICAL ENGINEERING:

a. Expository articles, written in text-book form and readily understood by practicing electrical engineers and engineering students, on important recent developments in electrical engineering. The paper by Travis on per-unit quantities and Prentice's paper on fundamental concepts of synchronous machine reactances, form excellent illustrations of this class.

b. Articles on economic and social problems of the engineer. Hellmund's paper in the April 1937 issue of ELECTRICAL ENGINEERING and Spooner's paper in the December 1934 issue are typical of this class. I might also here include the excellent addresses of Doctor Bush and Doctor Flanders which were delivered at the Milwaukee convention.

c. Articles on recent developments in the basic sciences such as physics and chemistry written from the point of view of possible application in the practice of electrical engineering. The services of the committee on basic sciences (formerly the electrophysics committee) may be pressed for obtaining such papers.

d. Educational articles. Several articles on education have appeared in Institute publications. These articles have always aroused interest not only among educators but also among practicing engineers.

e. Descriptive articles of recent engineering projects. Such articles are of interest to utility and power engineers.

f. Articles on the professional aspects of engineering including engineering ethics, licensing of engineers, etc.

g. News of the Institute, its activities and its members. This section in ELECTRICAL ENGINEERING is read by practically all members.

h. Short abstracts, prepared by the authors, of all technical papers presented at Institute conventions and published only in the TRANSACTIONS.

i. Student papers which are awarded prizes or/and honorable mention.

j. Letters to the editor.

Having at last arrived at the letter *j* my imagination ceases to function so I shall close with the hope that the readers of this letter will take enough interest in this vital matter to communicate to you their reaction to this proposal.

Very truly yours,

MICHEL G. MALTI (M'34)

Assistant Professor of Electrical Engineering,
Cornell University, Ithaca, N. Y.

Most Economical Voltages For Incandescent Lamps

To the Editor:

Until recently the writer has seen nothing written explaining how to determine the most economical voltages at which to operate incandescent lamps. When purchasing Mazda lamps the consumer has the 110-115-, and 120-volt lamps from which to select. If the socket voltage happens to be, say 115 volts, it most certainly does not follow that the most economical lamp to buy will be the 115-volt lamp. This fact, and a method to determine which voltage lamp the purchaser should buy, the writer will attempt to explain in this article.

It is well known that if the operating voltage of a lamp is raised, the life of the lamp will be materially shortened, and the lumens output will be increased accordingly. If the operating voltage is reduced, the opposite effect will take place, that is, the life will be lengthened and the lumens output diminished. If the socket voltage is very low, the cost of lamp renewals will be almost eliminated, but the cost of electric power per lumen will be high. On the contrary, if the socket voltage is high, the lumens output will be increased, but the total cost per lumen-hour will be prohibitive because of the cost of lamp renewals. At some voltage in between these extremes (for any given lamp with a specified cost for power) lies the point at which the user can get the most lumen-hours for his money.

In the Bureau of Standards Research Paper No. 502, characteristic equations are given for tungsten-filament vacuum and gas-filled lamps. They are of the form

$$y_n = A_n x^2 B_n + C$$

The equations express the relation of voltage to lumens output, to current, to watts input, and to the efficiency. In these equations the variable x is the logarithm of the ratio of any voltage to that voltage at which a lamp operates when it emits light at an assumed "normal" efficiency. The variable y_n is the logarithm of the ratios of currents, lumen outputs, or watts at these voltages. A, B, and C are constants.

These equations have proved to be quite accurate, but are cumbersome to use. Professor Parry Moon ("The Scientific Basis of Illuminating Engineering," McGraw-Hill Book Company, Inc.) has reduced these equations to a simpler form, for use where great accuracy is not required. However, by calculation, the errors do not exceed 5 per cent, even though the voltage varies from 80 per cent to 130 per cent normal. These equations, in part, are as follows:

$$F/F_0 = (V/V_0)^{B_2} \text{ luminous flux} \qquad (1)$$

$$P/P_0 = (V/V_0)^{B_3} \text{ power} \qquad (2)$$

$$L/L_0 = (V/V_0)^{-B_5} \text{ life} \qquad (3)$$

where V_0 is the rated voltage of the lamp and F_0, P_0, and L_0 are the values at rated voltage. The constants are obtained from table I.

With the ratio of socket voltage to lamp voltage as the independent variable, Professor Moon has set up another equation giving the cost per lumen-hour. Differentiating, a minimum voltage ratio is found, which is the most economical operating point. The proof will not be given here, but the general equation arrived at giving the voltage ratio for most economical operation is

$$\left(\frac{V}{V_0}\right)_m = \left[\frac{bP_0}{c}\,\frac{B_2 - B_3}{B_5 - B_2}\right]^{\frac{1}{B_5 - B_2}}$$

where
c equals cost of one lamp plus installation charge (cents)
b equals cost of power in cents per kilowatt-hour
V equals actual operating voltage
V_0 equals rated voltage of lamp
P_0 equals rated power of lamp

Constants are taken from table I.

For each division of lamps in table I the writer has reduced equation 4 so that curves may be plotted, making it easier to determine the most economical voltage. These curves are shown in figure 1. As an example, consider a residence where b equals 5 cents per kilowatt-hour, P_0 equals 60 watts, and c equals 15 cents.

$$\frac{bP_0}{c} = \frac{5 \times 60}{15} = 20$$

From figure 1 the corresponding value of V/V_0 equals 1.126. If the circuit is operating at V = 115 volts, the lamp rating for most economical operation should not be 115 volts, but

$$V_0 = \frac{V}{1.126} = 102 \text{ volts (approximately)}$$

As the cost of power becomes lower, V_0 and V become more nearly equal. As a general rule, it is more economical to operate lamps above rather than below their rated voltage. It is of interest to note from the curves that when bP_0/c = 4.5 or less, then V/V_0 becomes less than unity, and in that case, lamps should be operated below their rated voltage.

Table I

Symbol	Gas-Filled Lamps 40-50 Watts	Gas-Filled Lamps 60-150 Watts	Gas-Filled Lamps 200 Watts and up
B_2	3.685	3.613	3.384
B_3	1.523	1.523	1.543
B_5	13.5	13.5	13.1

After determining from figure 1 what voltage lamp to buy, one can compute the savings to be made by using equations 1, 2, and 3. If it is desired to make a comparison on the basis of cost per 1,000 lumen-hour hours, the additional formula may be used,

$$\text{Cost per 1,000 lumen-hours} = \frac{Ct + c}{Lt/1{,}000} \qquad (5)$$

where
C equals cost of power in cents per hour
t equals life of lamp in hours
L equals average lumens output of lamp

A very interesting example met with in actual practice, involving all of the foregoing formulas, is as follows: A certain firm was using 200-watt lamps for general illumination, which was considered sufficient. The cost of electricity was approximately $2^1/_2$ cents per kilowatt-hour. The average operating voltage was close to 113 volts. Lamps rated 120 volts were bought at a 20 per cent discount, and the labor cost for installation was estimated at 8 cents per lamp.

$$\frac{bP_0}{c} = \frac{2.5 \times 200}{48 + 8} = 8.93$$

From figure 1, V/V_0 equals 1.053. Therefore, the lamp rating for most economical operation is

$$V_0 = \frac{113}{1.053} = 107 \text{ volts (approximately)}$$

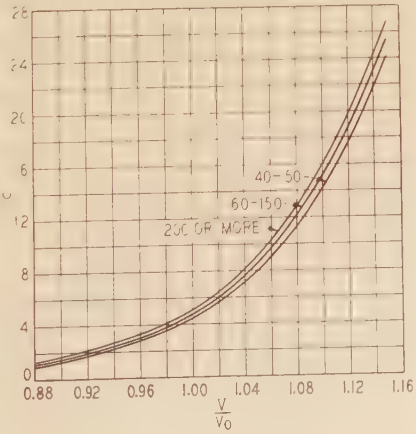

Figure 1. Curves for determining most economical voltage

Numbers on curves are ratings of lamps in watts

so 110-volt lamps should be used. But it has been stated that the illumination was considered sufficient, and using 200-watt lamps of this voltage rating would increase the illumination, the number of burn outs, and the power.

A comparison may now be made with 150-watt lamps. Using equations 1, 2, and 3 for the 200-watt 120-volt lamp, and obtaining the value of F_0 from the manufacturer's table,

Lumens output at 113 volts
Luminous flux = F =

$$F_0\left(\frac{V}{V_0}\right)^{B_2} = 3{,}400\left(\frac{113}{120}\right)^{3.384} = 2{,}770 \text{ lumens.}$$

Life of lamp = L =

$$L_0\left(\frac{V}{V_0}\right)^{-B_5} = 1{,}000\left(\frac{113}{120}\right)^{-13.1} = 2{,}190 \text{ hours.}$$

Power = P =

$$P_0\left(\frac{V}{V_0}\right)^{1.543} = 200\left(\frac{113}{120}\right)^{1.543} = 182.5 \text{ watts.}$$

Similarly, for the 150-watt 110-volt lamp on a 113-volt circuit, the following values are obtained:
F equals 2,660 lumens
L equals 698 hours
P equals 156.2 watts
Using equation 5 for the 200-watt 120-volt lamp,

Cost per 1,000 lumen-hours =

$$\frac{2.5 \times \frac{182.5}{1{,}000} \times 2{,}190 + 56}{2{,}770 \times 2{,}190 \div 1{,}000} = 0.174 \text{ cents.}$$

For the 150-watt 110-volt lamp,
Cost per 1,000 lumen-hours =

$$\frac{2.5 \times \frac{156.2}{1{,}000} \times 698 + 40}{2{,}660 \times 698 \div 1{,}000} = 0.168 \text{ cents}$$

Now since the 150-watt lamp in this case has a lumen output almost equal to that of the 200-watt lamp, and the cost per lumen-hour is less for the 150-watt lamp, then the latter is the one to use.

However, the most interesting part is that there was another electric rate which the firm wanted to get on. The cost for power on this more favorable rate would amount to about 2 cents per kilowatt-hour, while the firm was paying around $2^1/_2$ cents per kilowatt-hour on the rate which it was on. In the availability clause of this better rate was this requirement, "the lighting load shall not exceed 10 per cent of the total connected load." This requirement made the firm ineligible for the better rate, although their percentage of lighting load did not exceed 10 per cent by a great margin. However, it so happened that changing over to the 150-watt 110-volt lamps made it possible to get more lumens per kilowatt-hour, practically the same amount of illumination, and at the same time, to reduce the lighting load so as to get on this more favorable rate and net a savings of almost $2,000 a year.

Very truly yours,
CAMPBELL McCORD
Engineer, The Knoxville
News-Sentinal, Knoxville, Tenn.

Personal Items

S. P. MacFadden (A'19, M'30) vice-president of the Puget Sound Power & Light Company, Seattle, Wash., has assumed new duties as executive vice-president of the company. Mr. MacFadden was born June 17, 1894, at Laredo, Texas and received the degree of bachelor of science in electrical engineering at the Texas Agricultural and Mechanical College in 1916. Following his graduation, he entered the student engineering course of the Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa., and upon completion of the course in 1917 became affiliated with the Eastern Texas Electric Company, Beaumont. In the same year, however, he received a commission as lieutenant in the Coast Artillery Corps of the United States Army, and served with the American Expeditionary Force in France until 1919. At the end of the World War, Mr. MacFadden returned to the employ of the Eastern Texas Electric Company, but remained there only briefly before being designated assistant to the district manager of the southwestern district of Stone and Webster, Inc., with offices at Houston, in 1920. From 1924 until 1930 he was associated with several companies, including the Eastern Texas Electric Company, Port Arthur; Western Public Service Company, Scottsbluff, Nebr.; and Stone and Webster Service Corporation, Boston, Mass. Mr. MacFadden has been vice-president of the Pugent Sound Power & Light Company since 1930.

F. B. Corey (A'93, F'12, member for life) plant engineer, Pittsburgh Valve and Fittings Division, The Pitcairn Company, Barberton, Ohio, has retired from active service. Mr. Corey was born September 28, 1869, at Homer, N. Y., and was graduated from Cornell University with the degree of mechanical engineer in 1892, following which he entered the employ of the Elektron Manufacturing Company as a sales engineer in the Boston, Mass., offices. After several other brief affiliations he became a designing engineer in the railway engineer department of the General Electric Company in 1900. Mr. Corey remained with that company until 1911 before becoming engineer of inspection and tests for the Union Switch and Signaling Company Swissvale, Pa. In 1914 he established his own consulting-engineering offices in Pittsburgh, Pa., but in 1916 was appointed efficiency engineer for the Pittsburgh Valve and Fittings Company, Barberton. He was employed by that company in various capacities for more than 20 years before his retirement.

B. T. Anderson (A'35) who has been director of the bureau of railway signaling economics of the Union Switch & Signal Company, New York, N. Y., has been appointed general sales manager of the company. Mr. Anderson, a native (1884) of Alexis, Ill., received the degree of bachelor of science in electrical engineering at the University of Illinois in 1907, and upon graduation entered an apprenticeship course in railway signaling in the Union Switch and Signaling Company, Swissvale, Pa. In 1910 he became a draftsman for the Atchison, Topeka, and Santa Fe Railway Company, Topeka, Kans., but later was made construction foreman and assistant signal engineer and remained with that company until 1914. In that year he was appointed assistant signal engineer with the Delaware, Lackawanna, and Western Railway Company, Hoboken, N. J., where he remained until 1923. For 4 years (1923–27) Mr. Anderson was superintendent of signals for the Chesapeake and Ohio Railway Company, with offices at Richmond, Va., and during that period became active in the work of the American Railway Association. Since 1927 he has been affiliated with the Union Switch and Signaling Company as assistant to the vice-president, assistant to the president, publicity representative, and director of the bureau of railway signaling economics.

M. M. Kenneally (M'29) for the last 7 years manager of the power utilities department of the Ohio Brass Co., Mansfield, has been appointed sales manager of the Porcelain Insulator Corporation, Lima, N. Y. Mr. Kenneally was born March 16, 1894, at Oelwein, Iowa, and received the degrees of bachelor of science in electrical engineering (1915) and electrical engineer (1920) at Iowa State College. Following brief associations with several public utility companies, he was appointed electrical engineer on operation and construction for the Iowa Public Service Company, at Humboldt, in 1924. In 1926 Mr. Kenneally became assistant to the electrical engineer of the United Gas Improvement Company, Philadelphia, Pa., in which capacity he assisted in designing and supervising the construction of transmission equipment for that system. In the following year he became affiliated with the Ohio Brass Co. in its high-voltage laboratory, and after a few months' training there, was transferred to the New York, N. Y., offices of the company. In 1928 he was named district sales manager in the New York area, and in 1930 was returned to the Mansfield offices as manager of the power utilities department, in charge of sales, advertising, and engineering.

H. F. Boe (M'30) eastern district manager, Westinghouse Electric & Manufacturing Company, New York, N. Y., has assumed additional duties as commercial manager of the company. Mr. Boe, a native (1884) of Mansfield, Ohio, attended Carnegie Institute of Technology, and has been associated with the Westinghouse company since 1900, continuously except for one year (1916–17) as chief engineer and manager of the Vaile Kimes Company, Dayton, Ohio. Until 1916 Mr. Boe's work was in general engineering and sales in the East Pittsburgh offices of the Westinghouse company; in 1918 he was transferred to the Rochester, N. Y., offices as a sales engineer, and in 1922 he went to Buffalo, N. Y., as district industrial division manager. Later he was made manager of that office, and in 1935 was transferred to New York, N. Y., where in 1936 he became eastern district manager.

H. W. Tenney (M'36) section engineer in the materials and process engineering department of the Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa., has been appointed engineering manager of the recently created new products division of the company. Mr. Tenney, a native (1899) of Leominster, Mass., received the degrees of bachelor of science in electrical engineering (1920) and electrical engineer (1922) at Worcester Polytechnic Institute, and has been associated with the

An aerial view of the Nela Park Plant of the General Electric Company, Cleveland, Ohio which may be visited by those attending the Middle Eastern District meeting, Akron, Ohio, October 13-15

Westinghouse company continuously since his graduation in 1920. He entered the employ of the company as a laboratory assistant, but later became successively oscillograph specialist, foreman in charge of the engineering laboratory, assistant section engineer in the materials and process engineering section, and section engineer.

S. L. NICHOLSON (A'00, F'13, member for life) assistant to vice-president, Westinghouse Electric & Manufacturing Company, New York, N. Y., has relinquished his duties in that organization, and is retiring from active business. Mr. Nicholson was born November 10, 1870, at Germantown, Pa., and received his formal education in private schools in Philadelphia. After having held several engineering positions in Philadelphia and New York, he was engaged by the Westinghouse company as a sales engineer in 1898. In 1909 Mr. Nicholson was made sales manager, and in 1917 became assistant to the vice-president. He has been active in the affairs of the National Electrical Manufacturers Association and the American Standards Association.

VANNEVAR BUSH (A'15, F'24, Lamme Medalist '35, director) vice-president and dean of engineering, Massachusetts Institute of Technology, Cambridge, has been appointed a member of a special committee of the National Research Council to make a comprehensive survey of the applications of radio, sound recording and reproduction, motion pictures, photography, and other scientific achievements in the field of education.

C. L. COLLENS (A'07) president of the Reliance Electric and Engineering Company of Cleveland, Ohio, has been elected to membership on the board of directors of the American Standards Association, on nomination of the National Electrical Manufacturers Association. Mr. Collens is a past member of the ASA board (1929–32) and a past-president of NEMA in various activities of which he long has been active.

F. B. JEWETT (A'03, F'12, Edison Medalist '28, past-president) vice-president American Telephone and Telegraph Company, New York, N. Y., has been appointed a member of a special committee of the National Research Council to make a comprehensive survey of how radio, sound recording and reproduction, motion pictures, photography, and other scientific achievements may be applied to learning.

W. R. VAN BOKKELEN (A'12, M'26) has been appointed chief of the division of electricity for the 1939 Golden Gate International Exposition at San Francisco, Calif. Mr. Van Bokkelen was electrical engineer and chief engineer of the Coast Counties Gas and Electric Company, San Francisco, for 18 years; has been in consulting practice for the past few years.

G. V. HARRAP (A'34) assistant engineer in the power station of the Hull (England) Corporation Electricity Department, has been awarded a premium by the Institution of Electrical Engineers (Great Britain) for his paper "Taxation of Electricity Undertakings," recently presented before a session of the Institution.

O. A. BECKLUND (Enrolled Student) has received the AIEE Great Lakes District prize for branch paper for his paper "An Artificial Ear for Sound Measurements." Mr. Becklund is a native (1915) of St. Paul, Minn., and received the degree of bachelor of engineering in June 1937, at the University of Minnesota.

R. E. LYLE (A'32) who has been an electrical engineer in the engineering experiment station of Washington State College, Pullman, recently joined the engineering division of the Westinghouse Electric & Manufacturing Company, Seattle.

FRANK WALSH (A'26, M'30) district manager, Puget Sound Power and Light Company, Bellingham, Wash., has been elected a vice-president of the Northwest Electric Light and Power Association for the state of Washington.

P. J. AMBROSE (A'36) who has been a radio-engineering instructor at Indiana Technical College, Fort Wayne, now is employed in the power distribution department of the Consumers Power Company, Jackson, Mich.

E. A. RICKER (A'36) formerly a demonstrator on the faculty of applied science of the University of Toronto (Ont., Canada), now is a draftsman for the Canadian Westinghouse Company, Hamilton, Ont.

W. H. MCGRATH (A'02, M'20) has retired from active duty as executive vice-president of the Puget Sound Power & Light Company, Seattle, Wash. He will continue as a director of the company.

F. L. WEST (A'37) assistant engineer, Niagara, Lockport, and Ontario Power Company, Buffalo, N. Y., has been transferred to the Avon, N. Y., offices of that company in the capacity of superintendent.

H. E. PEARSON (A'26, M'32) formerly district power maintainer for the Board of Transportation of the City of New York, N. Y., now is employed by the Roller-Smith Company, Bethlehem, Pa.

Obituary

JOHN WHITE HOWELL (A'87, M'88, F'12, Edison Medalist '24, member for life, past manager) retired chief engineer of the lamp works of the General Electric Company, died July 28, 1937. Mr. Howell was born at New Brunswick, N. J., December 22, 1857. In 1881, he finished a special course in electrical engineering at Stevens Institute of Technology, and received the honorary degree of electrical engineer in 1899. He became identified with the Edison Lamp Company as an electrician in 1881, and became technical adviser to the manager of works in 1890; in 1912, when the company became a part of the General Electric Company, he was engineer and assistant manager of the lamp works. He retired in 1931 as chief engineer. He was recognized as one of the most distinguished of electrical pioneers in the production of incandescent lamps. Among his important achievements were the development of a successful portable voltmeter, a Wheatstone bridge type of potential indicator, and the comparative indicator. He determined for the first time the relation between the life and candle power of an incandescent lamp (1886); introduced a carbonaceous clamp that decreased clamping and filament costs and increased the quality of the lamp (1887); and later made changes to effect an increase in the speed of exhausting the envelope. He improved the Thomson-Houston method of treating carbon filaments, and devised a machine that made great advances in the treating process. He assisted in the design of the first stem-making machine and in the derivation of metalized filaments. Mr. Howell was a manager of the Institute, 1888–90, and a member of The American Society of Mechanical Engineers, Franklin Institute, and other technical societies. He was a member of the Edison Medal Committee from 1926 to 1931.

CHARLES BROSSMAN (M'28) consulting engineer, Indianapolis, Ind., died June 30, 1937. Mr. Brossmann was born January 17, 1877, at Philadelphia, Pa., and received his formal technical education at the Philadelphia Manual Training High School. In 1896 he became a draftsman and designer for the Baldwin Locomotive Works at Philadelphia, were he remained until 1898, when he served in the Spanish-American War. In 1900, he was engaged by the Indianapolis Water Company for some special design work, and in 1904 he established the consulting-engineering company of Brossman and King, with offices at Indianapolis. Mr. Brossman maintained his consulting practice continuously for more than 32 years, and was active in the affairs of the Indiana Engineering Society. He was a member of The American Society of Mechanical Engineers, American Society of Civil Engineers, and American Water Works Association.

FRANK BENJAMIN RAE (A'84, M'20, member for life) retired consulting engineer, Berkeley, Calif., and a charter member of the Institute, died July 28, 1937. Mr. Rae was born July 25, 1854, at Elmira, N. Y., and was self-educated. From 1870 until 1881 he was engaged in the telegraph business, first as messenger and operator, and later as manager of the Atlantic and Pacific Telegraph Company and the Western Union Telegraph Company. In 1881, he resigned the position of electrician at San Francisco for the Western Union company and became engineer for the Brush Electric Light Company, San Francisco, and manager of the San Jose (Calif.) Brush Electric Company. In 1883 he became engineer for the Commercial Telegram Company, an organization devoted to dispatching stock quotations, at New York, N. Y. From 1885 to 1887 he was a consulting engineer in New York, as a member of the firm of Mailloux and Rae. In 1887 Mr. Rae was consulting engineer for the Barker Syndicate in China, and from 1887 until 1892 was engineer for the Detroit (Mich.) Electrical Works, engaged on the Rae Electric Railway System. Since 1892 he had been a consulting electrical engineer in various cities throughout the United States. In 1923 he became electrical engineer for the city of Berkeley, Calif., but resigned and retired from active service in 1930. Mr. Rae was the holder of about 100 patents on various electrical devices and systems.

William Godfrey Heptinstall (A'26) manager of Waikato Carbonisation, Ltd., Rotowaro, New Zealand, died October 1, 1936, according to word just received at Institute headquarters. Mr. Heptinstall was born July 2, 1883, at Yorkshire, England, and attended London (England) University. Following a period of service with the Royal (British) Naval Volunteer Reserve he went to Canada, where he took charge of the operation of a small municipal gas and electric plant for the town of Minnedosa, Man. In 1920 he became plant superintendent of the Lignite Utilization Board, Bienfait, Sask., and remained there for almost 10 years before going to New Zealand to become manager of Waikato Carbonisation, Ltd.

Robert King (A'35) electrical engineer in the port department, City of Oakland, Calif., died recently. Mr. King was born March 4, 1885, in Alaska, and received his technical education by serving an apprenticeship in an electrical shop. In 1901 he established an electrical-contracting business, which he maintained until 1925, when he started a wholesale electrical-supply business. From 1927 until 1929 he served as a consulting engineer during the construction of several public schools in or near Oakland. He was appointed electrical engineer in the port department of that city in 1929.

Membership

Addresses Wanted

A list of members whose mail has been returned by the postal authorities is given below, with the addresses as they now appear on the Institute record. Any member knowing of corrections to these addresses will kindly communicate them at once to the office of the secretary at 33 West 39th St., New York, N. Y.

Charlton, H. C., 4827 Wilson Avenue, Montreal, Que., Canada.
Evans, Maldwyn F., Canadian Comstock Company, Ltd., Toronto, Ont., Canada.
Gregory, G. A., 1217 Jefferson Street, Olympia, Wash.
Hardy, John E., 1741 Pennsylvania Avenue, Denver, Colo.
Jones, K. B., Trumbull Electric Manufacturing Company, Inc., Ludlow, Ky.
Roberts, Fred A., 2010 West Michigan Street, Indianapolis, Ind.
Schaefer, Philip E., 5154 St. Paul Avenue, Chicago, Ill.
Sikofsky, George L., 1206 East New York Avenue, Brooklyn, N. Y.

8 Addresses Wanted

Applications for Election

Applications have been received at headquarters from the following candidates for election to membership in the Institute. If the applicant has applied for direct admission to a grade higher than Associate, the grade follows immediately after the name. Any member objecting to the election of any of these candidates should so inform the national secretary before September 30, 1937 or November 30, 1937, if the applicant resides outside of the United States or Canada.

Barbee, J. E., Humble Pipe Line Company, Houston, Texas.
Barnett, H. E., Cincinnati Suburban Bell Telephone Company, Cincinnati, Ohio.
Beck, R. W. (Member), 121 City Light Bldg., Seattle, Wash.
Birdsong, T. S., Alabama Power Co., Birmingham, Ala.
Casselman, C. H., Buffalo Niagara Electric Corp., Tonawanda, N. Y.
Conklin, W. A., Commonwealth and Southern Corporation, Jackson, Mich.
Curtis, C. E. (Member), University of Texas, Austin, Texas.
Daugherty, W. W., Bethlehem Steel Corporation, Cornwall, Pa.
Davis, R. L., General Electric Company, New York, N. Y.
De Arman, C. R., The Bristol Company, Waterbury, Conn.
DeVaughn, W., Forty Wall Street Corp., New York, N. Y.
Ellis, H. W. G., Ferranti Electric, Limited, Toronto, Ont., Canada.
Hall, E. S., Kansas Gas and Electric Company, Newton.
Harris, C. A., Southwestern Bell Telephone Co., St. Louis, Mo.
Hay, J. V., Southern Bell Telephone and Telegraph Company, New Orleans, La.
Johnson, L. C., Washington Water Power Company, Spokane.
Kight, M. B., Bureau of Reclamation, Denver, Colo.
Knight, N. (Member), Bell Telephone Co. of Canada, Toronto 2, Ont., Can.
Masso, A., Hertner Electric Company, Cleveland, Ohio.
MacGregor, D. M., Phoenix Engineering Corp., New York, N. Y.
Nannini, F., 445 Sierra Street, Reno, Nev.
Neuhardt, F. H., General Electric Company, New York, N. Y.
Pahl, J. G., K-P-F Electric Company, San Francisco, Calif.
Parri, I. W., Perfex Controls Co., Milwaukee, Wis.
Rose, A. M., American Telephone and Telegraph Company, Cleveland, Ohio.
Rothschild, H., Cline Electrical Mfg. Co., Chicago, Ill.
Sigman, F. I., McCarthy Brothers and Ford, Buffalo, N. Y.
Siegel, R., 393 Belmont Ave., Newark, N. J.
Smith, L. L., Pacific Tel. & Tel. Co., Portland, Ore.
Taylor, J. J., Ohio Brass Co., Barberton, Ohio.
Titus, C. F. (Member) Florida Power and Light Company, Daytona Beach.
Turner, J. T., Jr., National Light & Power Co. Ltd., Moose Jaw, Saskatchewan, Canada.
Walker, R. J., General Motors Corporation, Detroit, Mich.
Wilson, S. E., National Geophysical Company, Jennings, La.

34 Domestic

Foreign

Das Gupta, S. C., c/o J. K. Oil Mills & Ice Factory Cawnpore, India
Larard, F. J. (Member), Malayan Electrical Service Kuala, Lumpur, Federated Malay States.
Mackie, E. G. (Member), Punjab Public Works Dept., Joginder Naggar, Mandi State, India.
Milward, F. A. (Member), 59 Windermere Park, Rangoon, Burma, India.
Mundkur, B. S., 27 Bungalow Tatapuram, Cochin State, India.
Reid, J. H. (Member), Trinidad Electric Company, Ltd., Port-of-Spain, Trinidad, B. W. I.
Schuepp, R. H. (Member), Volkart Brothers, Lahore, India.
Souza, B. R., Compania Cubana de Electricidad Havana, Cuba.
Thirivengadam, M., Guntur Power & Light Ltd., Tenali, Madras, S. India.

9 Foreign

Engineering Literature

New Books in the Societies Library

Among the new books received at the Engineering Societies Library, New York, recently are the following which have been selected because of their possible interest to the electrical engineer. Unless otherwise specified, books listed have been presented gratis by the publishers. The Institute assumes no responsibility for statements made in the following outlines, information for which is taken from the preface of the book in question.

ALIGNING PHILCO RECEIVERS. By J. F. Rider. New York, John F. Ryder, 1440 Broadway, 1937. 136 pages, diagrams, 8x5 inches, cloth, $1.00. A practical manual for the repairman, containing diagrams and tables for every model of Philco receiver from the earliest to those for 1937.

FUNDAMENTALS of ENGINEERING ELECTRONICS. By W. G. Dow. New York, John Wiley, 1937. 604 pages, illustrated, 10x6 inches, cloth, $5.00. Emphasizes chiefly internal operating principles. A large part of the text is devoted to a study of the use of various geometrics and materials in electronic devices, and a relatively small part to circuit studies. The point of view is that of the engineer.

SOUND. By A. T. Jones. New York, D. Van Nostrand Company, 1937. 450 pages, illustrated, 9x6 inches, cloth, $3.75. An elementary book on sound. Sound production and transmission, free and forced vibrations, musical scales and instruments, speech and hearing are considered with their modern technical applications.

Engineering Societies Library

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MAINTAINED as a public reference library of engineering and the allied sciences, this library is a co-operative activity of the national societies of civil, electrical, mechanical, and mining engineers.

Resources of the library are available also to those unable to visit it in person. Lists of references, copies or translation of articles, and similar assistance may be obtained upon written application, subject only to charges sufficient to cover the cost of the work required.

A collection of modern technical books is available to any member residing in North America at a rental rate of five cents per day per volume, plus transportation charges.

Many other services are obtainable and an inquiry to the director of the library will bring information concerning them.

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Calif., Univ. of, Berkeley, Calif	8	Abe Tilles
Carnegie Inst. of Tech., Pittsburgh, Pa	2	G. R. Patterson
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Colorado, Univ. of, Boulder, Colo	6	H. B. Palmer
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North Carolina	4	W. J. Seeley	Otto Meier, Jr	Duke University, Dept. of Elec. Engg., Durham, N. C.
Oklahoma City	7	E. H. Dinwiddie	J. L. Jones	1006 Telephone Bldg., 405 No. Broadway, Oklahoma City, Okla.
Philadelphia	2	J. B. Harris, Jr	H. S. Phelps	Philadelphia Elec. Co., 2301 Market St., Philadelphia, Pa.
Pittsburgh	2	Paul Frederick	W. A. Furst	Westinghouse Elec. & Mfg. Co., E. Pittsburgh, Pa.
Pittsfield	1	A. Boyajian	J. L. Cantwell	General Electric Co., Pittsfield, Mass.
Portland	9	C. C. Boozier	C. B. Carpenter	Pacific Tel. & Tel. Co., Portland, Ore.
Providence	1	E. Carlson, Jr	O. E. Sawyer	59 Lawrence St., Cranston, R. I.
Rochester	1	B. M. Werly	C. L. Smith	Rochester Gas & Elec. Co., 89 East Ave., Rochester, N. Y.
St. Louis	7	B. T. McCormick	L. L. Crump	James R. Kearney Corp., 4236 Clayton Ave., St. Louis, Mo.
San Antonio	7	M. H. Lovelady	Louis Thein	St. Mary's University, San Antonio, Texas
San Francisco	8	R. O. Brosemer	C. A. Andrews	Westinghouse E. & M. Co., 1 Montgomery St., San Francisco, Calif.
Saskatchewan	10	F. E. Estlin	James Smith	Canada Wire & Cable Co. Ltd., P. O. Box 426, Regina, Sask.
Schenectady	1	S. A. Holme	E. M. Hunter	General Electric Co., Schenectady, N. Y.
Seattle	9	C. H. Hoge	R. E. Lindblom	University of Washington, 203 Engg. Hall, Seattle, Wash.
Sharon	2	H. S. Smith	F. L. Snyder	Westinghouse E. & M. Co., Sharon, Pa.
Spokane	9	M. F. Hatch	Neal Eaden	East 303 Glass Ave., Spokane, Wash.
Springfield	1	F. R. Longley	R. E. Curtis	Western Massachusetts Companies, Springfield, Mass.
Syracuse	1	W. H. Lawrence	E. J. Gibbons	206 Elizabeth St., Syracuse, N. Y.
Toledo	2	W. M. Campbell	H. R. Daykin	3701 Larchmont Parkway, Toledo, Ohio
Toronto	10	R. E. Jones	O. W. Titus	Canada Wire & Cable Co. Ltd., Box 340, Toronto, Ont.
Urbana	5	C. A. Keener	J. T. Tykociner	University of Illinois, Elec. Engg. Dept., Urbana, Ill.
Utah	9	C. A. Wolfrom	A. C. Kelm	Utah Power & Light Co., Salt Lake City, Utah
Vancouver	10	C. Arnott	J. H. Steede	B. C. Electric Railway Co. Ltd., 425 Carrall St., Vancouver, B. C
Virginia	4	W. F. Nimmo	J. S. Miller	University of Virginia, University, Va.
Washington	2	J. F. Meyer	N. B. Ames	George Washington University, Washington, D. C.
Worcester	1	V. Siegfried	W. W. Locke	Worcester Polytechnic Institute, Elec. Engg. Dept., Worcester, Mass.
Total 62				

Industrial Notes

Pittsburgh Flood Prevention Project.—The first major flood prevention project to reach the construction stage in the Pittsburgh district will protect the Turtle Creek Valley in which lies the huge works of the Westinghouse Elec. & Mfg. Co. and other major industries. In the future when high water from the Monongahela river threatens the valley, two huge gates will be lowered, forming a barrier against rising waters and the normal flow of Turtle Creek will be continued by means of powerful pumps. The project announced by Westinghouse vice-president Ralph Kelly, will cost $500,000. Installation of the gates will be made just below the main Westinghouse works. One of the steel gates, 80 feet long by 30 feet high will form a dam in Turtle Creek; the other, 40 feet long and 20 feet high will close the street. When not in use the gates will be held approximately 20 feet above the street level to accommodate traffic. The three pumps, of the vertical type, were designed by the Westinghouse condenser engineering department at South Philadelphia, and will have a combined capacity of 5000 to 7500 cubic feet per second. Each will be 11 feet 11 inches in diameter and will be driven at 112 rpm by 5000 hp electric motors. Each of the motors will be 23 feet in diameter.

Progress in Building Construction.—With the July record, once again construction has established a new high level for the recovery. According to figures of F. W. Dodge Corp., the July total for all classes of construction amounted to $321,602,700 in the 37 eastern states, topping the previous high figure of the recovery reported in June of this year in the amount of $317,842,100 and representing an increase of 8 per cent over the total of $294,734,500 reported in July 1936. Of the July 1937 volume, $81,046,000 was for residential building, $138,063,500 for non-residential building, and $102,493,200 for civil engineering projects, i.e., public works and public utilities.

New Coil Winding Company.—Electrical Windings, Inc., 16 No. May St., Chicago, has been organized to specialize in the design and manufacture of all types of transformers, solenoids, chokes, magnets, etc. The new company is set up to design coils and mountings for any specific purpose, or to work from manufacturers' specifications. Orland Murphy, president, was formerly associated with Utah Radio Products and Standard Transformer Corp. and was recently chief engineer for Robertson-Davis.

Empire Sheet Appointment.—Joseph B. Montgomery, Jr. became associated with Empire Sheet & Tin Plate Company's organization August 1st and assumed charge of sales. At a meeting of the Board of Directors held on Monday, August 9, he was elected vice-president in charge of sales, according to an announcement made by J. M. Hill, president. Mr. Montgomery was previously employed in the sales organization of the Bethlehem Steel Corp., in the sheet and strip sales department.

American Rolling Mill Appointment.—Marvin Marsh, special Armco sales representative in the Kansas City territory, has been named manager of the company's newly created district office at 7100 Roberts St., Kansas City, Mo., according to W. W. Sebald, vice-president in charge of commercial activities of The American Rolling Mill, Middletown, O.

Absorbs E-M Capacitor Division.—The Cornell-Dubilier Electric Corp., South Plainfield, N. J., has taken over the condenser manufacturing equipment and sales staff of the Electric Machinery Mfg. Co., Co., Inc., Minneapolis. According to Octave Blake, president of Cornell-Dubilier, the additional equipment supplements the modern machinery already housed in the company's 11 buildings covering more than 33 acres.

Trade Literature

Transformers.—Bulletin 2259, 4 pp. Describes industrial substation service transformers. Allis-Chalmers Mfg. Co., Milwaukee, Wis.

Motors.—Bulletin GEA 1195B, 4 pp. Describes large synchronous motors for driving metal-rolling mills. General Electric Co., Schenectady, N. Y.

Switchgear.—Bulletin 1187, 8 pp. Describes indoor type cubicle switchgear, listing outstanding advantages, applications, and details of construction as well as complete dimensions. Allis-Chalmers Mfg. Co., Milwaukee, Wis.

Aircraft Wires and Cables.—Bulletin A-832, 28 pp. Describes mechanical control cords and aircraft strand, galvanized wire and fittings; also electrical cables for use in airplane construction. John A. Roebling's Sons Co., Trenton, N. J.

Switchboards.—Bulletin GEA 2617, 8 pp. Describes safety enclosed, air circuit breaker switchboards, with trip-free air circuit breakers rated at 250 volts d-c, 600 volts a-c; up to 10,000 amperes, manually or electrically operated. General Electric Co., Schenectady, N. Y.

Lift Magnet Controller.—Bulletin, 4 pp. Describes a new automatic-discharge type controller enabling high speed lifting magnet operation, as well as quick release of the load. The Electric Controller & Mfg. Co., Cleveland, O.

Silent Gears.—Bulletin, 8 pp. Describes Celeron (laminated plastic) silent gears. Recommended practises for machinery, gear tooth data and tables on horsepower rating per inch of face are included. Continental-Diamond Fibre Co., Newark, Del.

Arc Welder.—Bulletin GEA 2447C, 10 pp. Describes new Mutator arc welder (electronic rectifier) specially designed to provide direct current at the low values required for welding of light-gauge metal. General Electric Co., Schenectady, N. Y.

Booster Regulators.—Catalog, Sec. 47–230, 8 pp. Describes application, operation, construction and ratings of the four step feeder-voltage booster-regulator, designed for use on rural lines or distribution circuits where the load changes over a wide range. Westinghouse Elec. & Mfg. Co., E. Pittsburgh, Pa.

Cable Testing Equipment.—Catalog E-54 (3), 8 pp. Describes apparatus for power factor measurements by the phase defect compensation method for high voltage production tests of power cables. Leeds & Northrup Co., 4962 Stenton Ave., Philadelphia, Pa.

Porcelain in Wiring.—Bulletin, "The Fact Manual of Standard Electrical Porcelain," second edition, rearranged and revised to illustrate the 1937 National Electric Code Revisions. Includes wall chart depicting proper use of porcelain knobs, tubes and cleats, as well as the newly developed all porcelain outlet, switch and receptacle boxes. Standard Electrical Porcelain Manufacturers, 201 No. Wells St., Chicago, Ill.

Diesel Generating Sets.—Bulletin 3600-A2, 16 pp. Describes the construction and application of model 36-A Diesel generating sets in industrial plants, either as independent units carrying the entire load or for parallel or auxiliary operation in conjunction with other electrical service. Alternating-current sets are offered in ratings from 5.3 to 100 kva and direct current sets from 5 to 80 kw. Fairbanks, Morse & Co., 900 So. Wabash Ave., Chicago, Ill.

Condensers.—Catalog, 32 pp. Lists a large and diversified line of condensers, together with essential resistors, in a manner which permits of finding any required condenser in minimum time. Condensers are grouped first under their general type classification, and then under working voltage, and finally by capacity. Eight pages of exact duplicate replacement condensers are included, covering practically all standard set requirements. Aerovox Corp., 70 Washington St., Brooklyn, N. Y.

Distribution System.—Catalog, 40 pp. Describes Universal type "Trol-E-Duct," a mobile electrical distribution system designed for general lighting and light duty power applications. This innovation consists essentially of section lengths of formed steel duct enclosing copper bus bars partially wrapped in insulating material. Movable current-carrying trolleys and stationary twistout plugs to which lights, tools and appliances can be wired are inserted in a narrow continuous slot extending along one side of the duct. The new system supplements Industrial "Trol-E-Duct," for heavy duty power applications. Bull Dog Electric Products Co., 7610 Jos. Campau Ave., Detroit, Mich.